Tokenomics: Dynamic Adoption and Valuation

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We develop a dynamic asset pricing model of cryptocurrencies/tokens that allow users to conduct peer-to-peer transactions on digital platforms. The equilibrium price of tokens is determined by aggregating heterogeneous users' transactional demand, rather than discounting cash flows as is done in standard valuations models. Endogenous platform adoption builds on user network externality and exhibits an *S*-curve: it starts slow, becomes volatile, and eventually tapers off. The introduction of tokens lowers users' transaction costs on the platform by allowing users to capitalize on platform growth. The resultant intertemporal feedback between user adoption and token price accelerates adoption and dampens user-base volatility. (*JEL* E42, G12, L86)

Blockchain-based applications and cryptocurrencies have recently taken a center stage among technological breakthroughs in FinTech. The global market capitalization of cryptocurrencies has grown to hundreds of billions of U.S. dollars. Nevertheless, academics, practitioners, and regulators have divergent views on how cryptocurrencies derive value.

In this paper, we provide a fundamentals-based dynamic valuation model for cryptocurrencies and platform tokens (generically referred to as tokens). We focus on the endogenous formation of a digital marketplace or network ("the platform") where its native cryptocurrency or token settles transactions and derives its value from underlying economic activities on the platform. We focus on platform-specific tokens rather than cryptocurrencies as a general-purpose medium of exchange. In contrast to financial assets whose values depend on cash flows, tokens derive value by enabling users to conduct economic transactions on the digital platform, making them a hybrid of money and investable assets.

Our model captures two key features shared by a majority of tokens. First, they are the means of payment on platforms that support specific economic transactions. For example, Filecoin is a digital marketplace that allows users to exchange data storage space for its tokens (FIL). Another example is basic attention token (BAT): advertisers use BATs to pay for their ads, publishers receive BATs for hosting these ads, and web browser users are rewarded BATs for viewing these ads. Second, user adoption exhibits network effects. In both examples, the more users the platform has, the easier it is for any user to find a transaction counterparty, and the more useful the tokens are. Our model also applies to tokens used on centralized platforms, such as those being developed by platform businesses.[1]

Consequently, the market price of tokens and the platform user base (i.e., the total number of platform users) naturally arise as two key endogenous variables in our model. Our equilibrium token pricing formula exhibits three desirable features. First, token value depends on the platform's productivity, which captures the platform's functionality and other related factors (e.g., technology and regulatory environment). Second, the user base enters positively into the pricing formula, capturing the positive network externality of user adoption. Third, user heterogeneity matters for both platform adoption and token pricing.

Moreover, we clarify the roles of tokens in platform adoption by comparing token-based platforms and platforms that settle transactions with the numeraire consumption goods. Using the numeraire goods as means of payment incurs a carry cost, the forgone return from investing in financial assets. In contrast, introducing tokens encourages early adoption of productive platforms, because agents expect token price appreciation and, thus, effectively face a lower carry

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[1] Examples include online social networks (e.g., QQ coins on Tencent's messaging platform) and online games (e.g., Linden dollar for *Second Life* and WoW Gold for *World of Warcraft*).

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cost when using tokens for transactions. The expectation of future token price change also stabilizes user adoption in the presence of platform productivity shocks.

Specifically, we consider a continuous-time economy with a continuum of agents who differ in their transaction needs (i.e., types) on the platform. We broadly interpret transactions as including value transfers (e.g., Filecoin and BAT) and smart contracting (e.g., Ethereum). Accordingly, we model an agent's utility flow from platform transactions as a function of platform productivity that evolves exogenously, her type, the user base, and the real (numeraire good) value of the agent's token holdings ("real token balance"). Importantly, this flow utility increases with the user base, capturing the network effects. We characterize the Markov equilibrium with platform productivity being the state variable. The model has two key endogenous variables, the user base and token price.

In our model, agents make a two-step decision on (1) whether to adopt by paying a participation cost to become a platform user, and, if so, (2) the real token balance. The token market clears by equating the agents' demand and the fixed supply. A key insight of our model is that users' adoption decision exhibits not only static complementarity through the flow utility of platform transactions but also an *intertemporal* complementarity via the carry cost of holding tokens that depends on the endogenous variation of token price.

Holding tokens as means of payment incurs a carry cost that is the forgone return from investing in financial assets. However, on a promising platform with a positive productivity drift, such cost is partly offset by the expected token price appreciation. Given a fixed supply of tokens, the prospective growth of user base driven by productivity growth leads agents to expect more users in the future and thus a stronger demand for tokens, which implies an increase of token price. As such, even though users forgo the financial assets' returns by holding tokens to transact, they are compensated by the expected appreciation of tokens.

The introduction of tokens also stabilizes the user base, making it less sensitive to platform productivity shocks. Consider a platform with growing yet stochastic productivity. A negative productivity shock reduces users' transactional utility and thus lowers the user base. This negative effect is mitigated by an increase in the expected token price appreciation. A lower current level of adoption implies a larger expected token price appreciation because more users can be brought onto the platform in the future. Consequently, the effective token carry cost declines, encouraging adoption. Similarly, a positive productivity shock directly increases adoption, but its effect is dampened because the pool of potential newcomers shrinks and the expected token price appreciation declines, resulting in a greater effective carry cost.

