Dynamic Investment, Capital Structure, and Debt Overhang*

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We develop a dynamic contingent-claim framework to model S. Myers's idea that a firm is a collection of growth options and assets in place. The firm's composition between assets in place and growth options evolves endogenously with its investment opportunity set and its financing of growth options, as well as its dynamic leverage and default decisions. The firm trades off tax benefits with the potential financial distress and endogenous debt-overhang costs over its life cycle. Unlike the standard capital structure models of Leland, our model shows that financing and anticipated endogenous default decisions have significant implications of firms' growth-option exercising decisions and leverage policies. The firm's ability to use risky debt to borrow against its assets in place and growth options substantially influences its investment strategies and its value. Quantitatively, we find that the firm consistently chooses conservative leverage in line with empirical evidence in order to mitigate the debt-overhang effect on the exercising decisions for future growth options. Finally, we find that debt seniority and debt priority structures have both conceptually important and quantitatively significant implications on growth-option exercising and leverage decisions as different debt structures have very different debt-overhang implications. (*JEL* E2, G1, G3)

Models of truly intertemporal investments with irreversibility and models of dynamic financing with endogenous defaults have proceeded relatively

independent of each other. The literature on intertemporal investments with multiple rounds of investments often ignores the financing flexibility possessed by the firms. On the contrary, models of dynamic financing have tended to ignore the investment opportunity set to a point whereby proceeds from each round of new financing are paid out to equityholders. Even though considerable insights have emerged from each strand of literature, much remains to be done in integrating investment theory with dynamic financing. Our paper builds on the insights of the real-options and contingent-claims/credit-risk literature with the objective of showing the important link between the optimal exercise of growth options and corporate leverage in a parsimonious and tractable way. By using our dynamic model of investment and financing, we show that a rational firm significantly lowers its leverage anticipating its future growth-option exercising decisions. Our numerical exercise generates empirically observed low leverage (e.g., around 1/3) once we incorporate multiple rounds of growth options, indicating the important interaction of growth-option exercising, leverage, and valuation.

Integration of multiple rounds of investments with multiple rounds of financing presents many modeling challenges: first, the firm must solve endogenously the upper threshold of its value when it must undertake new investments. In making this decision the firm must take into account the level of debt it must use to optimally trade off the expected tax benefits with the possibility of premature termination of the firm when default occurs, taking into account all future investment and financing possibilities. The optimal default-decision constitutes a lower threshold level of the value of the firm, which must also be decided endogenously. This paper provides an analytically tractable framework to examine dynamic endogenous corporate investment, financing, and default decisions.[1] We provide a tractable model of real options in which the firm makes these endogenous lower (default) and upper (investment) decisions over time, while choosing its optimal debt level along the way. In so doing, we provide a methodological framework for assessing how the life cycle of the firm may influence the manner in which it makes inter-temporal investment, and financing decisions. Broadly speaking, we use the term financing to encompass both the level of debt and the optimal default decisions.

Several new insights emerge from our analysis. In thinking about of inter-temporal investments and financing, we start with the intuitive premise that the firm starts its life as a collection of growth options, much as in Myers (1977). For simplicity, we assume that the collection of growth

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[1] See Stein (2003) for a survey on corporate investment, agency conflicts, and information. See Caballero (1999) for a survey on aggregate dynamic investment. See Harris and Raviv (1991) for a survey on theories of capital structure.

options possessed by the firm is known and does not change over time. Then, as the firm moves through time, it optimally decides when to exercise each growth option and how to finance each growth option, keeping in mind that several additional growth options may be available to the firm in the future. When the firm has exercised all its growth options, it is left only with assets in place. However, it starts its life with no assets in place. At all other times, it has some future growth options and some assets in place. The composition of growth options and assets is endogenously determined in a dynamic optimizing framework. Thus, our model captures the life cycle of the firm in a natural way.

There is an important economic distinction between assets in place and growth option in terms of what fraction of each is available to residual claimants upon default. It is reasonable to argue that assets in place are "hard assets," with values that are verifiable and hence may provide greater liquidation value upon default compared with growth options, which may have embedded human capital and hence may possess a different (and possibly much lower) liquidation value along the lines of Hart and Moore (1994). We explicitly incorporate the potential differences of recovery values for growth options and assets in place. The modeling of this difference is a new contribution to the real-options literature, as well, because it requires the values of foregone growth options upon default (which are non-linear functions of primitive states) to bear on optimal exercise boundaries. Indeed, we provide explicit quantitative and qualitative characterization of the effect of embedded human capital in future growth options on optimal investment thresholds, default thresholds, and the level of debt used by the firm at each stage of its life cycle. We believe that this is a unique contribution of our study.

Our research provides a natural bridge between structural credit risk/capital structure models, and the dynamic irreversible investment theory.[2] We find that even for firms with only one growth option, integrating investment and financing decisions generates new insights, not captured by either the standard real-options models (e.g., McDonald and Siegel 1986), or credit risk/capital structure models (e.g., Leland 1994). For example, Leland (1994) shows that the default threshold decreases in volatility for the standard (put) option argument in a contingent-claim framework based on the standard trade-off theory of Modigliani and

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[2] McDonald and Siegel (1985, 1986) and Brennan and Schwartz (1985) are important contributions to modern real-options approach to investment under uncertainty. Dixit and Pindyck (1994) is a standard reference on real-options approach toward investment. Abel and Eberly (1994) provide a unified framework integrating the neoclassical adjustment cost literature with the literature on irreversible investment. Grenadier (1996) studies strategic interactions among agents in real-options settings. Grenadier and Wang (2005) analyze the effect of informational frictions and agency on investment timing decisions. Grenadier and Wang (2007) study the effect of time-inconsistent preferences on real option exercising decisions. In our earlier draft, Sundaresan and Wang (2006), we provide additional detailed analysis on the impact of alternative debt structures on corporate investment and leverage policies.

Miller (1963). However, the default threshold in our model may either decrease or increase in volatility. The intuition is as follows: (*i*) a higher volatility raises the investment threshold in our model for the standard (call option) value of waiting argument; (*ii*) a higher investment threshold naturally leads to a greater amount of debt issuance. That is, the firm issues more debt (but at a later time), when volatility is higher. Larger debt issuance raises the default threshold, *ceteris paribus*. As a result, unlike Leland (1994), we have two opposing effects of volatility on the default threshold owing to endogenous investment in our model.

In developing our analysis, we have made the analytically convenient assumption that the firm uses its financing flexibility only at times when it makes its optimal investments. At a first glance, the reader may think that this is a strong assumption. Nevertheless, it turns out that this assumption proves to be innocuous for the following reasons: First, when growth options are economically meaningful, investments occur over time at frequent (but stochastic) intervals. Hence the real cost of the assumption is rather slight. In addition, it is well known (Strebulaev 2007) that even the introduction of low-costs financing leads to the result that firms will choose to adjust their capital structure at periodic intervals rather than continuously. Dudley (2012) shows that when there are fixed costs of adjustment, it is optimal for firms to synchronize capital structure adjustment with the financing of large investment projects. In our model, the primary reason for financing is investment, and investments require a lump-sum cost. Hence, it is natural to model financing adjustments when investments occur. Moreover, the key focus of our paper is on the effect of financing on growth-option exercising decisions.

In addition, our study provides several additional insights on the valuation of equity and credit spreads at different stages in the life cycle of the firm. We have a natural benchmark to assess of our results: after all the growth options are optimally exercised, the firm is left with only assets in place. At this final stage, our results are exactly the same as either Leland (1994) (when no dynamic financing adjustments are allowed) or Goldstein, Ju, and Leland (2001) (when dynamic financing adjustments are allowed). At all previous stages, the firm has a mixture of assets in place and growth options, and they influence both equity valuation and credit spreads. The key insight is that the incremental financial flexibility at times other than actual investments is less important when there are growth options.

Related literature.   Recently, there is a growing body of literature that extends Leland (1994) to allow for strategic debt service,[3] and dynamic

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[3] Anderson and Sundaresan (1996) use a binomial model to study the effect of strategic debt service on bond valuation. See Mella-Barra and Perraudin (1997), Fan and Sundaresan (2000), and Lambrecht

capital structure decisions. Fischer, Heinkel, and Zechner (1989); Goldstein, Ju, and Leland (2001); and Strebulaev (2007) formulate dynamic leverage decisions with exogenously specified investment policies.[4] Leary and Roberts (2005) empirically find that firms rebalance their capital structure infrequently in the presence of adjustment costs. Following Leland (1994), most contingent-claims models of credit risk/ capital structure assume that the firm's cash flows are exogenously given and focus on the firm's financing and default decisions.[5] Unlike these work, our model endogenizes growth-option exercising decisions and induces dynamic leverage decisions via motives of financing investment.[6] Titman and Tsyplakov (2007) also build a model that allows for dynamic adjustment of both investment and capital structure. Their model is based on continuous investment decisions, whereas our model focuses on the irreversibility of growth-option exercising.[7] We solve the model in closed form (up to coupled nonlinear equations), whereas Titman and Tsyplakov (2007) have three state variables and numerically solve the decision rules. Ju and Ou-Yang (2006) show that the firm's incentive to increase firm risk ex post is mitigated if the firm wants to issue debt periodically. In the interest of parsimony, we abstract from stochastic interest rates.[8] Guo, Miao, and Morellec (2005) develop a model of irreversible investment with regime shifts. Hackbarth, Miao, and Morellec (2006) study the effects of macro conditions on credit risk and firms' financing policies. Tserlukevich (2008) studies the effect of real options on financing behavior.

1. Model

We first set up a dynamic formulation where the firm is a collection of growth options and assets in place. Assume that the firm behaves in the

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(2001) for continuous-time contingent-claim treatment. Sundaresan and Wang (2007) study the interactions between investment and financing decisions when equityholders may ex post strategically force concessions from debtholders.

[4] Early important contributions towards building dynamic capital structure models include Kane, Marcus, and McDonald (1984, 1985).

[5] Leland (1998) extends Leland (1994) by incorporating risk management with capital structure, and also allows the firm to engage in asset substitution by selecting volatility of the project.

[6] Our model ties the investment and financing adjustments to occur at the same time. This assumption is made for analytical convenience. We leave extensions to allow for separate adjustments of investment and financing for future research.

[7] Brennan and Schwartz (1984) is an early, important contribution, which allows for the interaction between investment and financing.

[8] See Kim, Ramaswamy, and Sundaresan (1993); Longstaff and Schwartz (1995); and Collin-Dufresne and Goldstein (2001) for extensions of Merton (1974), to allow for a stochastic interest rate and other features.

interests of existing equityholders at each point in time.[9] At time zero, the firm starts with no assets in place, and knows that it has N growth options. These growth options can only be exercised sequentially. One way to view these growth options is as the discretized decisions for capital-accumulation decisions.[10]

The firm observes the demand shock Y for its product, where Y is given by the following geometric Brownian motion (GBM) process:

$$dY(t) = \mu Y(t)dt + \sigma Y(t)dW(t), \tag{1}$$

and W is a standard Brownian motion.[11] Equivalently, we may also interpret Y as the (stochastic) price process for the firm's output.[12] The risk-free interest rate r is constant. For convergence, assume that the (risk-neutral) expected growth rate μ is lower than the interest rate, in that $r > \mu$. Assume no production cost after the asset is in place.[13] When the firm exercises its n-th growth option, it creates the n-th asset in place, which generates profit at the rate of $m_n Y$, where $m_n > 0$ is a constant. We may interpret m_n as the production capacity, or equivalently the constant rate of output produced by the firm's n-th asset in place. Let the firm's total profit rate from its first n assets in place be $M_n Y$, where $M_n = \sum_{j=1}^{n} m_j$.

Let T_n^i denote the endogenously chosen time at which the firm exercises its n-th growth option, where $1 \leq n \leq N$. Let I_n denote the fixed cost of exercising its n-th growth option. These exercising costs I_n are constant and known at time 0. At each endogenously chosen (stochastic) investment time T_n^i, the firm issues a mixture of debt and equity to finance the exercising cost I_n. As in standard trade-off models of capital structure, debt has a tax advantage. The firm faces a constant tax rate $\tau > 0$ on its income after servicing interest payments on debt. To balance the tax benefits, debt induces deadweight losses when the firm does poorly. The firm dynamically trades off the benefits and costs of issuing debt. For analytical convenience, assume that debt is perpetual and is issued at

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[9] We ignore the conflicts of interests between managers and investors and leave them for future research.

[10] One could certainly visualize growth options arriving with some intensity at random times in the future. In such an economy, the optimal investment decisions would reflect the arrival intensity in addition to the factors that we consider in our formulation. Extension of random arrivals of growth options is clearly an interesting topic for future research.

[11] Let W be a standard Brownian motion in $\mathbb{R}$ on a probability space $(\Omega, F, \mathbb{Q})$ and fix the standard filtration $\{F_t : t \geq 0\}$ of W. Since all securities are traded here, we work directly under the risk-neutral probability measure $\mathbb{Q}$. Under the infinite horizon, additional technical conditions such as uniform integrability are assumed here. See Duffie (2001).