In the Markov equilibrium, our tokenized economy features an *S*-curve of user adoption: as the platform productivity grows, the user base slowly expands, and then expansion speeds up, before eventually tapering off near full adoption.

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We show that the user base is larger and more stable than that of a tokenless platform that has the same productivity process but uses the numeraire goods as means of payment.

We derive an equilibrium token pricing formula that incorporates endogenous network effects in an otherwise canonical Gordon growth formula. Token valuation boils down to solving an ordinary differential equation subject to intuitive boundary conditions. Just as tokens affect the user-base dynamics, endogenous adoption is critical for understanding major asset pricing issues surrounding tokens.

First, the network effects imply a large cross-sectional variation in token price among platforms in the early stages of adoption. Second, adoption externality amplifies the impact of productivity shocks on token price, creating "excess volatility" (e.g., Shiller 1981). The amplification is even stronger when we allow the productivity drift to increase with the user base. This is a form of community bootstrapping that practitioners emphasize. Finally, by allowing the productivity beta (systematic risk) to increase with adoption, our model generates an initial rise of token price followed by a decline and eventual stabilization, broadly consistent with the observed "bubbly" price dynamics. These results are in line with evidence in the literature (e.g., Liu and Tsyvinski 2018; Shams 2019).

In sum, our model features rich interactions between financial markets and the real economy: the financial side operates through the endogenous determination of token prices, whereas the real side manifests itself in the user adoption. By trading in the token market, users profit from platform growth and effectively bear a reduced carry cost of conducting transacting.[2] For payment platforms, the prominent adoption problem in platform economics (e.g., Rochet and Tirole 2006) is naturally connected with the carry cost in the classical models of money as a transaction medium (e.g., Baumol 1952; Tobin 1956). Compared with traditional user subsidies, we demonstrate the advantages of tokens in accelerating and smoothing adoption. This new solution is often based on blockchains: decentralized consensus allows the token supply to be credibly fixed and, thus, anchors the token price to users' demand.[3]

Among early economic studies on blockchain games and consensus generation mechanisms, Biais et al. (2019) and Saleh (2020) analyze mining/minting games in Proof-of-Work- and Proof-of-Stake-based public blockchains; Easley et al. (2019), Huberman, Leshno, and Moallemi (2019), and Cong, He, and Li (2020) study the market structure and industrial organization of miners; Cong and He (2018) examine the impact of decentralized consensus

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[2] Our model has complete information. Tokens capitalize on the otherwise nontradable growth of the platform's user base and thereby accelerate adoption under network effects. This is distinct from the informational effects of financial markets that feed back into real activities (see Bond, Edmans, and Goldstein 2012).

[3] As pointed out by Hinzen, John, and Saleh (2020), decentralized consensus often comes at the cost of payment delays for proof-of-work blockchains. Computer scientists and economists are actively studying alternative protocols (e.g., Fanti, Kogan, and Viswanath 2019; Saleh 2020).

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on industrial organization; and Cao, Cong, and Yang (2018) and Cao et al. (2020) analyze blockchain architecture for financial reporting without using tokens. We differ by focusing on token users' trade-off and the resultant dynamic interaction between platform adoption and token pricing that apply to both centralized and decentralized platforms.

We also connect platform economics and asset pricing by demonstrating that the token price is anchored to the platform-specific convenience yield (transactional value). Our treatment of convenience yield is related to Krishnamurthy and Vissing-Jorgensen (2012) and earlier studies. Our contribution is to incorporate user network effects in the convenience yield and study the implications on platform adoption and token pricing. We emphasize agent heterogeneity and its asset pricing implications. Instead of balance sheet crises (e.g., Brunnermeier and Sannikov 2014; He and Krishnamurthy 2011, 2013), we model the heterogeneity of platform users and the resultant endogenous growth of user base. In our model, platform users expect profits from selling tokens to future users. This mechanism is related to, but different from, that in Harrison and Kreps (1978) and Scheinkman and Xiong (2003), as the expected capital gain in our model is due to the growth of user base rather than heterogeneous beliefs.

We do not analyze the implications of blockchain technology on general purpose currencies and monetary policies (e.g., Balvers and McDonald 2017; Raskin and Yermack 2018; Garratt and Wallace 2018; Schilling and Uhlig 2019). Instead, we focus on the endogenous interaction between token pricing and user adoption on platforms that serve niche markets with time-varying productivity. Therefore, token pricing is anchored to the platform's productivity and the popularity of specific economic transactions it supports, such as advertisement (BAT) and digital storage (Filecoin).