[12] In our model as in many other investment and capital structure models, the process Y captures both demand and productivity shocks.

[13] Our model ignores operating leverage. We may extend our model to allow for operating leverage by specifying the firm's profit from its n-th asset in place as $m_n Y - w_n$, where w_n is the operating cost for the n-th asset in place.

par. The assumption of perpetual debt simplifies the analysis substantially and has been widely adopted in the literature.[14] Note that we have assumed that the firm can only issue debt at investment times $\{T_n^i : 1 \leq n \leq N\}$. At a first glance, this may appear to be a strong assumption. In fact, our assumption is actually rather mild. Strebulaev (2007) has shown that in the presence of frictions firms adjust their capital structure only infrequently. Therefore, in a dynamic economy that we model, the leverage of the firm is likely to differ from the "optimum" leverage predicted by models that permit costless adjustment of leverage. Given this finding it is more natural to recapitalize when optimal investment decisions are warranted. In addition, models that permit re-leveraging in good times, implicitly or explicitly use the debt proceeds to pay dividends, which is at odds with the basic provision that senior claims (such as debt) may not be issued to finance junior claims (such as equity).

Let C_n and F_n denote the coupon rate and the par value of the perpetual debt issued to finance the exercising of the n-th growth option at T_n^i. Let T_n^d denote the endogenously chosen stochastic default time after the firm exercises n growth options, but before exercising the $(n+1)$-th growth option, where $1 \leq n \leq N$. When exercising the new growth option at T_{n+1}^i, the firm calls back its outstanding debt with par F_n and coupon C_n, and issues new debt with par F_{n+1} and coupon C_{n+1}. That is, at each point in time, there is only one class of debt outstanding.[15]

Figure 1 describes the decision-making process of the firm over its life cycle. The firm has $(N+1)$ stages. In stage 0, the firm has no assets in place. We assume that the initial value of the demand shock is sufficiently low such that the firm always starts with waiting, the economically most interesting case. If the demand shock $\{Y(t) : t \geq 0\}$ rises sufficiently high (i.e., greater or equal to an endogenous threshold Y_1^i to be determined in Section 2) at the stochastic (endogenous) time T_1^i, the firm exercises its first growth option by paying a one-time fixed cost I_1 at time T_1^i as in McDonald and Siegel (1986). Note that since $Y(0)$ is sufficiently low, we have $T_1^i > 0$. Notation-wise, we use $Y_1^i = Y(T_1^i)$. To finance the first growth-option exercising cost I_1, the firm issues a mixture of equity and perpetual debt. This completes the description of the firm's decision in its initial stage (stage 0). Next turn to stage 1.

After the first asset is in place, the firm collects profit flow $m_1 Y$ until it decides to either default on its debt or exercise its (second) growth option. If the firm defaults before exercising the second growth option ($T_1^d < T_2^i$),

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[14] We may extend the model by allowing for a finite average maturity for debt as in Leland (1994b) at the cost of additional modeling complexity.

[15] See Section 5 and also Hackbarth and Mauer (2012) for analysis where more than one class of debt are outstanding. The design of priority structure of debt and its implications for real-options exercise is a topic worthy of further research.
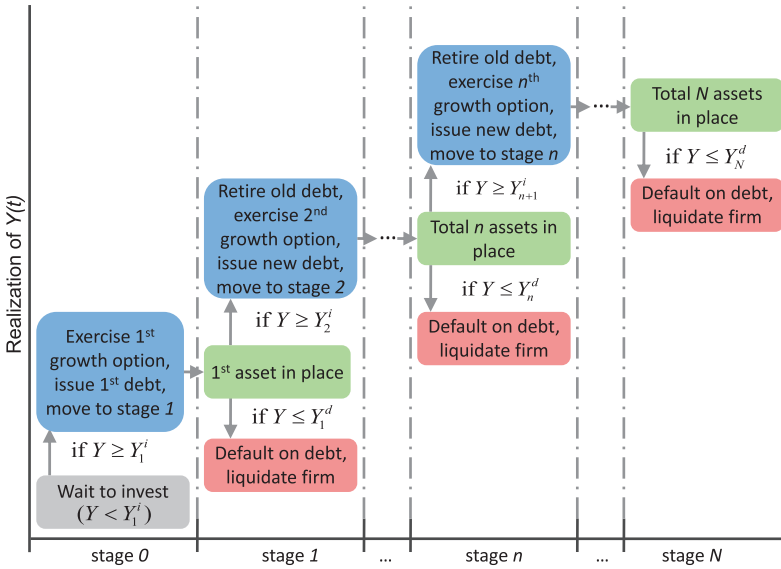
Figure 1
The firm's decision-making process over its life cycle.
The firm starts with N sequentially ordered growth options. We divide its decision making over its life cycle into $(N+1)$ stages. In stage 0, the firm exercises its first growth option when the stochastic process Y given in Equation (1) rises sufficiently high (i.e., $Y \geq Y_1^i = Y(T_1^i)$). The firm waits otherwise. When exercising, the firm issues a mixture of equity and the first perpetual debt with coupon C_1 to finance the exercising cost I_1. This completes the description of the firm's stage 0 decision. Now move to stage 1. Provided that $Y_1^d < Y < Y_2^i$, the firm generates cash flow $m_1 Y$ from its operation. If its cash flow drops sufficiently low, (i.e., $Y \leq Y_1^d$), the firm defaults. If the cash flow rises sufficiently high (i.e. $Y \geq Y_2^i$), the firm exercises its second growth option, and issues a mixture of equity and the second perpetual debt with coupon C_2 to finance the exercising cost I_2. After the second growth option is exercised, the firm generates stochastic cash flow $(m_1 + m_2)Y$, provided that $Y \geq Y_2^d$. This process continues. If the firm reaches the final stage N, the firm has total N assets in place and collects total cash flow $M_N Y$, where $M_N = \sum_{n=1}^{N} m_n$. The decision variables include N investment thresholds Y_n^i, N default thresholds Y_n^d, and N coupon policies C_n, where $n = 1, 2, \ldots, N$. Notation-wise, we define the n-th stage as Y, such that $Y_n^d \leq Y < Y_{n+1}^i$ where $Y_{n+1}^i = Y(T_{n+1}^i)$ and $Y_n^d = Y(T_n^d)$.

it ceases to exist. All proceeds from liquidation go to creditors. However, liquidation is inefficient because it induces value losses from both the existing assets in place and foregone growth options. We will specify the liquidation payoff in the next paragraph when discussing the firm's general stage n problem. Intuitively, if the demand shock Y is sufficiently high, then it is optimal for the firm to exercise its second growth option. By incurring the fixed investment cost I_2, the firm exercises its second growth option at endogenously chosen time T_2^i. At the second investment time T_2^i, the firm calls back its outstanding debt with par F_1, and issues a mixture of equity and the new perpetual debt with par F_2 to finance the second growth option exercising cost I_2. This concludes the firm's decision in stage 1.

It is straightforward to describe the firm's stage-*n* decision problem. After immediately exercising the *n*-th growth option, the firm operates its existing *n* assets in place until the demand shock Y either rises sufficiently high, which triggers the firm to call back debt with par F_n, issue a mixture of new perpetual debt and equity to finance I_{n+1} to exercise the $(n+1)$-th growth option, or the demand shock Y drops sufficiently low, which leads the firm to default on its outstanding debt with par F_n.

Let $A_n(Y)$ denote the after-tax present value of all *n* existing assets in place (under all equity financing), in that

$$A_n(Y) = \left(\frac{1-\tau}{r-\mu}\right) M_n Y, \qquad 1 \le n \le N, \tag{2}$$

where M_n is the production capacity for all existing *n* assets in place and is given by

$$M_n = \sum_{j=1}^{n} m_j, \qquad 1 \le n \le N. \tag{3}$$

When equityholders default on debt, the firm is liquidated. Let $L_n(Y)$ denote the proceeds from liquidation in stage *n*. Liquidation proceeds $L_n(Y)$ has two components: one from the existing assets in place, and the other from foregone growth options. Following Leland (1994), we assume that the firm uncovers $(1 - \gamma_A)$ fraction of the present value $A_n(Y)$ from existing *n* assets in place. Unlike Leland (1994), our model has growth options. Even though growth options may be less tangible, they still have scrap value upon liquidation. We calculate the liquidation value for unexercised growth options in an analogous way as we do for the liquidation value from existing assets in place. That is, we assume that the firm collects $(1 - \gamma_G)$ fraction of the present value of unexercised foregone growth options. We use the workhorse real-option model to assess values for unexercised growth options, as if these options were stand-alone and financed solely by equity. Let $G_k(Y)$ denote the present value of a stand alone growth option (with exercise cost I_k and cash-flow multiplier $m_k > 0$) under all-equity financing. The following lemma summarizes the main results (McDonald and Siegel 1986).

Lemma 1

Consider an all-equity financed firm with a single growth option. The firm may exercise its stand-alone growth option by paying a one-time fixed cost I_k, and then generate a perpetual stream of after-tax stochastic cash flow $(1 - \tau)m_k Y$, where $m_k > 0$ is a constant and the stochastic

process Y is given by Equation (1). The firm (option) value is given by

$$G_k(Y) = \left(\frac{Y}{Y_k^{ae}}\right)^{\beta_1} \left[\left(\frac{1-\tau}{r-\mu}\right) m_k Y_k^{ae} - I_k\right], \qquad Y < Y_k^{ae}, \tag{4}$$

where Y_k^{ae} is the optimal growth-option exercising threshold and is given by

$$Y_k^{ae} = \frac{r-\mu}{1-\tau} \frac{\beta_1}{\beta_1 - 1} \frac{I_k}{m_k}, \tag{5}$$

and β_1 is the (positive) option parameter and is given by

$$\beta_1 = \frac{1}{\sigma^2}\left[-\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right] > 1. \tag{6}$$

The firm's liquidation value in stage n, $L_n(Y)$, is then given by

$$L_n(Y) = (1 - \gamma_A)A_n(Y) + (1 - \gamma_G)\sum_{k=n+1}^{N} G_k(Y), \tag{7}$$

where $A_n(Y)$ given in Equation (2) is the after-tax present value of the existing n assets in place, and $G_k(Y)$ given in Equation (4) is the after-tax present value of k-th unexercised growth options. The specification of our liquidation payoff is reasonably general and also intuitive. We allow for different loss rates γ_A and γ_G for assets in place and growth options, respectively. For example, the growth options may reflect the embedded human capital of current owners, which the new owners may not be able to replicate after liquidation. This may suggest that γ_G is greater than γ_A, although our model specification does not require this condition. In addition to being realistic, the specification for liquidation payoffs is also quite tractable, and we have closed-form solutions for both liquidation values of assets in place and of growth options, as shown above. Finally, we tie liquidation values for assets in place and growth options to their respective stand-alone values under all-equity financing.

Having detailed the firm's decision making in stage n, we introduce a few value functions, and leave the formal mathematical definition of these value functions to the Appendix. Let $E_n(Y)$ denote equity value in stage n, (i.e., the present discounted value of all future cash flows accruing to the existing equityholders after servicing debt and paying taxes). Even though equity value $E_n(Y)$ does not internalize the benefits and costs of debt in stage n, it does include the tax benefits and distress costs of debt in future stages. Let $D_n(Y)$ denote debt value in stage n. Recall that debt coupon C_n is serviced until debt is either called back at par F_n at

investment time T_{n+1}^i, or is defaulted at T_n^d. At default, creditors collect $L_n(Y(T_n^d))$ given in Equation (7), which is less than C_n/r in equilibrium.[16] Let $V_n(Y)$ denote stage-n firm value, which is the sum of equity and debt values, in that $V_n(Y) = E_n(Y) + D_n(Y)$.

The firm follows the decision-making process sketched earlier during each stage of its life cycle until it defaults. If the firm survives to exercise its last growth option (i.e., $t \geq T_N^i$), then the firm has exercised all of its growth options. The firm then collects $M_N Y$ in profit flow from its N assets in place, servicing debt payment C_N and paying taxes, until profit drops sufficiently low, which triggers the firm to default on its outstanding debt with par F_N at time T_N^d. The last-stage default-decision problem for the "mature" firm is the one analyzed in Leland (1994).

Having described the decision-making process over the life cycle of the firm, we next solve the model using backward induction.

2 Solution

We solve our model in four steps. The first three steps take debt coupon levels $\{C_n : 1 \leq n \leq N\}$ and liquidation payoff $\{L_n(Y) : 1 \leq n \leq N\}$ in each stage n as given, and analyze the firm's growth option and default-option-exercising decisions. To be specific, first, we study the default decision in stage N (no investment decision in the last stage). This is effectively the classic capital structure/default problem treated in Leland (1994). Second, we characterize the firm's optimal growth-option and default-option exercising decisions when the firm is in the intermediate stages of its life cycle (stage 1 to stage $N-1$). Third, we analyze the firm's initial growth-option exercising decision (no default decision in stage 0). After solving the investment and default decisions, we provide optimality conditions for the firm's financing policies $\{C_n : 1 \leq n \leq N\}$ over its life cycle (stage 0 to stage $N-1$).