Among contemporary theories featuring token valuation in static settings, Sockin and Xiong (2020) study tokens as indivisible membership certificates for agents to match and trade with each other; Li and Mann (2020) argue that initial token offering allows agents to coordinate by costly signaling through token acquisition; Pagnotta and Buraschi (2018) study Bitcoin pricing on exogenous user networks; Catalini and Gans (2019) examine developers' pricing of tokens to fund projects and aggregate information; Chod and Lyandres (2018) contrast security token offerings with traditional financing.[4] Our paper is the first to clarify the role of tokens in capitalizing the endogenous platform growth, and thereby, reducing agents' effective carry cost of holding the transaction medium.[5] Empirically, besides the corroborating evidence in Liu and Tsyvinski

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[4] Tokens in our model facilitate transactions. They should be distinguished from security tokens that represent claims on issuers' cash flows or rights to redeem products and/or services (e.g., Slice and Siafund tokens).

[5] The carry cost prominently features in classical models of money demand (e.g., Baumol 1952; Tobin 1956) and recent literature of cash management (e.g., Alvarez and Lippi 2013; Bolton, Chen, and Wang 2011; Décamps et al. 2011; Li 2018; Lucas and Nicolini 2015).

(2018) and Shams (2019), the findings in Liu, Sheng, and Wang (2020) that technological fundamentals affect cryptocurrency valuations are consistent with our model too.

In dynamic settings, Athey et al. (2016) emphasize the role of learning in agents' decisions to use Bitcoin absent stochastic platform productivity and user network externality; Biais et al. (2018) emphasize the fundamental value of Bitcoin from transactional benefits; Fanti, Kogan, and Viswanath (2019) provide a valuation framework for Proof-of-Stake (PoS) payment systems; and Goldstein, Gupta, and Sverchkov (2019) study initial token offerings that allow monopolistic platforms to credibly commit to long-run competitive pricing of services. We differ by studying the joint determination of user adoption and token valuation in a framework that highlights user heterogeneity, network externalities, and, most importantly, intertemporal feedback effects. Moreover, our model is applicable to platforms owned by trusted third parties and permissioned blockchains as well as permissionless blockchains. Cong, Li, and Wang (2020) builds on the framework in this paper but focuses on platform financing, endogenous productivity growth, and dynamic token allocation among founding entrepreneurs, users, and resource contributors.

1. A Model of Platform Economy

Consider a continuous-time economy where a continuum of agents of unit measure conduct peer-to-peer transactions on a blockchain platform or a general digital marketplace.

1.1 Platform and agents

The platform is characterized by A_t, the productivity that evolves according to a geometric Brownian motion:

$$\frac{dA_t}{A_t} = \widehat{\mu}^A dt + \sigma^A d\widehat{Z}_t^A, \tag{1}$$

where $\widehat{Z}_t^A$ is a standard Brownian motion under the physical measure and $\widehat{\mu}^A$ and σ^A are constant parameters. We interpret A_t broadly. A positive shock to A_t can reflect technological advances, favorable regulatory changes, growing users' interests, and increasing variety of activities feasible on the platform.

1.1.1 Preferences for platform transactions. The platform allows agents to conduct transactions that are settled via a medium of exchange. We consider two cases for the medium of exchange: the generic good, which is the numeraire, and the local platform currency (token). We first lay out the common features and then compare these two cases.

We use $x_{i,t}$ to denote the value of agent i's holdings of medium of exchange in units of the numeraire good. Conditioning on participating on the platform,

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a platform user i derives a utility flow from her holdings of the medium of exchange, $x_{i,t}$,

$$dv_{i,t} = (x_{i,t})^{1-\alpha} (N_t A_t e^{u_i})^{\alpha} dt - \phi dt - x_{i,t} r dt. \tag{2}$$

Next, we explain the three terms in (2) and later discuss agents' participation decision.

The first term,

$$x_{i,t}^{1-\alpha} (N_t A_t e^{u_i})^{\alpha} dt, \tag{3}$$

is the transactional benefits of $x_{i,t}$, where N_t is the platform user base, u_i is the agent's type, A_t measures platform productivity, and $\alpha \in (0,1)$ is a constant.[6] Let $\mathcal{U}_t$ denote the set of participating agents (with $x_{i,t} > 0$), so formally, $N_t \in [0,1]$ is the measure of $\mathcal{U}_t$. We choose this specification of utility flow with the following considerations. First, the utility flow increases in N_t. It captures the positive user network effect, as it is easier to find a transaction counterparty in a larger community. Second, the marginal utility decreases with $x_{i,t}$, captured by $\alpha > 0$. The exponents of $x_{i,t}$ and $N_t A_t e^{u_i}$ sum to one for analytical convenience. Third, agents' transaction needs (or types), u_i, are heterogeneous.[7] Let $G(u)$ and $g(u)$ denote the cross-sectional cumulative distribution function and density function of u_i, respectively. Both are continuously differentiable over the finite support $[\underline{U}, \overline{U}]$.

To realize the transaction benefits given in (3), the agent needs to incur a flow cost ϕ per unit of time for platform participation corresponding to the second term in (2). For example, transacting on the platform requires effort and attention. At any time t, agents may choose not to participate and then collect no utility. Naturally, agents with sufficiently high u_i choose to join the platform, while agents with sufficiently low u_i do not participate.

In addition to ϕ, the agent has to incur an opportunity cost of $r x_{i,t}$ per unit of time to realize the transactional benefits in (3). This cost is similar to those induced by the cash-in-advance constraints in monetary models where the opportunity costs of conducting transactions is the forgone interest payments on the money balance (e.g., Galí 2015; Walsh 2003). It also resembles the carry cost of cash holdings in corporate finance models that emphasize external financing frictions (e.g., Bolton, Chen, and Wang 2011; Li 2018).