2.1 The final stage (stage N) in the firm's life cycle
In the final stage, the firm has exercised all its growth options and hence operates N assets in place, which generate the profit flow at the rate of $M_N Y$ with $M_N = \sum_{k=1}^{N} m_k$ being the total production capacity. Conditioning on the optimal choice of the investment threshold Y_N^i, we have the classic Leland (1994) formulation. In addition, the corresponding details of proof are provided in the Appendix.

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[16] Because default is endogenous, equityholders will never default when liquidation value exceeds the risk-free value of debt.

Leland (1994) and Goldstein, Ju, and Leland (2001) show that equity value $E_N(Y)$ is given by

$$E_N(Y) = \left(A_N(Y) - \frac{(1-\tau)C_N}{r} \right) + \left[\frac{(1-\tau)C_N}{r} - A_N(Y_N^d) \right] \left(\frac{Y}{Y_N^d} \right)^{\beta_2}, \quad Y \geq Y_N^d,$$

(8)

where $A_N(Y)$ given in Equation (2) is the after-tax present value of N existing assets in place, and the optimal default threshold Y_N^d for a given coupon C_N is given by

$$Y_N^d = \frac{r-\mu}{M_N} \frac{\beta_2}{\beta_2 - 1} \frac{C_N}{r},$$

(9)

and β_2 is given by

$$\beta_2 = -\frac{1}{\sigma^2} \left[\left(\mu - \frac{\sigma^2}{2} \right) + \sqrt{\left(\mu - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} \right] < 0.$$

(10)

Equity value $E_N(Y)$ is given by the after-tax present value of all N assets in place, $A_N(Y)$, minus the after-tax present value of the risk-free perpetual debt $(1-\tau)C_N/r$, plus the option value of default, the last term in Equation (8). The standard option-value argument implies that the default threshold Y_N^d decreases with volatility σ, and equity value $E_N(Y)$ is convex in Y. Naturally, when $Y \leq Y_N^d$, equity is worthless, i.e., $E_N(Y) = 0$.

Similarly, the market value of debt $D_N(Y)$ is given by

$$D_N(Y) = \frac{C_N}{r} - \left[\frac{C_N}{r} - L_N(Y_N^d) \right] \left(\frac{Y}{Y_N^d} \right)^{\beta_2}, \quad Y \geq Y_N^d,$$

(11)

where the second term captures the default risk. In stage N, the firm only has assets in place, and therefore, $L_N(Y_N^d) = (1 - \gamma_A)A_N(Y_N^d)$. Note that $D_N(Y)$ is concave in Y because the creditor is short a default option. The second term in Equation (11) measures the discount on debt owing to the risk of default, which has two components: the loss given default $(C_N/r - L_N(Y_N^d))$ for the creditor, and $(Y/Y_N^d)^{\beta_2}$, the present discounted value for a unit payoff when the firm hits the default boundary Y_N^d.

Intuitively, firm value is $V_N(Y) = E_N(Y) + D_N(Y)$ which is given by

$$V_N(Y) = A_N(Y) + \frac{\tau C_N}{r} - \left[\gamma_A A_N(Y_N^d) + \frac{\tau C_N}{r} \right] \left(\frac{Y}{Y_N^d} \right)^{\beta_2}, \quad Y \geq Y_N^d. \quad (12)$$

Firm value $V_N(Y)$ is given by the after-tax value of the N assets in place $A_N(Y)$, plus the perpetuity value of the risk-free tax shield $\tau C_N/r$, minus the cost of liquidation. Importantly, firm value $V_N(Y)$ is concave in Y, because the firm as a whole is short in a liquidation option.

Intuitively, after T_N^i, the firm is long in the N assets in place and the risk-free tax shield perpetuity $\tau C_N/r$, and short in the liquidation option. Upon liquidation, the firm as a whole loses γ_A fraction of assets in place value $A_N(Y_N^d)$ and also the perpetual value of tax shields, the sum of the two terms in the square bracket in Equation (12).

2.2 Intermediate stages (stage $(N-1)$ to stage 1)

Having analyzed the firm's optimization problem in stage N, we now use backward induction to analyze the firm's decision problem in stage $(N-1)$. As Figure 1 indicates, the key decisions are (*i*) the N-th growth-option exercising and (*ii*) the default decision on the existing debt with par F_{N-1}. For generality, we solve the firm's decision problem for its intermediate stage n, including stage $(N-1)$ as a special case.

2.2.1 Equityholders' decisions and equity pricing.
For given investment threshold Y_{n+1}^i and default threshold Y_n^d in stage n, equity value $E_n(Y)$ solves the following ODE:

$$rE_n(Y) = (1-\tau)(M_n Y - C_n) + \mu Y E_n'(Y) + \frac{\sigma^2}{2} Y^2 E_n''(Y), \quad Y_n^d \le Y \le Y_{n+1}^i. \tag{13}$$

Now consider boundary conditions for investment. When exercising the $(n+1)$-th growth option, equityholders are required to call back the old debt at the par value F_n. Importantly, we will determine the value of F_n as part of the model solution that depends on the firm's endogenous investment, default, and coupon decisions.

Note that since the firm has to call back the debt at its par, the total cost of exercising the $(n+1)$-th growth option is given by $(I_{n+1} + F_n)$, the sum of investment cost I_{n+1} and the face value of the debt F_n. And part of this exercising cost is financed by new debt, which has market value $D_{n+1}(Y_{n+1}^i)$ at issuance time T_{n+1}^i. The remaining part $(I_{n+1} + F_n - D_{n+1}(Y_{n+1}^i))$ is financed by equity. Therefore, the net payoff to equitysholders right after exercising is $E_{n+1}(Y_{n+1}^i) - (I_{n+1} + F_n - D_{n+1}(Y_{n+1}^i))$. The value-matching condition for the threshold Y_{n+1}^i is then given by

$$E_n(Y_{n+1}^i) = V_{n+1}(Y_{n+1}^i) - (I_{n+1} + F_n), \tag{14}$$

where $V_{n+1}(Y) = E_{n+1}(Y) + D_{n+1}(Y)$ is firm value in stage $(n+1)$. Because equityholders optimally choose the threshold Y_{n+1}^i, the following smooth-pasting condition holds:

$$E_n'(Y_{n+1}^i) = V_{n+1}'(Y_{n+1}^i). \tag{15}$$

Now turn to the default boundary conditions. Using the same arguments as those for equity value $E_N(Y)$ in the last stage, equityholders choose the

default threshold Y_n^d to satisfy the value-matching condition $E_n(Y_n^d) = 0$ and the smooth-pasting condition $E_n'(Y_n^d) = 0$.

Unlike the decision problem in the last stage, we now have a double (endogenous) barrier option exercising problem, where the upper boundary is primarily about the real-option exercising decision as in McDonald and Siegel (1986), and the lower boundary is effectively the financial default-option-decision as in Leland (1994). Of course, the upper (investment) and the lower (default) boundaries are interconnected. This is precisely how the investment and default decisions affect each other. Next, we formally characterize this interaction between investment and default decisions.

Let $\Phi_n^i(Y)$ denote the present discounted value of receiving a unit payoff at T_{n+1}^i if the firm invests at T_{n+1}^i, namely, $T_{n+1}^i < T_n^d$. Similarly, let $\Phi_n^d(Y)$ denote the present discounted value of receiving a unit payoff at T_n^d if the firm defaults at T_n^d, namely $T_n^d < T_{n+1}^i$. The closed-form expressions for $\Phi_n^i(Y)$ and $\Phi_n^d(Y)$ are given by

$$\Phi_n^i(Y) = \mathbb{E}_t\left[e^{-r(T_{n+1}^i - t)}\mathbf{1}_{T_n^d > T_{n+1}^i}\right] = \frac{1}{\Delta_n}[(Y_n^d)^{\beta_2} Y^{\beta_1} - (Y_n^d)^{\beta_1} Y^{\beta_2}], \qquad (16)$$

$$\Phi_n^d(Y) = \mathbb{E}_t\left[e^{-r(T_n^d - t)}\mathbf{1}_{T_n^d < T_{n+1}^i}\right] = \frac{1}{\Delta_n}[(Y_{n+1}^i)^{\beta_1} Y^{\beta_2} - (Y_{n+1}^i)^{\beta_2} Y^{\beta_1}], \quad (17)$$

and

$$\Delta_n = (Y_n^d)^{\beta_2}(Y_{n+1}^i)^{\beta_1} - (Y_n^d)^{\beta_1}(Y_{n+1}^i)^{\beta_2} > 0. \qquad (18)$$

Using these formulas, we may write equity value $E_n(Y)$ as follows:

$$E_n(Y) = A_n(Y) - \frac{(1-\tau)C_n}{r} + e_n^i\Phi_n^i(Y) + e_n^d\Phi_n^d(Y), \qquad Y_n^d \le Y \le Y_{n+1}^i, \tag{19}$$

where

$$e_n^i = V_{n+1}(Y_{n+1}^i) - (I_{n+1} + F_n) - \left(A_n(Y_{n+1}^i) - \frac{(1-\tau)C_n}{r}\right) > 0, \qquad (20)$$

$$e_n^d = -\left[A_n(Y_n^d) - \frac{(1-\tau)C_n}{r}\right] > 0. \tag{21}$$

Equity value $E_n(Y)$ is given by the after-tax present value of assets in place $A_n(Y)$ minus the after-tax perpetuity value of risk-free debt with coupon C_n, (i.e. $(1-\tau)C_n/r$) plus two option values: the (real) growth option and the (financial) default option. The third term in Equation (19) measures the present value of the growth option, which is given by the

product of $\Phi_n^i(Y)$, and the *net* payoff e_n^i from exercising the growth option. The *net* payoff e_n^i is the difference between the payoff from growth-option exercise $V_{n+1}(Y_{n+1}^i) - (I_{n+1} + F_n)$ and $(A_n(Y_{n+1}^i) - (1-\tau)C_n/r)$, the forgone unlevered equity value when investing at the threshold Y_{n+1}^i. Note that the forgone "un-levered" equity value appears as an additional cost term in the net payoff e_n^i because the option payoff $V_{n+1}(Y_{n+1}^i) - (I_{n+1} + F_n)$ contains cash flows from the existing assets in place. Similarly, the fourth term in Equation (19) is the present value of the (financial) default option, which is given by the product of $\Phi_n^d(Y)$ and the *net* payoff e_n^d upon default. Because equityholders receive nothing at default, the *net* payoff e_n^d is given by the savings, $-(A_n(Y_n^d) - (1-\tau)C_n/r) > 0$, from avoiding the loss of running the "un-levered" equity value at the default threshold Y_n^d.

2.2.2 Debt pricing.

In the Appendix, we show that debt value $D_n(Y)$ in stage n where $Y_n^d \leq Y \leq Y_{n+1}^i$ is given by:

$$D_n(Y) = \frac{C_n}{r} - \left[\frac{\Phi_n^d(Y_n^i)}{1 - \Phi_n^i(Y_n^i)}\Phi_n^i(Y) + \Phi_n^d(Y)\right]\left(\frac{C_n}{r} - L_n(Y_n^d)\right), \quad (22)$$

where $\Phi_n^i(Y)$ and $\Phi_n^d(Y)$ are given in Equation (16) and Equation (17), respectively. Creditors incur losses when the firm default (i.e., $C_n/r > L_n(Y_n^d)$). The second term in Equation (22) gives the value discount on debt due to the risk of default. We may obtain the par value F_n of this debt by evaluating $D_n(Y)$ at the investment threshold Y_n^i.

Because debt is priced at par F_n at issuance time T_n^i, we have the following valuation equation for the par value F_n:

$$F_n = \frac{C_n}{r} - \frac{\Phi_n^d(Y_n^i)}{1 - \Phi_n^i(Y_n^i)}\left(\frac{C_n}{r} - L_n(Y_n^d)\right). \quad (23)$$

Default is costly in that $C_n/r > L_n(Y_n^d)$. The second term in Equation (23) gives the value discount of debt at issuance due to default risk.

2.2.3 Firm valuation.

Now, we may calculate firm value $V_n(Y)$ as the sum of debt value $D_n(Y)$ and equity value $E_n(Y)$, in that

$$V_n(Y) = A_n(Y) + \frac{\tau C_n}{r} + v_n^i \Phi_n^i(Y) + v_n^d \Phi_n^d(Y), \quad Y_n^d \leq Y \leq Y_{n+1}^i, \quad (24)$$

where

$$v_n^i = V_{n+1}(Y_{n+1}^i) - I_{n+1} - \left(A_n(Y_{n+1}^i) + \frac{\tau C_n}{r}\right), \quad (25)$$

$$v_n^d = L_n(Y_n^d) - \left(A_n(Y_n^d) + \frac{\tau C_n}{r}\right). \tag{26}$$

Having described the details to solve for the default threshold Y_n^d and the investment threshold Y_{n+1}^i for stage $n \geq 1$, we now turn to the investment decision for the initial stage. Unlike the intermediate stages, the initial stage (stage 0) has no default decision, and hence simplifies the analysis.