1.1.2 Valuing utilities from platform transactions.
Because we consider a dynamic economy of infinite horizon, the valuation of flow utilities from platform activities requires a stochastic discount factor (SDF). For tractability,

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[6] Appendix A provides a theoretical foundation for this reduced-form flow utility.

[7] For payment blockchains (e.g., Ripple), a high value of u_i can reflect agent i's urge to conduct an international remittance. For smart-contracting blockchains (e.g., Ethereum), u_i captures the productivity of agent i's project that is being financed. For decentralized computation (e.g., Dfinity) and data storage (e.g., Filecoin) applications, u_i corresponds to users' need for secure and fast access to computing power and data.

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we consider the following widely used specification of SDF, which we denote by Λ:

$$\frac{d\Lambda_t}{\Lambda_t} = -r\,dt - \eta\,d\widehat{Z}_t^\Lambda, \tag{4}$$

where r is the risk-free rate and η is the price of risk for systematic shock $\widehat{Z}_t^\Lambda$ under the physical measure.[8] The SDF is typically linked to agents' consumption dynamics, so by specifying an exogenous SDF, we assume that agents' utility from platform activities constitutes only a small part of their total utility. This echoes the models of an individual firm's valuation where an exogenous SDF is directly parameterized (chap. 7, Campbell 2017).

Let ρ denote the instantaneous correlation between the SDF shock and the platform productivity shock A_t. A positive ρ implies a positive beta. Indeed, the usefulness and quality of a particular platform evolves with the economy, as agents discover new ways to utilize the technology, which in turn depends on the progress of complementary technologies. Macroeconomic and regulatory events may also affect the usage of a platform.

It is more convenient to solve our model via the change-of-measure technique widely used in the derivatives pricing literature (Duffie 2001). By Girsanov's theorem, we know that under the risk-neutral measure, the Brownian motion driving the SDF is Z_t^Λ that satisfies

$$dZ_t^\Lambda = d\widehat{Z}_t^\Lambda + \eta\,dt. \tag{5}$$

Therefore, under the risk-neutral measure, A_t follows

$$\frac{dA_t}{A_t} = \widehat{\mu}^A\,dt + \sigma^A\,d\widehat{Z}_t^A = \left(\widehat{\mu}^A - \eta\rho\sigma^A\right)dt + \sigma^A\,dZ_t^A \equiv \mu^A\,dt + \sigma^A\,dZ_t^A. \tag{6}$$

Let $y_{i,t}$ denote agent i's (undiscounted) *cumulative* payoff from platform activities, which depends on $dv_{i,t}$ (the transactional benefits defined in (2)) and may differ on platforms with and without token as a medium of exchange as will be specified shortly. Let $\widehat{\mathbb{E}}$ and $\mathbb{E}$ denote the expectation under the physical and risk-neutral measure, respectively. Agent i maximizes her life-time payoff,

$$\widehat{\mathbb{E}}\left[\int_0^\infty \frac{\Lambda_t}{\Lambda_0}\,dy_{i,t}\right] = \mathbb{E}\left[\int_0^\infty e^{-rt}\,dy_{i,t}\right]. \tag{7}$$

This equality follows from the change-of-measure technique. Throughout the remainder of our paper, we conduct our analysis under the risk-neutral measure, unless stated otherwise.

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[8] The standard no-arbitrage argument implies that the drift of the SDF has to equal the negative interest rate. See Duffie (2001) for a textbook treatment.

1.2 Tokenless and tokenized economies

1.2.1 Tokenless economy. On platforms with transactions settled by the numeraire good, agent i's utility flow only depends on whether she chooses to be a platform user, and if she does, the transactional benefits of holding $x_{i,t}$ units of goods as means of payment:

$$dy_{i,t} = \max\left\{0, \max_{x_{i,t}} dv_{i,t}\right\} \tag{8}$$

$$= \max\left\{0, \max_{x_{i,t}} \left[(x_{i,t})^{1-\alpha}\left(N_t A_t e^{u_i}\right)^{\alpha} - \phi - r x_{i,t}\right]\right\} dt. \tag{9}$$

Here, the inner "max" operator gives the conditional demand $x_{i,t}$ if participating and the outer "max" operator reflects agent i's option to leave the platform and obtain zero profit.

This tokenless model is analogous to the standard models of money holdings (e.g., Ljungqvist and Sargent 2004), given that the real value of money balance, $x_{i,t}$, generates a flow utility via $(x_{i,t})^{1-\alpha}(N_t A_t e^{u_i})^{\alpha} dt$ but induces a cost of forgone interests via $x_{i,t} r dt$. Our novel modeling ingredient is the user network effect—the positive externality of agents' adoption decision—which is a defining feature of digital platforms.