2.3 The initial stage (stage 0) in the firm's life cycle
As in standard real-option models, equity value $E_0(Y)$ in stage 0 solves the following ODE:

$$rE_0(Y) = \mu Y E_0'(Y) + \frac{\sigma^2}{2} Y^2 E_0''(Y), \qquad Y \leq Y_1^i, \tag{27}$$

subject to the following boundary conditions

$$E_0(Y_1^i) = V_1(Y_1^i) - I_1, \tag{28}$$

$$E_0'(Y_1^i) = V_1'(Y_1^i). \tag{29}$$

The intuition behind the value-matching Condition (28) builds and extends the one in McDonald and Siegel (1986). Without any assets and liability, the firm raises $D_1(Y_1^i)$ in debt to partially finance the exercising cost I_1. Immediately after exercising the first growth option at the threshold Y_1^i, equityholders collect $E_1(Y_1^i) - (I_1 - D_1(Y_1^i))$ giving rise to the value-matching Condition (28). The smooth-pasting Condition (29) states that the investment threshold Y_1^i is chosen optimally. Finally, equity value $E_0(Y)$ also satisfies the standard absorbing barrier condition at the origin, in that $E_0(Y) \to 0$, when $Y \to 0$.

Equity value $E_0(Y)$, the solution to the above optimization problem, is given by

$$E_0(Y) = \left(\frac{Y}{Y_1^i}\right)^{\beta_1} \left(V_1(Y_1^i) - I_1\right), \ Y \leq Y_1^i, \tag{30}$$

where β_1 is given by Equation (6), and the investment threshold Y_1^i solves the following implicit equation:

$$Y_1^i = \frac{1}{1-\tau} \frac{r-\mu}{m_1} \frac{\beta_1}{\beta_1 - 1} \left[\left(I_1 - \frac{\tau C_1}{r}\right) + \frac{\beta_1 - \beta_2}{\beta_1 \Delta_1}(Y_1^i)^{\beta_2}\left((Y_1^d)^{\beta_1} v_1^i - (Y_2^i)^{\beta_1} v_1^d\right)\right], \tag{31}$$

and Δ_1 is a strictly positive constant given in Equation (18) with $n = 1$.

Unlike in the standard equity-based real-options models (e.g., McDonald and Siegel 1986), the payoff from investment in our model is total firm value $V_1(Y)$, which includes the present values of cash flows from both operations and financing. Note that equity value $E_0(Y)$ is convex in Y, a standard result in the real-options literature.

Having analyzed the firm's investment and default thresholds, we now analyze the firm's dynamic financial (debt) policies, and summarize the firm's integrated dynamic decision making over its life cycle.

2.4 Dynamic debt policies and a summary of the firm's life cycle decisions

First, review the decision problem in stage N. The firm chooses its last default threshold Y_N^d as a function of coupon C_N by maximizing equity value $E_N(Y; C_N)$. The solution for Y_N^d as a function of C_N is given by Equation (9), a well-known problem treated in Leland (1994). Then, equityholders choose C_N to maximize $V_N(Y)$ and then evaluate the first-order condition (FOC) for C_N at $Y = Y_N^i$. Intuitively, equityholders internalize all benefits and costs of debt issuance at T_{n+1}^i and pay fair market value $D_N(Y_N^i) = F_N$ when choosing coupon C_N.[17] Because firm value $V_N(Y)$ given by Equation (12) is known in closed form, we obtain the following explicit solution for C_N in terms of Y_N^i:

$$C_N = \frac{r}{r-\mu}\frac{\beta_2-1}{\beta_2}\frac{1}{h}M_N Y_N^i, \tag{32}$$

where h is given by

$$h = \left[1 - \beta_2\left(1 - \gamma_A + \frac{\gamma_A}{\tau}\right)\right]^{-1/\beta_2} > 1. \tag{33}$$

Using Formula (9) for Y_N^d for a given coupon C_N, we obtain the relationship between the last default threshold Y_N^d and the last growth-option exercising: $Y_N^d = Y_N^i/h$.

Now consider stage $(N-1)$. Equityholders choose the thresholds Y_N^i and Y_{N-1}^d to maximize equity value $E_{N-1}(Y; C_{N-1})$, taking the default threshold Y_N^d in Equation (9) and optimal coupon C_N in Equation (32) in stage N as given. Because equityholders internalize the tax benefits from issuing debt at T_{N-1}^i, equityholders choose coupon C_{N-1} to maximize $V_{N-1}(Y)$ and then evaluate at Y_{N-1}^i.

Next turn to stage n, where $1 \leq n < (N-1)$. As in stage $(N-1)$, the firm chooses thresholds Y_{n+1}^i and Y_n^d to maximize equity value $E_n(Y; C_n)$, taking into account the firm's future optimality conditions described earlier. Then, the firm chooses the optimal coupon policy C_n to maximize $V_n(Y)$ and then evaluate at Y_n^i.

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[17] The optimality for C_N and Y_N^i and the envelope condition jointly imply that we do not need to consider the feedback effects between the investment threshold Y_N^i and the coupon policy C_N.

Finally, stage 0 is a special case of stage n. The firm chooses the first investment threshold Y_1^i to maximize equity value $E_0(Y)$. Note that $Y_0^d = 0$ (no debt and no default). We have shown that equity value $E_0(Y)$ is given by Equation (30) and the investment threshold is given by the implicit nonlinear Equation (31).

Our model thus have predictions for the dynamics of leverage choice by the firm, and how the leverage dynamics relate to the life cycle of the firm. Unlike most existing dynamic financing models, which ignore investments, our model explicitly incorporates investment frictions, which are potentially important. The leverage dynamics under investment frictions will reflect the importance of remaining future growth options, and the potential for premature liquidation from excessive leverage. We explore this tension later in the paper.[18]

Having outlined the solution methodology for the general model specification, we next summarize the setting where the firm is all-equity financed (i.e., McDonald and Siegel 1986) with multiple growth options and taxes.

2.5 All-equity-financing model

We now consider an all-equity setting with multiple rounds of growth options. Recall that Lemma 1 gives the growth option value $G_k(Y)$ and the exercise threshold Y_k^{ae} for a firm with a stand-alone growth option. When the firm has N *sequentially* ordered growth options, the technology constraint requires that growth option k can only be exercised if and only if all previous $(k-1)$ growth options have been exercised. Intuitively, this sequential exercising constraint binds when future growth options are worthy immediately exercising after growth options are exercised. We can show that simultaneously exercising growth options k and $(k-1)$ is optimal if and only if $m_k/I_k \geq m_{k-1}/I_{k-1}$. Then we may combine these two consecutive growth options into one, with an exercise cost $(I_k + I_{k-1})$ and a "new" cash flow multiplier $(m_k + m_{k-1})$ for the combined growth option. By redefining growth options, we can always focus on the setting where m_n/I_n strictly decreases in n, in that

$$\frac{m_1}{I_1} > \frac{m_2}{I_2} > \ldots > \frac{m_N}{I_N}. \tag{34}$$

Under this condition, the option value of waiting is strictly positive between any two consecutive growth options in the first-best (all-equity financing) benchmark. The following lemma summarizes the main results for the equity financing benchmark.

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[18] Our characterization of leverage dynamics only requires us to solve a system of non-linear equations for investment and default thresholds, and coupon policies. This substantially simplifies our analysis, in that we have solved out the *endogenous* default and investment thresholds up to a set of nonlinear equations.

Lemma 2

The firm's investment decisions follow stopping time rules $T_n^i = \inf\{ t \geq 0 : Y(t) = Y_n^{ae}\}$ for $n = 1, 2, \ldots, N$, where the investment threshold Y_n^{ae} are given by Equation (5), and the constant β_1 is given by Equation (6). Firm (equity) value $E_n(Y)$ in stage n is given by the sum of assets in place $A_n(Y)$ and its unexercised growth options, in that

$$E_n(Y) = A_n(Y) + \sum_{k=n+1}^{N} G_k(Y), \qquad Y \leq Y_n^{ae}, 1 \leq n \leq N, \qquad (35)$$

where $G_k(Y)$ is the k-th growth option value and is given in Equation (4).

For any stage n, the investment threshold Y_n^{ae} is the same as the one if the n-the growth option were stand-alone. Taxes reduce cash flows but do not provide benefits under all-equity financing. This explains the factor $1/(1 - \tau)$ for the investment threshold Y_n^i given in Equation (5). As in standard real-options models (McDonald and Siegel 1986), the investment threshold Y_n^i increases in volatility. For the ease of future reference, let Y_n^* denote the n-th investment threshold without taxes ($\tau = 0$). We have

$$Y_n^* = (r - \mu)\frac{\beta_1}{\beta_1 - 1}\frac{I_n}{m_n}, \qquad 1 \leq n \leq N. \qquad (36)$$

Next, we analyze the investment and financing decisions for the one-growth-option setting ($N = 1$).

3 Benchmark: One-Growth-Option Setting

Before delving into the details of the general model where the firm has multiple rounds of growth options and leverage choices, we first provide explicit solutions for the one-growth-option setting in Subsection 3.1 and then highlight important economic insights in Subsection 3.2. Importantly, we show that this simple one-growth-option setting yields novel insights that can only be obtained by *jointly* analyzing the firm's investment and default decisions.

3.1 Closed-form solution

When the firm has only one growth option, we obtain closed-form formulas for the joint investment, leverage, and default decisions. Our one-growth-option setting can be viewed as a model setting, where McDonald and Siegel (1986), the seminal real-options model in a Modigliani-Miller world, meet Leland (1994), the classic contingent-claim tradeoff model of

capital structure. The following proposition summarizes the main results.[19]

Proposition 1

The firm's investment decision follows a stopping time rule $T_1^i = \inf\{t : Y(t) \geq Y_1^i\}$, where the investment threshold Y_1^i is given by

$$Y_1^i = \left[1 + \frac{1}{h}\left(\frac{\tau}{1-\tau}\right)\right]^{-1} Y_1^{ae}, \tag{37}$$

where the constant h is given in Equation (33) and Y_1^{ae} is the all-equity investment threshold given in Equation (5). The default time T_1^d is given by $T_1^d = \inf\{t > T_1^i : Y(t) \leq Y_1^d\}$, where the default threshold Y_1^d is given by $Y_1^d = Y_1^i/h < Y_1^i$. The optimal coupon C_1^* for debt issued at the investment time T_1^i is given by

$$C_1^* = \frac{r}{1-\tau}\left(\frac{\beta_2 - 1}{\beta_2}\right)\left(\frac{\beta_1}{\beta_1 - 1}\right)\left(h + \frac{\tau}{1-\tau}\right)^{-1} I_1. \tag{38}$$

Firm value $V_1(Y)$ (after investing at T_1^i) is given by

$$V_1(Y) = A_1(Y) + \frac{\tau C_1^*}{r} - \left(\gamma_A A_1(Y_1^d) + \frac{\tau C_1^*}{r}\right)\left(\frac{Y}{Y_1^d}\right)^{\beta_2}, \qquad Y \geq Y_1^d. \tag{39}$$

Firm (equity) value $E_0(Y)$ (before investing at T_1^i) is given by Equation (30).

We make the following observations. The investment threshold Y_1^i, the default threshold Y_1^d, and the optimal coupon C_1^* are all proportional to the investment cost I_1. At investment time T_1^i, equity value $E_1(Y_1^i)$, debt value $D_1(Y_1^i)$, and firm value $V_1(Y_1^i)$ are all proportional to the investment cost I_1. This implies that the market leverage at the moment of investment T_1^i, $D_1(Y_1^i)/V_1(Y_1^i)$ is independent of the size of the investment cost I_1. Next, we turn to the model analysis.