1.2.2 Tokenized economy. In what follows, we introduce a native (crypto)currency on the platform, that is, the token. To conduct transactions on the platform, we require users to use tokens.[9]

The value of agent i's token holdings, $x_{i,t}$, satisfies the following identity:

$$x_{i,t} = P_t k_{i,t}, \tag{10}$$

where P_t is the token price in terms of the numeraire good and $k_{i,t}$ is the units of tokens. The real (numeraire) value, $x_{i,t} = P_t k_{i,t}$, rather than the token units, $k_{i,t}$, appears in the transactional benefits because the transaction utility depends on the numeraire value of goods and services that are transacted as in the standard monetary economic models.

Without loss of generality, we write the equilibrium token price process as follows under the risk-neutral measure:

$$dP_t = \widetilde{\mu}_t dt + \widetilde{\sigma}_t dZ_t^A \tag{11}$$

$$= P_t \mu_t^P dt + P_t \sigma_t^P dZ_t^A. \tag{12}$$

Here, $\widetilde{\mu}_t$ and $\widetilde{\sigma}_t$ can be any admissible stochastic processes. (12) is a rewriting of (11), where

$$\widetilde{\mu}_t = P_t \mu_t^P \qquad \text{and} \qquad \widetilde{\sigma}_t = P_t \sigma_t^P, \tag{13}$$

and μ_t^P and σ_t^P can follow any admissible processes.

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[9] In Appendix D, we generalize our model to incorporate agents' endogenous decisions on whether to use the numeraire good or token as a medium of exchange and show that our results are robust.

We choose to work with μ_t^P and σ_t^P rather than $\widetilde{\mu}_t$ and $\widetilde{\sigma}_t$, because the former set of notations is more convenient especially when we analyze our model's asset pricing predictions, such as the expected token return and token return volatility. But we could have used $\widetilde{\mu}_t$ and $\widetilde{\sigma}_t$ to obtain the same solution.[10]

Conditional on participating on the platform, agent i derives a total payoff that includes the transactional benefits of token holdings, that is, $dv_{i,t}$, and also the investment payoff due to endogenous token price change:

$$dv_{i,t} + k_{i,t}\mathbb{E}_t[dP_t]. \tag{14}$$

By substituting $k_{i,t} = x_{i,t}/P_t$ into (14), we rewrite the total payoff:

$$dv_{i,t} + x_{i,t}\frac{\mathbb{E}_t[dP_t]}{P_t}. \tag{15}$$

Using the definition $\mu_t^P dt \equiv \mathbb{E}[dP_t]/P_t$, we can write the total payoff as

$$dy_{i,t} = \max\left\{0, \max_{x_{i,t}}\left[dv_{i,t} + x_{i,t}\mu_t^P dt\right]\right\}, \tag{16}$$

where the outer max accounts for agent i' option to leave the platform and achieve zero profit. In contrast to (8) of the tokenless economy that contains only $dv_{i,t}$ (the transactional benefits), token introduces an additional term $x_{i,t}\mu_t^P dt$ due to the endogenous price variation.

By substituting (2) into (16), we express $dy_{i,t}$ explicitly as follows:

$$dy_{i,t} = \max\left\{0, \max_{x_{i,t}}\left[\left(x_{i,t}\right)^{1-\alpha}\left(N_t A_t e^{u_i}\right)^{\alpha} - \phi - \left(r - \mu_t^P\right)x_{i,t}\right]\right\}dt. \tag{17}$$

Introducing tokens changes the unit carry cost for holding the transaction medium from $r\,dt$ to $\left(r - \mu_t^P\right)dt$. Agents must hold a medium of exchange (the numeraire good in the tokenless economy, or token in the tokenized economy) for dt to conduct transactions.[11] During this holding period, in addition to incurring the opportunity cost of forgone interests, agents in the tokenized economy are exposed to the endogenous variation of token price. This feature is absent in our tokenless economy.

1.2.3 Markov equilibrium. We study a Markov equilibrium with A_t, the platform productivity, as the state variable whose dynamics generate the rational-expectation agents' information filtration. To focus on the dynamics

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[10] Proposition B1 in Appendix B shows that $P_t > 0$ in equilibrium. Therefore, since $P_t > 0$, we can uniquely infer $\left(\mu_t^P, \sigma_t^P\right)$ from $(\widetilde{\mu}_t, \widetilde{\sigma}_t)$ or infer $(\widetilde{\mu}_t, \widetilde{\sigma}_t)$ from $\left(\mu_t^P, \sigma_t^P\right)$, so these two sets of notations are equivalent.

[11] In blockchain-based systems, the holding period naturally arises because forging the ledger of transactions takes time. For example, consensus on the Bitcoin blockchain requires 10–11 minutes. This confirmation period is necessary for the finality of transactions as shown by Chiu and Koeppl (2017).

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of user adoption and demand, we fix the token supply to a constant M.[12] The market-clearing condition is

$$M = \int_{i \in [0,1]} k_{i,t} \, di, \tag{18}$$

where for those who do not participate, $k_{i,t} = 0$.

Definition 1. A Markov equilibrium with state variable A_t is described by agents' decisions and equilibrium token price such that the token market-clearing condition given by (18) holds and agents optimally decide to participate (or not) and choose token holdings.

2. Solution: Tokenless Economy

In our tokenless economy, the numeraire good is the medium of exchange. Conditional on joining the platform (i.e., $x_{i,t} > 0$), the agent chooses $x_{i,t}$ to maximize (2), which implies:

$$(1-\alpha)\left(\frac{N_t A_t e^{u_i}}{x_{i,t}}\right)^\alpha = r, \tag{19}$$

the marginal benefit from transaction is equal to the carry cost of forgone interests on $x_{i,t}$. Rearranging the optimality condition, we have

$$x_{i,t}^* = N_t A_t e^{u_{i,t}} \left(\frac{1-\alpha}{r}\right)^{1/\alpha} \tag{20}$$

and the maximized profit from participating on platform is then:

$$dv_{i,t} = \left[N_t A_t e^{u_i} \alpha \left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} - \phi\right] dt. \tag{21}$$

An agent participates only when $dv_{i,t}$ given in (21) is positive. That is, agent i participates if and only if $u_i \geq \underline{u}_t^{NT}$, where $\underline{u}_t^{NT}$ is the endogenous threshold given by:

$$\underline{u}_t^{NT} = -\ln\left(N_t^{NT}\right) + \ln\left(\frac{\phi}{A_t \alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r}\right). \tag{22}$$

Here, the superscript "NT" refers to the equilibrium value of the "no-token" (tokenless) economy. Given the distribution of u_i, $G(u_i)$, the user base is thus given by:

$$N_t^{NT} = 1 - G\left(\underline{u}_t^{NT}\right) \tag{23}$$

(22) and (23) jointly determine $\underline{u}_t^{NT}$ and N_t^{NT} as functions of A_t.

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[12] This captures the majority real-world applications. More generally, blockchain technology allows supply to be based on explicit rules. This can be accommodated in the model by adding in exogenous token inflation or deflation rates that are orthogonal to the endogenous adoption and token demand dynamics.

Proposition 1 (Tokenless Equilibrium). In the Markov equilibrium with A_t being the state variable, the user base N_t^{NT} and user type threshold $\underline{u}_t^{NT}$ solve (22) and (23). Additionally, the user base, N_t^{NT}, increases in A_t if the cross-sectional distribution of agent type, $G(u)$, has an increasing hazard rate.

3. Solution: Tokenized Economy

Now consider a tokenized platform. As platform productivity A_t is the state variable, all endogenous variables are functions of A_t in equilibrium. For example, $P_t = P(A_t)$. By applying Itô's lemma to $P_t = P(A_t)$, we obtain:

$$dP_t = dP(A_t) = \left[P'(A_t)A_t\mu^A + \frac{1}{2}P''(A_t)\left(A_t\sigma^A\right)^2 \right]dt + P'(A_t)A_t\sigma^A dZ_t^A. \tag{24}$$

By matching the coefficients of dt and dZ_t^A to (12), we obtain μ_t^P and σ_t^P as functions of A_t:

$$\mu_t^P = \mu^P(A_t) = \frac{P'(A_t)}{P(A_t)}A_t\mu^A + \frac{1}{2}\frac{P''(A_t)}{P(A_t)}\left(A_t\sigma^A\right)^2. \tag{25}$$

and

$$\sigma_t^P = \sigma^P(A_t) = \frac{P'(A_t)}{P(A_t)}A_t\sigma^A. \tag{26}$$

With tokens, agent i's decision on the real balance of medium of exchange, inside the inner max operator in (17), is similar to (19) of the tokenless economy, but with tokens, the effective carry cost is now $r - \mu_t^P$ and the optimality condition agents' token balance is:

$$(1-\alpha)\left(\frac{N_t A_t e^{u_i}}{x_{i,t}^*}\right)^\alpha = r - \mu^P(A_t), \tag{27}$$

where the marginal benefit of transaction is equal to the carry cost.

Rearranging the equation, we have

$$x_{i,t}^* = N_t A_t e^{u_i}\left(\frac{1-\alpha}{r-\mu^P(A_t)}\right)^{\frac{1}{\alpha}}. \tag{28}$$

the maximized profit from participating on platform is then

$$\max_{x_{i,t}}\left\{dv_{i,t} + x_{i,t}\mathbb{E}_t\frac{[dP_t]}{P_t}\right\} = \left[N_t A_t e^{u_i}\alpha\left(\frac{1-\alpha}{r-\mu^P(A_t)}\right)^{\frac{1-\alpha}{\alpha}} - \phi \right]dt. \tag{29}$$

Substituting $x_{i,t}^*$ given in (28) into (17), we obtain the following user-type cutoff threshold:

$$\underline{u}_t = -\ln(N_t) + \ln\left(\frac{\phi}{A_t\alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r-\mu^P(A_t)}\right), \tag{30}$$

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and because only agents with $u_i \geq \underline{u}_t$ will participate, the user base is then given by

$$N_t = 1 - G\left(\underline{u}_t\right). \tag{31}$$

Equations (30) and (31) solve $\underline{u}_t = \underline{u}(A_t)$ and $N_t = N(A_t)$ as functions of A_t.