3.2 Model analysis and insights
One of the most important results of real-options analysis is that both the investment hurdle and option value increase with volatility (by drawing the analogy to the standard Black-Scholes-Merton option pricing insight.) We show that debt financing invalidates this well-known result

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[19] Mauer and Sarkar (2005) derive similar results for the one-growth-option setting. Their focus on the results and economic interpretations is very different. We derive explicit formulae and provide explicit link between investment and default thresholds, while they do not. They contain operating leverage (variable production costs), and we do not.
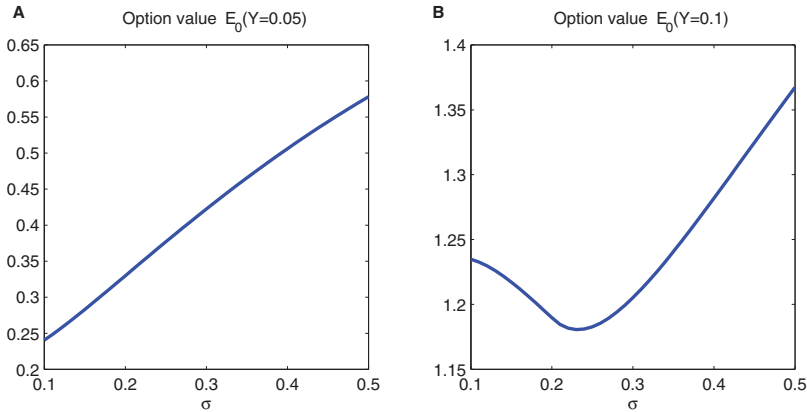
Figure 2
Two opposing effects of volatility on the firm's option value $E_0(Y)$.
Panel A pots the monotonic relation of $E_0(Y)$ in σ for $Y=0.05$. However, for a higher value of Y where the growth option is deeper in the money, we find that volatility σ has a non-monotonic relation on the value of real option $E_0(Y)$. Panel B shows that $E_0(Y)$ first decreases and then increases with σ for $Y=0.1$.

in the real-options literature, in that the value of growth option may decrease with volatility. The intuition is as follows. Before debt financing and exercising the growth option, the firm is a growth option. What is the underlying asset for this growth option? Unlike in the standard real-options setting such as McDonald and Siegel (1986), the value of the underlying asset is given by the sum of (*i*) the value of the "unlevered" equity and (*ii*) the (stochastic annuity) value of tax shields (prior to default) minus (*iii*) the present value of financial distress because of equi-tyholders' optimal exercising of the ex post default option as in Leland (1994). The underlying asset value, given by the sum of these three components of the firm's value, is therefore *concave* in Y as we have shown (because of the short position in the inefficient liquidation loss.) Therefore, increasing volatility may *lower* the *payoff value* from growth-option exercising. This negative effect on the payoff (upon the real-option exercising) partially mitigates the standard positive-volatility effect on the real-option value, causing the total effect of volatility σ on the firm's option value $E_0(Y)$ to be non-monotonic.

Figure 2 highlights these two opposing effects of volatility σ on the option value $E_0(Y)$. Panel A shows that the option value $E_0(Y)$ is increasing in volatility σ for sufficiently low values of Y (e.g., $Y=0.05$). In this case, the standard real-option positive-volatility effect on the option value dominates, because the option value is deep out of the money and hence the standard real-option convexity argument applies. However, as Y increases, the growth option becomes sufficiently in the money. In this case, the standard real-option volatility effect becomes less
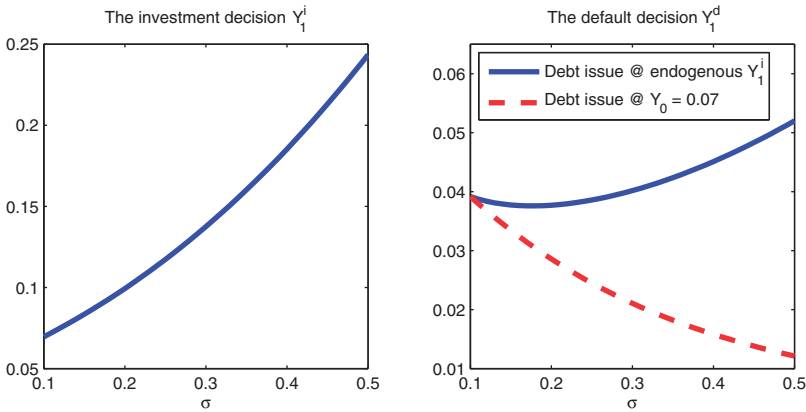
Figure 3
Volatility effects on the investment threshold Y_1^i and the default threshold Y_1^d.
This figure shows that even though the investment threshold Y_1^i is monotonically increasing in σ as in standard real-option models, the default threshold Y_1^d is non-monotonic in σ unlike in the standard contingent-claim tradeoff model of capital structure due to the endogenous choice of the investment threshold Y_1^i. To demonstrate the endogenous investment threshold effect on Y_1^d, we plot the dashed line in Panel B, which corresponds to the default threshold in Leland (1994), where leverage is chosen at time zero at a fixed value of Y_0 for all values of volatility σ. For the Leland case, we set $Y_0 = 0.07$, which is the optimal investment threshold for the case with $\sigma = 10\%$ in our one-growth-option setting (i.e., $Y_1^i(\sigma = 0.1) = Y_0 = 0.07$).

important compared with the negative effect of volatility on the option-exercise payoff becomes more important. As a result, we see that the option value $E_0(Y)$ is non-monotonic in volatility σ in Panel B of Figure 2, where $Y = 0.1$ is sufficiently large. For our example, we find that $E_0(Y = 0.1)$ decreases in σ for values of $\sigma < 0.23$ and increases in σ for $\sigma > 0.23$ (i.e., when volatility is sufficiently high).[20]

Next, we turn to the volatility effects on the investment threshold Y_1^i and the default threshold Y_1^d. Panel A of Figure 3 shows that the investment hurdle Y_1^i is increasing with volatility σ, which can be shown by using the closed-form solution (37). Moreover, this result is consistent with the standard real-options intuition that the higher the volatility σ, the longer the firm waits before investing, and hence a higher threshold Y_1^i is the result.

However, the default threshold Y_1^d is non monotonic in σ as shown in Panel B of Figure 3 because the two opposing effects of volatility on the firm's default threshold Y_1^d. First, for a given value of Y_0, Leland (1994) and Goldstein, Ju, and Leland (2001) show that the default threshold Y_1^d decreases with σ, consistent with the standard real-option result, despite an intuitive but involved argument.[21] The dashed line in Panel B of Figure 3

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[20] Miao and Wang (2007) show the opposing effects of volatility on real options in an incomplete-markets setting where the entrepreneur cannot fully diversify the idiosyncratic risk of the underlying project.

[21] To be precise, in Leland (1994), volatility σ also has two opposing effects on the default threshold Y_1^d. Given coupon C, the higher the volatility σ, the lower the default threshold due to the standard

shows the monotonically decreasing function of Y_1^d in volatility σ. Second, as debt coupon C is chosen at the moment when the firm exercises its growth option and the optimal investment threshold Y_1^i is increasing in volatility σ as shown in Panel A, the firm's optimal debt coupon C is also increasing with σ, thus providing a channel for the default boundary Y_1^d to potentially increase with σ. Combining the standard Leland mechanism with the newly introduced channel (via the endogenous Y_1^i at which the firm issues debt), we see that the default threshold Y_1^d first decreases in σ (when the Leland mechanism dominates) but then increases in σ as the investment threshold Y_1^i significantly increases.

Our results, that real option value does not necessarily increase with volatility and the timing of exercising (default/put) options may not decrease with volatility are more than theoretical possibilities. Indeed, we think that our results have important practical implications for firms' capital budgeting. Growth option/capital investments are often financed by a mixture of debt and equity, but current real-options-based capital budgeting recommendations covered in standard MBA textbooks and practitioners' journals hold only under all-equity-financed firms. When the M&M theorem does not hold and there is room for debt financing, we need to be careful in providing real-option-style capital-budgeting recommendations. It is no longer clear that we should emphasize the positive effect of volatility on the value of real options nor should we emphasize that the higher the volatility, the longer we should wait before exercising the real options.

Having analyzed the two benchmarks, we next turn to the feedback effects between investment and financing when the firm has multiple rounds of growth options.

4 Analysis for the General Model

To highlight the rich interactions between a firm's investment and financing decisions over its life cycle, we first analyze the two-growth-option setting ($N = 2$), and then generalize our model to settings with multiple growth options (e.g., $N = 3, 4, 5, 6$).

4.1 Parameter choices
For the baseline calculation, we use the following annualized parameter values summarized in Table 1. As in Leland (1994) and the follow-up dynamic capital structure literature, we choose the risk-free interest rate

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real-option convexity argument. However, coupon C is endogenous. Indeed, the higher the volatility σ, the lower the debt coupon C. Therefore, the second (endogenous coupon) effect mitigates the first option effect on the default threshold Y_1^d, but the overall, the standard real-option effect still dominates the endogenous coupon effect causing the firm's default threshold Y_1^d to *decrease* with σ.

Table 1
Parameter values (annualized whenever applicable)

r	τ	μ	σ	I_n	m_n	γ_A	γ_G
0.05	0.2	0.01	0.2	1	0.8^{n-1}	0.25	0.5

$r = 0.05$, the expected growth rate $\mu = 0.01$, annual volatility $\sigma = 0.2$, and the effective tax rate $\tau = 0.2$. The default cost parameters are $\gamma_A = 0.25$ for assets in place and $\gamma_G = 0.5$, implying that the recovery rate for assets in place is 75% of the unlevered asset value and 50% for the (unlevered) unexercised growth option value, respectively. Whenever applicable, all parameter values are annualized.

Without loss of generality, we normalize the cost of exercising the growth option to unity, in that $I_n = 1$, for all stages $1 \leq n \leq N$. Instead we capture the net value of each sequential growth option via the the production capacity (the rate at which output is generated from each asset in place) m_n. We normalize the production capacity in the first stage to be unity, $m_1 = 1$, and decrease the production capacity m_n at an exponential rate (i.e., $m_n = m_{n-1} \times (1 - g_n)$). We choose $g_n = 0.2$ for $n = 2, 3, \ldots$, which implies that the profitability of each new asset in place decays at 20%. Therefore, $m_2 = 0.8$ and $m_3 = 0.8^2 = 0.64$, $m_4 = 0.8^3$, and $m_5 = 0.8^4$.

4.2 The settings with $N = 1, 2, 3$ growth options

Our framework is general enough that we can allow the firm to have multiple growth options. Earlier, we treated the case of a firm with two growth options. We now extend our analysis to the case of a firm that has three growth options. Panels A, B, and C in Table 2 report the firm's decisions in three models (with 1, 2, and 3 growth options in total, respectively.)

One growth option ($N = 1$). Panel A of Table 2 summarizes the closed-form solution for the one-growth-option setting ($N = 1$ and $m_1 = 1, 0.8, 0.64$). First, we analyzes the baseline case with $m_1 = 1$. The firm optimally chooses to invest when Y exceeds $Y^i = 0.099$, and defaults when its earning Y falls below $Y^d = 0.038$. When exercising its growth option at $Y^i = 0.099$, the firm issues perpetual risky debt with coupon rate $C = 0.082$ at a credit spread of 108 basis points. The implied initial leverage is 62.1%. For lower production capacity m_1, the firm chooses a higher investment threshold and also a higher default threshold. For the case with $m_1 = 0.8$, we have $Y^i = 0.124$ and $Y^i = 0.047$, and for the case with $m_1 = 0.64$, we have $Y^i = 0.155$ and $Y^i = 0.059$. Importantly, the production capacity has no effect on the optimal coupon C and leverage.

Table 2
Models where the number of growth options $N=1, 2, 3$

Stage	Production capacity m	Investment threshold Y^i	Default threshold Y^d	Coupon rate C	Leverage Lev	Credit spreads cs (bps)
Panel A. $N=1$						
1st	1	0.099	0.038	0.082	62.1%	108
1st	0.8	0.124	0.047	0.082	62.1%	108
1st	0.64	0.155	0.059	0.082	62.1%	108
Panel B. $N=2$						
1st	1	0.095	0.030	0.077	43.8%	129
2nd	0.8	0.126	0.048	0.188	62.1%	108
Panel C. $N=3$						
1st	1	0.093	0.028	0.080	39.1%	127
2nd	0.8	0.117	0.036	0.156	46.6%	123
3rd	0.64	0.158	0.060	0.319	62.1%	108

This table reports results for three settings with $N=1, 2, 3$. For the setting with $N=1$, we consider three subcases with $m_1 = 1, 0.8, 0.64$. For the setting with $N=2$, we set $m_1 = 1$ and $m_2 = 0.8$. Finally, for the case with $N=3$, we set $m_1 = 1, m_2 = 0.8$, and $m_3 = 0.64$.

This is due to the endogenous adjustment of investment and default thresholds such that the firm achieves its optimal leverage ratio at 62.1% at the moment of debt issuance.