Next, we derive an equilibrium token pricing formula. By using $k_{i,t} = x_{i,t}/P_t$ and substituting (28) into the market-clearing condition (18), we obtain:

$$P_t = \frac{N(A_t)S(A_t)A_t}{M} \left(\frac{1-\alpha}{r-\mu^P(A_t)}\right)^{\frac{1}{\alpha}}, \tag{32}$$

where

$$S_t \equiv S(A_t) = \int_{\underline{u}(A_t)}^{\overline{U}} e^u dG(u) \tag{33}$$

is the sum of all participating agents' e^{u_i}, measuring the aggregate transaction needs. Industry practices broadly corroborate the pricing formula (32). For example, as proxies for N_t and S_t respectively, daily active addresses (DAA) and daily transaction volume (DTV) are featured prominently in practitioners' token valuation framework.[13] But instead of heuristically aggregating such inputs into a pricing formula, we derive the token pricing formula as equilibrium outcome together with endogenous user adoption.

By substituting (25) into the market-clearing condition (32), we obtain

$$r P(A_t) = P'(A_t)A_t\mu^A + \frac{1}{2}P''(A_t)\left(A_t\sigma^A\right)^2 + (1-\alpha)\left(\frac{N(A_t)S(A_t)A_t}{P(A_t)M}\right)^\alpha P(A_t). \tag{34}$$

While our token pricing equation (34) may appear similar to the Black-Scholes-type differential equation for derivatives pricing, the underlying economic force in our model is different. The "flow" term, $(1-\alpha)\left(\frac{N_t S_t A_t}{P_t M}\right)^\alpha P(A_t)$, in (34) comes from rearranging the market-clearing condition (32) and reflects the aggregation of agents' transactional demand for tokens. The Black-Scholes equation does not feature such a flow term. Moreover, the Black-Scholes equation has a "theta" term—the variation of derivative value over time—because of finite maturity, while (34) does not have such term because token does not have maturity. Finally, platform productivity, A_t, the underlying fundamental that drives token price, is not tradable, so the coefficient on $P'(A_t)$ is $\mu^A A_t$, the drift of A_t under the risk-neutral measure, instead of r in the case of derivatives on *tradable* underlying assets whose risk-adjusted return must be r.[14]

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[13] See, for example, the article on token valuation, *Today's Crypto Asset Valuation Frameworks*, by Ashley Lannquist at World Economic Forum (Blockchain and Digital Currency).

[14] Appendix F2 compares in detail our token pricing equation (34) with the Black-Scholes derivative pricing equation.

The preceding equation is an ordinary differential equation (ODE) for $P(A_t)$, the token price as a function of the state variable A_t. We solve this ODE with the following boundary conditions on $P(A_t)$. The lower boundary is given by:

$$\lim_{A_t \to 0} P(A_t) = 0. \tag{35}$$

As $A_t = 0$ is an absorbing state, the platform is not productive and no agent participates. Therefore, the token price is zero.

When A_t is sufficiently high, all agents participate with probability one at all $s \geq t$. In this case, the following Gordon Growth Formula for token price holds:

$$\lim_{A_t \to \infty} P(A_t) = \overline{P}(A_t) \equiv \frac{\overline{S} A_t}{M} \left(\frac{1-\alpha}{r - \mu^A} \right)^{\frac{1}{\alpha}}, \tag{36}$$

where the aggregate transaction needs, $\overline{S}$, is given by

$$\overline{S} \equiv \int_{\underline{U}}^{\overline{U}} e^u \, dG(u). \tag{37}$$

In our numerical solution, we choose a very large value $\overline{A}$, so that the following value-matching condition hold at $\overline{A}$:[15]

$$P(\overline{A}) = \overline{P}(\overline{A}). \tag{38}$$

Our method of solving continuous-time Markov equilibrium follows Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), and Li (2017).

The next proposition summarizes the equilibrium features of tokenized economy.

Proposition 2 (Tokenized Equilibrium). Under the increasing hazard rate condition for $G(u)$ and other regularity conditions, the Markov equilibrium with A_t as the state variable has the following properties:

1. Token price $P(A_t)$ solves the ODE (34) subject to boundary conditions (35) and (38). μ_t^P and σ_t^P in the token price dynamics (12) are given by (25) and (26), respectively.
2. Given the token price dynamics, agents participate if $u_i \geq \underline{u}_t$, given by (30), and the user base is $N_t = 1 - G(\underline{u}_t)$, which increases in A_t and μ_t^P.
3. Conditional on participation, the value of agents' token holdings $x_{i,t}^*$ is given by (28), which increases in A_t and μ_t^P.

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[15] Additionally, when we perturb $\overline{A}$ by a small amount ϵ, our solution with the new upper bound is numerically indistinguishable from the solution with $\overline{A}$ being the upper bound.