Two growth options ($N=2$). Panel B of Table 2 reports the results for the two-growth-option setting. First consider the decision rules in the second (last) stage. After the first debt is in place, the firm exercises its second growth option when its earning exceeds $Y_2^i = 0.126$, larger than 0.124, the investment threshold for the setting with $N=1$ and $m_1 = 0.8$ (see Panel A and B of Table 2). This reflects the effect of debt overhang, in that the firm exercises its investment option later than an otherwise identical firm with this stand-alone growth option (with $m_1 = 0.8$) does. The cost of exercising the second growth option is higher now because the equityholders need to call back the existing debt at par, which potentially involves the wealth transfer to creditors.[22] However, conditioning on calling back the existing debt and exercising the growth option, equity-holders optimally choose their leverage and maximize the firm value going forward. This again gives rise to 62.1% leverage ratio, the same level as in the stand-alone one-growth-option setting, which is due to the scaling invariance property of the leverage ratio for assets in place as in Leland (1994). Given that the equityholders are investing at a higher threshold and needs to call back the existing debt, the coupon is naturally much higher (i.e., $C_2 = 0.188$) than the coupon for stand-alone one-growth-option setting (i.e., $C = 0.082$). Despite a higher debt coupon,

---

[22] There is another truncation effect that effects the credit-spread calculation, but that effect is dominated.

the credit risk of the second-stage debt is the same as the one in the stand-alone one-growth option setting. This is in the spirit of Leland (1994) and our one-growth-option benchmark result.

Now, we turn to the first-stage decision making and see how the presence of the second growth option influences the optimal-exercising and financing decisions of the first growth option. First, note the anticipation effect of the subsequent debt overhang as we have discussed in the previous paragraph. Equityholders anticipate future debt overhang and thus rationally lowers the leverage and take into account the future conflicts of interest between equityholders and debtholders. This is reflected via a lower leverage, 43.8%, in stage 1 compared with 62.1% in stage 2. Moreover, the presence of the second growth option raises the firm's debt capacity. Therefore, benefits from exercising the first growth option are higher in the two-growth-option setting than in the stand-alone one-growth-option setting with $m_1 = 1$. Therefore, in the two-growth-option setting, the payoffs from investing in the first round are greater, and hence the firm optimally invests earlier (i.e., $Y_1^i = 0.095$ compared with $Y_1^i = 0.099$ for the stand-alone one-growth-option setting with $m_1 = 1$). Additionally, the firm defaults later, as we can see from $Y^d = 0.03$, which is lower than 0.038 for the setting with $N = 1$ and $m_1 = 1$.

Three growth options ($N = 3$). Here, we show that the exercising timing decisions for early-rounds growth options are even earlier with more growth options in the future. One effect of having more future-growth options is that the firm can raise more debt against future cash flows, which effectively raises the firm's immediate ability to issue debt and makes investment more attractive. This explains the result that $Y_1^i = 0.093$ in three-growth-option setting, which is lower than $Y_1^i = 0.095$ in the two-growth-option setting. Additionally, the leverage chosen by the firm when it exercises its first growth option is now lowered to 39.1% from 43.8%. We see the pattern for eventual convergence as we increase the number of growth options.

By comparing our results for the two-growth-option setting with those for the three-growth-option setting, we see that quantitatively the three-growth-option problem can be "somewhat approximately" decomposed into two two-growth-option optimization problems. The intuition is as follows. The additional effect of current growth-option exercising and financing effect on any future growth-option exercising and financing decisions beyond the immediate one is quantitatively less significant. Using this logic, we may simplify an N-stage growth-option exercising/financing problem into $(N − 1)$ two-growth-option exercising problem. Of course, our approximation based on the economic insight only holds for growth options that are sufficiently close to each other.

For growth options that are somewhat different from each other, our insights for this approximation may work better if we decompose the N-growth option problem into a collection of three-growth-option problems. Based on what we have shown and also what we will show in the following subsection, we conjecture that for some sophisticated real-world corporate-investment/financing-decision problem with many growth options, we may obtain a good understanding at first pass (and potentially also in terms of quantitative analyses) by using a tractable and plausible setting with only a few growth options (perhaps as few as three or four.)

4.3 Multiple growth options

In principle, we can extend our analysis to treat a firm with any number, N, of growth options. Such a treatment will allow us to see, how a firm optimally decide on its dynamic leverage strategy, while taking into its cognizance that it may have to issue additional debt to finance future growth options. Intuitively, we would expect such a firm to start with fairly low to moderate levels of debt in its early stages and slowly ramp up the debt level, By doing so, the firm can mitigate the debt-overhang effects on its growth options in earlier stages, and exploit its steadier cash flows from assets in place to service a higher level of debt in its later stages.

4.3.1 N growth options. In Table 3, we present the investment thresholds Y_1^i, default thresholds Y_1^d, the optimal coupon rate C_1 and optimal leverage Lev_1 when the firm is in its *first stage* and at the the moment of exercising its first growth option, in six setting where the firm faces one to as many as six growth options into the future. Note that the first growth option in all cases has the identical parameter values. Thus, the differences in these models only arise from the future growth options and assets in places across them.

Table 3
The first-stage decisions in models with N growth options

Model with N growth options	Investment threshold Y_1^i	Default threshold Y_1^d	Coupon rate C_1	Leverage ratio Lev_1
1	0.099	0.038	0.082	62.1%
2	0.095	0.030	0.077	43.8%
3	0.093	0.028	0.080	39.1%
4	0.093	0.027	0.082	37.0%
5	0.092	0.026	0.083	35.3%
6	0.092	0.026	0.083	34.5%

This table reports the first-stage decisions in a model with N growth options. We increase N from 1 to 6. Our parameter values are $m_1 = 1, m_2 = 0.8, m_3 = 0.64, m_4 = 0.512, m_5 = 0.410,$ and $m_6 = 0.328$. Others are reported in Table 1.

It is worth making the following observations. First, the investment threshold Y_1^i monotonically decreases from $Y_1^i = 0.099$ for the case with $N=1$ to $Y_1^i = 0.092$ for the case with $N=6$. Intuitively, the additional benefit of not distorting future investment options in models with more growth options (a higher value of N) encourages the firm to exercise its growth options in earlier stages sooner. Second, the optimal leverage level Lev_1 decreases from 62.1% for the case with $N=1$ to 34.5% for the case with $N=6$. This partly reflects the debt conservatism as the firm worries about the debt-overhang costs in the future if leverage is too high. Third, a firm with more growth options tends to default at a much later point than a firm with fewer growth options. For example, the default threshold Y_1^d decreases from $Y_1^d = 0.038$ for the case with $N=1$ to $Y_1^d = 0.026$ for the case with $N=6$. This makes intuitive sense: for the firm with many growth options, the cost of default should include the forgone opportunities associated with the loss of all future growth options, and hence the firm chooses to default much later in the case with $N=6$ than that with $N=1$, in line with the predictions from the leverage pattern that we discussed earlier.

To reiterate, leverage, default, and investment thresholds in early stages all monotonically decrease as the firm has more growth options. This life-cycle pattern driven by the endogenous composition between growth options and assets in place is important. Quantitatively, in our model with $N=3$ or more growth options, we obtain leverage in the empirically plausible range of 1/3 for U.S. corporations. Moreover, the optimal investment and default, as well as coupon decisions, essentially all converge as we increase the number of growth options to six (i.e., $N=6$). Intuitively, the additional growth option (most likely to be exercised in the distant future if the firm has not defaulted by then) has little if any effect on the decision making in stage 1 because of the discounting effect.

4.4 Growth-option liquidation recoveries $(1 - \gamma_G)$ and leverage

Growth options and assets in place generally have different recoveries during the liquidation process. Intuitively, growth options tend to have lower debt capacity than assets in place, for various reasons, such as different degrees of tangibility and also inalienability of human capital embedded in growth options. To capture this important feature and analyze its effect on leverage, we allow for growth options and assets in place to have different recovery rates in liquidation. For simplicity, we have assumed that the recovery value of an unexercised growth option is equal to a constant fraction, $(1 - \gamma_G)$, of $G_k(Y)$ given by Equation (4), which is the value for an otherwise identical (all-equity) stand-alone growth option.

Table 4
Effects of liquidation loss parameter γ_G on leverage

Model with N	$\gamma_G = 0.25$	$\gamma_G = 0.5$	$\gamma_G = 0.75$	$\gamma_G = 1$
1	62.1%	62.1%	62.1%	62.1%
2	45.5%	43.8%	42.6%	41.8%
3	40.1%	39.1%	38.0%	36.9%
4	37.6%	37.0%	36.5%	35.9%
5	35.9%	35.3%	34.8%	34.2%
6	34.9%	34.5%	33.8%	33.2%

This table reports the effect of the growth-option liquidation-loss parameter, γ_G, on leverage in the first stage for models with N growth options, where N ranges from 1 to 6. The liquidation loss parameter for assets in place, γ_A, is fixed at 0.25 for all six models. Note that the recoveries of assets in place are weakly greater than the recoveries of growth options for all six models. Other parameter values are reported in Table 1.

In this section, we explore the comparative static effects of varying liquidation loss parameter γ_G on leverage. First, we recall that in our baseline case (reported in Table 1), we fix the liquidation loss parameter for assets in place, γ_A, to be 25% and the loss parameter for growth options, γ_G, to be 50%. That is, the recovery value for growth option is 50% of its *market* value (if it were stand-alone and all-equity financed), which is significantly smaller than the recovery value for assets in place, which is 75% of its *market* value (if it were stand-alone and all-equity financed.)

Next, in Table 4, we report leverage in the first stage by considering four values for the growth-option liquidation-loss parameter γ_G: 25%, 50% (baseline case), 75%, and 100%. First, we note that for the case with only one growth option (i.e., $N = 1$), the parameter γ_G has no effect on leverage choice because the only growth option has already been exercised at the moment of debt financing and hence changing the effect o γ_G cannot have any effect on leverage. Second, the higher the loss parameter for the growth option, γ_G, the more costly the firm's default is, and hence the lower the leverage. For all five cases with $N \geq 2$ where the firm still has un-exercised growth options after making the first leverage decision, we see that leverage (in the first stage) clearly decreases with γ_G. For example, the first-stage leverage ratio decreases from 45.5% to 41.8% when γ_G increases from 0.25 to 1 for $N = 2$.

Third, the first-stage leverage decreases substantially with N. Intuitively, a firm with more growth options tends to take a lower leverage (especially at early stages) in order to mitigate the debt-overhang burdens in the future. The flexibility of increasing leverage in the future when the firm has more assets in place is a valuable option for the firm.[23]

---

[23] Additionally, by construction, a firm with more growth options in our model also has more financing options as we require the firm to choose leverage when exercising a growth option. However, quantitatively,

In a model with informational asymmetry and real options, Fulghieri, Garcia, and Hackbarth (2013) show that high-growth firms may prefer equity over debt, and then switch to debt financing as they mature. Their predictions under a different model formulation are in line with our model's prediction in that leverage increases over the firm's life cycle in our model, as the firm sequentially exercises its growth options and becomes one with more assets in place.

Does a firm with more growth options have a larger leverage response to the change of γ_G? The answer is not obvious. For example, for the case with $N=2$, leverage in the first stage decreases from 45.5% to 41.8% by 3.7%, and for the case with $N=6$, it decreases from 34.9% to 33.2% by 1.7% as we decrease recovery, $1-\gamma_G$, from 75% to zero. It is not easy to assess the significance of this difference (3.7% versus 1.7%).

5 Alternative Debt Structures

In our baseline model, we assume that the existing debt is called back and new debt is issued when the firm exercises its growth option. In reality, we often see different types of debt in terms of seniority and priority coexisting. To analyze the effects of debt seniority and priority, we next consider two widely used alternative debt structures: the APR and the *pari passu* structure. For simplicity, we only consider the cases with two rounds of growth options.

5.1 Formulation and solution under alternative debt structures

Let c_1 and c_2 denote the coupon rate on the first and second perpetual debt, respectively. And let $D_2^f(Y)$ and $D_2^s(Y)$ denote the market value of the first debt and that of the second debt issued at the second investment time T_2^i, respectively. These debt values (after the second growth option is exercised (i.e., $t \geq T_2^i$)) are given by

$$D_2^f(Y(t)) = \mathbb{E}\left[\int_t^{T_2^d} e^{-r(s-t)}c_1 ds + e^{-r(T_2^d-t)}D_2^f(Y(T_2^d))\right], \quad T_2^i \leq t \leq T_2^d,$$

(40)

---

given that it is not uncommon that investment and financing take place at the same time or within very close time window, this approximation (of tying a firm's investment and financing decisions at the same time) appears a second-order issue for a firm that has recurrent and sufficiently regular investment opportunities. Put differently, the additional flexibility of having a timing for financing different from that for investment may have a second-order effect on firm value (at least for some parameter values).

$$D_2^s(Y(t)) = \mathbb{E}\left[\int_t^{T_2^d} e^{-r(s-t)} c_2 ds + e^{-r(T_2^d-t)} D_2^s(Y(T_2^d))\right], \quad T_2^i \le t \le T_2^d,$$

(41)

where the residual values of the first and second debt, $D_2^s(Y(T_2^d))$ and $D_2^f(Y(T_2^d))$ are given by the debt structures to be discussed later.

The total market value of debt after exercising both growth options is then given by $D_2(Y) = D_2^f(Y) + D_2^s(Y)$. Let $D_1(Y)$ denote the market value of the first debt after the first growth option is exercised, but before the second growth option or the first default option is exercised (i.e., $T_1^i < t < (T_1^d \wedge T_2^i)$). We have

$$D_1(Y(t)) = \mathbb{E}\left[\int_t^{T_1^d \wedge T_2^i} e^{-r(s-t)} c_1 ds + e^{-r(T_1^d-t)} D_1(Y(T_1^d))\mathbf{1}_{T_1^d < T_2^i}\right.$$

$$\left. + e^{-r(T_2^i-t)} D_2^f(Y(T_2^i))\mathbf{1}_{T_1^d > T_2^i}\right].$$

(42)

Under the new debt structure, the total coupon level for all outstanding debt in stage 1 and 2 are $C_1 = c_1$ and $C_2 = c_1 + c_2$, respectively. The valuation for equity remains the same as that in the baseline model, given the specified coupon levels for all outstanding debt.

When the firm defaults, it splits the recovery values to the first and the second debt valued at $D_2^f(Y_2^d)$ and $D_2^s(Y_2^d)$, depending on the debt covenants. Assume that the debt covenants will be strictly enforced by the court without any deviation. Given these endogenous values at the chosen default boundary Y_2^d, we may write the market value of the seasoned debt issued at T_1^i and that of the second debt issued at T_2^i, before default at T_2^d, as follows:

$$D_2^f(Y) = \frac{c_1}{r} - \left[\frac{c_1}{r} - D_2^f(Y_2^d)\right]\left(\frac{Y}{Y_2^d}\right)^{\beta_2}, \quad Y \ge Y_2^d,$$

(43)

$$D_2^s(Y) = \frac{c_2}{r} - \left[\frac{c_2}{r} - D_2^s(Y_2^d)\right]\left(\frac{Y}{Y_2^d}\right)^{\beta_2}, \quad Y \ge Y_2^d,$$

(44)

where $D_2^f(Y_2^d)$ and $D_2^s(Y_2^d)$ are valued differently under different debt structures as we have noted. Thus, the total debt value is $D_2(Y) = D_2^f(Y) + D_2^s(Y)$. In addition, the total debt value at default $D_2(Y_2^d)$ is equal to the total firm's liquidation value at default, because equity is worthless at default.

Now we turn to analyze the effect of the debt structure on the financing decision in different stages. First, consider stage 2. Define $V_2^s(Y)$ as the

sum of equity value $E_2(Y)$ and $D_2^s(Y)$, the value of debt issued when the firm exercises its second growth option, in that $V_2^s(Y) = E_2(Y) + D_2^s(Y)$. Using Equation (8) and Equation (44), we have

$$
\begin{aligned}
V_2^s(Y) = A_2(Y) &+ \frac{\tau C_2 - C_1}{r} \\
&+ \left(D_2^s(Y_2^d) - A_2(Y_2^d) + \frac{C_1 - \tau C_2}{r} \right) \left(\frac{Y}{Y_2^d} \right)^{\beta_2}, \quad Y \geq Y_2^d. \quad (45)
\end{aligned}
$$

The distinction between $V_2(Y)$ and $V_2^s(Y)$ is essential for our analysis. Equityholders no longer care about the payoffs to the first-round debtholders after collecting the proceeds from the debt issuance at T_2^i. This creates conflicts of interests between equityholders and the existing (first-round) debtholders. Equityholders choose the investment threshold Y_2^i and the coupon policy c_2 to maximize $V_2^s(Y)$, not $V_2(Y)$. The first debt issued at T_1^i to finance the exercise of the first growth option generates a debt-overhang problem and distorts the exercising decision for the second growth option. Of course, the first-round debtholders anticipate the equityholders' incentives in stage 2 and hence price the debt accordingly in stage 1 at the moment of debt issuance. As a result, equityholders eventually bear the cost of this debt overhang induced by debt issued in stage 1. Unlike most papers in the literature on debt overhang, the amount of preexisting debt and hence the severity of debt-overhang in our model is determined endogenously. We show that different debt structures affect the debt overhang problem in different ways.

Next, we consider two commonly used debt structures: the absolute priority rule (APR) and the *pari passu* structure.

5.2 Absolute priority rule

The long maturity of debt allows us to generate debt overhang in a convenient way (Myers 1977; Hennessy 2004). Because debt is perpetual and not callable, the first debt continues to exist even after exercising the second growth option.

Now, we consider the APR. For expositional simplicity and concreteness, we assume that the first debt has seniority over the second debt, unless otherwise noted. Smith and Warner (1979) document that 90.8% of their sampled covenants contain some restrictions on future debt issuance. As in Black and Cox (1976), at default, the junior debtholders will not get paid at all until the senior debtholders are completely paid off. At the second default threshold Y_2^d, the senior debtholders collect

$$
D_2^f(Y_2^d) = \min\{F_1, (1 - \gamma_A)A_2(Y_2^d)\}, \quad (46)
$$

where F_1 is the par value of the first debt and is equal to $F_1 = D_1(Y_1^i)$.

The payoff Function (46) states that either the senior debtholders receive F_1 at T_2^d or collect the total recovery value of the firm $(1 - \gamma_A)A_2(Y_2^d)$ at T_2^d. It is immediate to see that under this seniority structure, the junior debt value at default time T_2^d is given by

$$D_2^s(Y_2^d) = \max\{(1 - \gamma_A)A_2(Y_2^d) - F_1, 0\}. \tag{47}$$

Let F_2 denote the par value of the second debt issued at T_2^i. The second debt is also issued at par, and thus we have $F_2 = D_2^s(Y_2^i)$. Equityholders receives nothing at default, and hence in equilibrium we have $(1 - \gamma_A)A_2(Y_2^d) \leq F_1 + F_2$. Even when the senior debtholders receive par F_1 at default time T_2^d, senior debtholders still prefer that the firm does not default. This is intuitive, because the par value $F_1 < C_1/r$.

Debt seniority structure matters not only for payoffs at default boundaries Y_2^d as in Black and Cox (1976), but also for the real investment and financial-leverage decisions. The costs and benefits of issuing debt depend on the seniority and payoff structures. Moreover, the equityholders' interests and incentives also change over time and after each financing and investment decisions. How equityholders' incentives change over time naturally depends on the debt seniority structure.

5.3 *Pari passu*
Now, we turn to another debt structure, *pari passu,* which requires that debt issued at T_1^i and that issued at T_2^i have equal priority in default at stochastic time T_2^d. The total debt recoveries at default $(Y = Y_2^d)$ are proportional to $(1 - \gamma_A)A_2(Y_2^d)$, the total liquidation value of the firm. Because both types of debt are perpetual, the residual values at the default threshold Y_2^d are thus given by

$$D_2^f(Y_2^d) = \frac{c_1}{c_1 + c_2}(1 - \gamma_A)A_2(Y_2^d), \tag{48}$$

$$D_2^s(Y_2^d) = \frac{c_2}{c_1 + c_2}(1 - \gamma_A)A_2(Y_2^d). \tag{49}$$

Here, we assume that the payments to debtholders are based on the debt values at the second investment time T_2^i. This assumption captures the key feature of the *pari passu* structure, and substantially simplify the analysis.[24]

Equityholders choose c_2 at stochastic time T_2^i to maximize $V_2^s(Y)$ given in Equation (45), the sum of equity value $E_2(Y)$ and newly issued debt value $D_2^s(Y)$. The following implicit function characterizes the optimal

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[24] Under this assumption, we do not need to carry the face values F_1 and F_2 for both classes of debt. A more realistic way to model *pari passu* seniority structure is to make the payment at default proportional to the face values F_1 and F_2.

Table 5
The effects of debt structures

Stage	Production capacity m	Investment threshold Y^i	Default threshold Y^d	Coupon rate C	Leverage Lev	Credit spreads cs (bps)
Panel A. Baseline						
1st	1	0.095	0.030	0.077	43.8%	129
2nd	0.8	0.126	0.048	0.188	62.1%	108
Panel B. APR						
1st	1	0.096	0.031	0.079	48.8%	75
2nd	0.8	0.138	0.044	0.173	54.8%	82
Panel C. *pari passu*						
1st	1	0.096	0.017	0.041	23.7%	120
2nd	0.8	0.115	0.053	0.208	71.1%	151

This table reports results for three commonly used debt restructures: the baseline, APR, and *pari passu*. We set $m_1 = 1$ and $m_2 = 0.8$ for a model with $N = 2$.

coupon c_2 for a given level of the first coupon c_1:

$$
c_2 = -c_1 + \frac{r}{r-\mu}\left(\frac{\beta_2 - 1}{\beta_2}\right)
$$

$$
\frac{1}{h}\left[1 - \frac{\beta_2}{\beta_2 - 1}\left(\frac{\tau^{-1} - \beta_2(1 - \gamma_A + \gamma_A/\tau)}{1 - \beta_2(1 - \gamma_A + \gamma_A/\tau)}\right)\frac{c_1}{c_1 + c_2}\right]^{1/\beta_2} M_2 Y_2^i.
$$

$$(50)$$

5.4 The effects of debt structures

Table 5 reports the effects of debt structures on investment and default decisions, optimal coupon and leverage choices, and the equilibrium credit spread.[25] We compare the results from the three debt structures: APR, *pari passu*, and our baseline case (where new debt is only issued if and only if existing debt is retired and paid back in full.)

First, we note that debt structures have significant implications on the second growth-option exercising timing. The investment threshold $Y_2^i = 0.138$ under the APR, which is significantly larger than $Y_2^i = 0.126$ under our baseline case, which in turn is also significantly larger than $Y_2^i = 0.115$ under the *pari passu* case. This sequencing is consistent with our intuition. As APR offers the strongest protection for the existing debtholders, the debt-overhang problem is most severe, and hence the investment threshold is the largest among the three. *pari passu* gives the most favorable seniority treatment for the new debt, and hence the

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[25] Hackbarth and Mauer (2012) also study the priority-structure choice by considering the trade-off between pari passu and APR.

equityholders are least concerned about the debt-overhang burden causing the investment threshold to be even lower than our baseline case.

Second, the optimal leverage *Lev* and coupon rate *C* also reflect the severity of debt overhang that depends on the underlying debt structure. Intuitively, as APR offers the strongest protection for the existing debt and *pari passu* offers least protection for debt issued in the first stage, the second-stage leverage and coupon rate are the lowest under APR (54.8% and 0.173, respectively) and the highest under *pari passu* (71.1% and 0.208, respectively). Because of the anticipated debt-overhang problem in the second stage, the equityholders choose the first-stage leverage and debt coupon in the reverse order, in that leverage and coupon in stage 1 are the highest under APR (48.8% and 0.079) and the lowest under *pari passu* (23.7% and 0.041).

Third, the protection of the existing debt also implies that credit spread for the risky debt is lowest under APR and highest under *pari passu* in stage 2.

In summary, we find that debt structures have substantial effect on investment timing decisions and leverage through the life cycle of the firm mostly through the important endogenous debt-overhang channel.

6 Conclusions

Our paper provides an integrated framework for thinking about multiple rounds of sequential investments simultaneously with dynamic financing. Our modeling approach is tractable and provides a coherent way to think about a complex, dynamic problem that is at the heart of both investments theory and dynamic financing. Importantly, we show that the firm substantially lower its leverage in order to take advantage of its growth options going forward, and indeed we obtain empirically plausible leverage (around 1/3) with as few as three growth options. Besides providing a natural way to model the life cycle of the firm, our model also highlights the need to distinguish between the residual values of "assets in place" and that of remaining live-growth options upon default.

In addition, we find that debt seniority and debt priority structures have conceptually important and quantitatively significant implications on growth-option exercising and leverage decisions, because different debt structures (e.g., APR versus *pari passu*) have very different endogenous debt-overhang implications.

Finally, mainly for tractability reasons, we have assumed that financing and investment timing decisions coincide and also side-stepped from some important frictions that may influence a firm's financing and investment decisions over its life cycle. We see generalizing our model to separate investment and financing decisions as a natural next step. More broadly, we expect that future research will incorporate

important frictions, such as dynamic corporate liquidity considerations, contractual frictions because of moral hazard, and informational frictions, into a corporate life cycle framework similar to ours. Over the last decade or so, we have seen significant progress in the development of dynamic corporate finance models that can be structurally estimated, such as Hennessy and Whited (2005, 2007). We have also seen a fast growing dynamic financial contracting literature, including DeMarzo and Fishman (2007), DeMarzo and Sannikov (2006), and DeMarzo and others (2012). Our model builds on the classic McDonald-Siegel-Leland contingent-claims real-options framework. We expect to see fruitful cross-overs among the three different but highly complementary approaches from which researchers can draw to further deepen our standing of the dynamics of corporate financial and investment decision making. For example, Asvanunt, Broadie, and Sundaresan (2007) and Bolton, Wang, and Yang (2014) study the effect of liquidity on real option decisions and valuation.

A Appendix

We provide derivations and proofs for various results used in the main text.

A.1 Derivations of Main Results in Section 2
We solve the firm's decision problem and its valuation equations for debt, equity, and firm using backward induction. First, note that the decision problem in the last stage is the standard model analyzed in Leland (1994).

A.2 The Final Stage (Stage N)
A.2.1 Equity pricing. Using the standard valuation argument, we may value equity $E_N(Y)$ using the following ordinary differential equation (ODE):

$$rE_N(Y) = (1-\tau)(M_N Y - C_N) + \mu Y E'_N(Y) + \frac{\sigma^2}{2} Y^2 E''_N(Y), \quad Y \geq Y_N^d, \quad \text{(A.1)}$$

subject to the following conditions at the endogenously chosen default boundary Y_N^d:

$$E_N(Y_N^d) = 0, \quad \text{(A.2)}$$

$$E'_N(Y_N^d) = 0. \quad \text{(A.3)}$$

The value-matching (A.2) states that equity value is zero when equityholders default. The smooth-pasting (A.3) implies that equityholders optimally choose the default boundary Y_N^d. Moreover, the default option is completely out of money when Y approaches ∞.

A.2.2 Debt pricing. Similarly, using the standard valuation argument, we may value debt $D_N(Y)$ using the following ODE:

$$rD_N(Y) = C_N + \mu Y D'_N(Y) + \frac{\sigma^2}{2} Y^2 D''_N(Y), \quad Y_N^d \leq Y, \quad \text{(A.4)}$$

subject to the following conditions:

$$D_N(Y_N^d) = L_N(Y_N^d), \tag{A.5}$$

$$\lim_{Y \to \infty} D_N(Y) = \frac{C_N}{r}. \tag{A.6}$$

A.3 Intermediate Stages (Stage $(N-1)$ to Stage 1)

Now consider the firm's intermediate stage n, where $1 < n < N$. Given the default threshold Y_n^d and the investment threshold Y_{n+1}^i, we may write down equity value as in Equation (19), using $\Phi_n^i(Y)$ and $\Phi_n^d(Y)$, which are the present discounted value of receiving a unit payoff contingent on the $(n+1)$-th growth option exercised before the firm defaults at T_n^d or not, respectively. Formally,

$$\Phi_n^i(Y) = \mathbb{E}_t[e^{-r(T_{n+1}^i - t)} \mathbf{1}_{T_n^d > T_{n+1}^i}], \quad \text{and} \quad \Phi_n^d(Y) = \mathbb{E}_t[e^{-r(T_n^d - t)} \mathbf{1}_{T_n^d < T_{n+1}^i}], \tag{A.7}$$

where $\mathbf{1}_{T_n^d > T_{n+1}^i}$ and $\mathbf{1}_{T_n^d < T_{n+1}^i}$ are the indicator functions. If $T_n^d > T_{n+1}^i$, we have $\mathbf{1}_{T_n^d > T_{n+1}^i} = 1$. Otherwise, $\mathbf{1}_{T_n^d > T_{n+1}^i} = 0$. It is important to see that $\Phi_n^i(Y_n^d) = \Phi_n^i(Y_{n+1}^i) = 1, \Phi_n^d(Y_{n+1}^i) = \Phi_n^i(Y_n^d) = 0$, and $\Phi_n^d(Y) > 0$, $\Phi_n^i(Y) > 0$, for $Y_n^d < Y < Y_{n+1}^i$.

Using the equity value Formula (19) and the firm value Formula (24), we have

$$YE_n'(Y) = A_n(Y) + e_n^i \Phi_n^{i\prime}(Y)Y + e_n^d \Phi_n^{d\prime}(Y)Y, \tag{A.8}$$

$$YV_{n+1}'(Y) = A_{n+1}(Y) + v_{n+1}^i \Phi_{n+1}^{i\prime}(Y)Y + e_{n+1}^d \Phi_{n+1}^{d\prime}(Y)Y. \tag{A.9}$$

Applying the smooth pasting condition $E_n'(Y_{n+1}^i) = V_{n+1}'(Y_{n+1}^i)$ to (A.8) and (A.9) gives

$$\left(\frac{1-\tau}{r-\mu}\right) m_{n+1} Y_{n+1}^i = \beta_1 (Y_{n+1}^i)^{\beta_1} \left[\frac{e_n^i(Y_n^d)^{\beta_2} - e_n^d(Y_{n+1}^i)^{\beta_2}}{\Delta_n} - \frac{v_{n+1}^i(Y_{n+1}^d)^{\beta_2} - v_{n+1}^d(Y_{n+2}^i)^{\beta_2}}{\Delta_{n+1}} \right]$$
$$- \beta_2 (Y_{n+1}^i)^{\beta_2} \left[\frac{e_n^i(Y_n^d)^{\beta_1} - e_n^d(Y_{n+1}^i)^{\beta_1}}{\Delta_n} - \frac{v_{n+1}^i(Y_{n+1}^d)^{\beta_1} - v_{n+1}^d(Y_{n+2}^i)^{\beta_1}}{\Delta_{n+1}} \right]. \tag{A.10}$$

Similarly, the smooth pasting condition $E_n'(Y_n^d) = 0$ gives

$$0 = A_n(Y_n^d) + \beta_1 (Y_n^d)^{\beta_1} \frac{e_n^i(Y_n^d)^{\beta_2} - e_n^d(Y_{n+1}^i)^{\beta_2}}{\Delta_n} - \beta_2 (Y_n^d)^{\beta_2} \frac{e_n^i(Y_n^d)^{\beta_1} - e_n^d(Y_{n+1}^i)^{\beta_1}}{\Delta_n}. \tag{A.11}$$

For given investment threshold Y_{n+1}^i and default threshold Y_n^d in stage n, debt value D_n (Y) solves the following ODE:

$$rD_n(Y) = C_n + \mu Y D_n'(Y) + \frac{\sigma^2}{2} Y^2 D_n''(Y), \quad Y_n^d \le Y \le Y_{n+1}^i, \tag{A.12}$$

subject the following boundary conditions:

$$D_n(Y_n^d) = L_n(Y_n^d), \tag{A.13}$$

$$D_n(Y_{n+1}^i) = F_n. \tag{A.14}$$

Using the ODE (A.12) for debt value $D_n(Y)$ and the corresponding boundary conditions (A.13) and (A.14), we have the debt value is given by:

$$D_n(Y) = \frac{C_n}{r} - \left(\frac{C_n}{r} - F_n\right)\Phi_n^i(Y) - \left(\frac{C_n}{r} - L_n(Y_n^d)\right)\Phi_n^d(Y), \qquad Y_n^d \le Y \le Y_{n+1}^i. \qquad (A.15)$$

Intuitively, debt value is given by the risk-free debt value C_n/r, minus the present value of the discount when debt is called back at T_{n+1}^i, and minus the present value of the loss when the firm defaults at T_n^d. Because debt is priced at par F_n at issuance time T_n^i, using the debt pricing Formula (A.15), we have the par value is given by Equation (23). And then using Equation (23), debt value $D_n(Y)$ is then given by Equation (22).

Now turn to the firm's decision making in stage 0. Substituting the conjectured equity value Equation (30) into the ODE Equation (27) and applying the endogenous default boundary conditions (28) and (29) give the following implicit equation for the first investment threshold Y_1^i:

$$A_1(Y_1^i) = \frac{\beta_1}{\beta_1 - 1}\left[I_1 - \frac{\tau C_1}{r} + \frac{\Phi_1^{i\prime}(Y_1^i)Y_1^i - \beta_1\Phi_1^i(Y_1^i)}{\beta_1}v_1^i + \frac{\Phi_1^{d\prime}(Y_1^i)Y_1^i - \beta_1\Phi_1^d(Y_1^i)}{\beta_1}v_1^d\right].$$

$$(A.16)$$

Simplifying the Equation (A.16) gives Equation (31). Note that we may also obtain the same results directly using the general Formulation (A.10) with the following properties: $Y_0^d = 0$, $e_0^d = 0$, $e_0^i = V_1(Y_1^i) - I_1$, and $\Delta_0 = (Y_0^d)^{-\beta_2}(Y_1^i)^{\beta_1}$.

A.4 Proof of Lemmas 1 and 2

Following the standard real-option analysis (e.g., McDonald and Siegel 1986), the value of the growth option, $G_k(Y)$, for a stand-alone investment opportunity with one-time exercise cost I_k that generates cash flow $(1 - \tau)m_k Y$, solves the following ODE:

$$rG_k(Y) = \mu Y G_k'(Y) + \frac{\sigma^2}{2}Y^2 G_k''(Y), \qquad Y \le Y_k^{ae}, \qquad (A.17)$$

subject to the following boundary conditions

$$G_k(Y_k^{ae}) = \frac{(1 - \tau)m_k Y_k^{ae}}{r - \mu} - I_k, \qquad (A.18)$$

$$G_k'(Y_k^{ae}) = \frac{(1 - \tau)m_k}{r - \mu}. \qquad (A.19)$$

In addition, we have the absorbing barrier condition $G_k(0) = 0$ because Y is a GBM process. Using the standard guess-and-verify procedure, we obtain the option value Formula (4) for $G_k(Y)$ for $Y \le Y_k^{ae}$, and the growth-option exercise threshold Y_k^{ae} given in Equation (5).

When the firm is all-equity financed and holds a sequence of decreasingly attractive growth option, the exercising decisions of each option is independent of the exercising decisions of other options. This is a robust result under all-equity financing. Therefore, when $m_1/I_1 > m_2/I_2 > \ldots > m_N/I_N$ holds, all-equity-financed firm value is given by the sum of assets in place and unexercised growth options. That is, in stage n, the firm has n existing assets in place valued at $A_n(Y)$ and $N - n$ unexercised growth options. Each growth option is valued at $G_k(Y)$ with exercising cost I_k and cash-flow multiple m_k. Total firm value is then given by Equation (35). Note that the growth options are ordered sequentially from the most attractive (1st growth option) to the least attractive is without loss of generality. See the main text for discussions on how we may redefine the growth options if the preceding growth option is more attractive. (For example, if $m_2/I_2 \ge m_1/I_1$, we can combine the first

two growth options and relabel the option with exercising cost $I_1 + I_2$ and cash-flow multiple $m_1 + m_2$.)

A.5 Proof of Proposition 1

When the firm has only one growth option, by definition, the exercise threshold for the second growth option is infinite, in that $Y_2^i = \infty$. Therefore, Equation (16) and Equation (17) imply $\Phi_1^i(Y) = 0$ and $\Phi_1^d(Y) = (Y/Y_1^d)^{\beta_2}$, for $Y \geq Y_1^d$. Since the firm only has one growth option, we have

$$v_1^d = L_1(Y_1^d) - \left(A_1(Y_1^d) + \frac{\tau C_1}{r}\right) = -\left(\gamma_A A_1(Y_1^d) + \frac{\tau C_1}{r}\right). \tag{A.20}$$

Equation (31) for the first investment threshold Y_1^i thus implies

$$Y_1^i = \frac{1}{1-\tau} \frac{r-\mu}{m_1} \frac{\beta_1}{\beta_1-1}\left[\left(I_1 - \frac{\tau C_1}{r}\right) + \frac{\beta_1 - \beta_2}{\beta_1}\left(\gamma_A A_1(Y_1^d) + \frac{\tau C_1}{r}\right)\left(\frac{Y_1^i}{Y_1^d}\right)^{\beta_2}\right]. \tag{A.21}$$

The optimal coupon policy is given by

$$C_1 = \frac{r}{r-\mu} \frac{\beta_2 - 1}{\beta_2} \frac{1}{h} m_1 Y_1^i. \tag{A.22}$$

Rearranging and simplifying Equation (A.21) gives the following implicit equation for the investment threshold:

$$(\beta_1 - 1)A_1(Y_1^i) = \beta_1 I_1 - \beta_1 \frac{\tau C_1}{r} + (\beta_1 - \beta_2)\frac{C_1}{r}\left[\gamma_A(1-\tau)\left(\frac{\beta_2}{\beta_2 - 1}\right) + \tau\right]$$

$$= \beta_1 I_1 - \beta_1 \frac{\tau C_1}{r} + (\beta_1 - \beta_2)\frac{\tau C_1}{r}\left(\frac{h^{-\beta_2}}{1 - \beta_2}\right)h^{\beta_2}, h^{\beta_2}, \tag{A.23}$$

$$= \beta_1 I_1 - (\beta_1 - 1)\tau\frac{1}{h}\frac{m_1 Y_1^i}{r-\mu},$$

where the first, second, and third line uses the explicit formulae for Y_1^d as a function of C_1 given in Equation (9) (when stage 1 is the last stage (i.e. $N = 1$)), h given in Equation (33), and coupon C_1 given in Equation (A.22), respectively. Finally, re-arranging the last expression gives Y_1^i in Equation (37). Substituting Equation (37) into Equation (A.22) gives the coupon policy Equation (38) and the default threshold $Y_1^d = Y_1^i/h$.

If the initial value Y_0 is below the investment threshold Y_1^i given in Equation (37), the firm will wait before investing. Equity value before investment $E_0(Y)$ is given by Equation (30).

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