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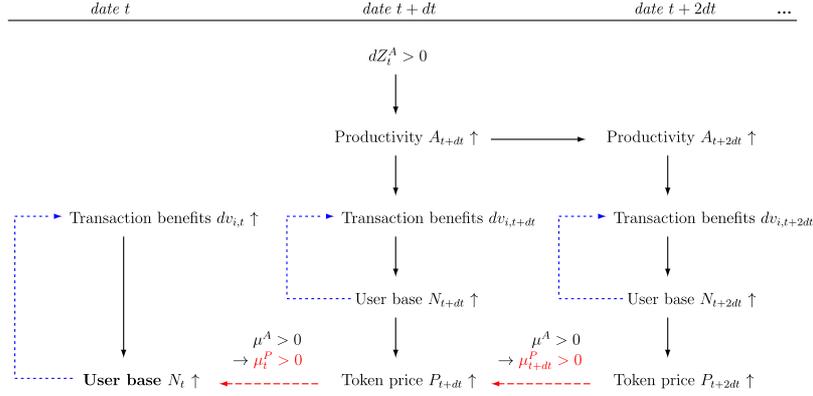
Figure 1
Adoption-accelerating effect of tokens
The black solid arrows point to the increases of the expected future productivity A, which lead to higher transaction benefits of tokens, and in turn, larger user bases N. The blue dotted arrows indicate that increases in user base result in even higher transaction benefits due to the user network externality. Finally, more users drive up the token prices P in future dates, which feed into a current expectation of price appreciation and greater adoption (red dash arrows).

The tokenless and tokenized economies differ in agents' means of payment. As in the cash-in-advance models (e.g., Galí 2015; Walsh 2003), the cost of conducting transactions comprises the forgone interests on the balance of the transaction medium.[16] In the tokenless economy, this carry cost is rdt, the risk-adjusted return of any tradable assets. In the tokenized economy, this carry cost is $(r - \mu_t^P)dt$ due to the expected token return.[17] Agents expect token price to appreciate when they expect higher future productivity (and thus larger user base and stronger token demand).[18] Figure 1 illustrates the intertemporal feedback mechanism from token price dynamics to user adoption.

In sum, tokens accelerate user adoption by capitalizing the expected user-base growth into an expected token return, reducing the effective carry cost. In contrast, the numeraire good as means of payment does not deliver any financial return to offset the carry cost.[19]

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[16] Appendix F1 compares our model setup with those used by standard monetary models.

[17] As shown by the optimality condition (27), tokens deliver a dominated risk-adjusted financial return, $\mu_t^P < r$, but compensation arrives in the form of transactional benefits. Therefore, holding token is still optimal for users.

[18] We note that a predetermined token supply schedule is important. If token supply can arbitrarily increase ex post, then the expected token price appreciation is delinked from the productivity growth and the resultant increase of user base and token demand. Predeterminacy or commitment is credibly achieved through the decentralized consensus mechanism empowered by the blockchain technology. In contrast, traditional monetary policy has a commitment problem (Barro and Gordon 1983).

[19] Admittedly, an expected capital loss, $\mu_t^P < 0$, discourages adoption by increasing the carry cost. In Appendix D, we extend our model: agents can choose between the numeraire good and token as a transaction medium rather than always use either the numeraire good (Section 3) or the token (Section 4). In this more general setting, agents only use tokens when $\mu_t^P > 0$, and, thus, by introducing tokens, platforms accelerate adoption by simply expanding agents' choice set, allowing them to pick the currency with the lowest carry cost.

4. Roles of Tokens

In this section, we first present the planner's solution and then use it as a benchmark to analyze the roles of tokens in the tokenless and tokenized economies.

4.1 The planner's solution

The planner's problem is subject to the same platform transaction technology faced by users in the tokenless economy. However, unlike the individual users, the planner solves the adoption problem by taking into consideration the positive externalities of agents' adoption decisions. The present value of aggregate platform transactional benefits is given by

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-rt}\int_{i\in\mathcal{U}_t} dv_{i,t}di\right], \tag{39}$$

where $\mathbb{E}[\cdot]$ is the expectation under the risk-neutral measure and $dv_{i,t}$ is given in (2).

We solve (39) as follows. First, we show that similar to the tokenless economy, the set of users, $\mathcal{U}_t$, is characterized by a cutoff threshold: $\mathcal{U}_t = \{i : u_{i,t} \geq \underline{u}_t^{PL}\}$, which implies $N_t = 1 - G(\underline{u}_t^{PL})$. This follows from that it is in the planner's interest to bring agents with higher u_i on the platform for any given N_t. Second, solving the socially optimal demand, $x_{i,t}$, boils down to maximizing $dv_{i,t}$ agent-by-agent, yielding the solution given by (20). By aggregating $dv_{i,t}$ over all participating agents, we obtain

$$\int_{i\in\mathcal{U}_t}\left[\alpha N_t A_t e^{u_i}\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} - \phi\right]di = N_t\left[\alpha\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}}A_t\int_{i\in\mathcal{U}_t}e^{u_i}di - \phi\right]. \tag{40}$$

As (40) is linear in N_t, the planner chooses full participation by setting $N_t = 1$, if the platform productivity is sufficiently high, that is, if and only if $A_t > \underline{A}^{PL}$, where

$$\underline{A}^{PL} = \phi\left[\alpha\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}}\overline{S}\right]^{-1} \tag{41}$$

and $\overline{S}$ is given by (37).

4.2 User adoption growth

Figure 2 reports the adoption dynamics from our numerical solutions for the tokenized economy, the tokenless economy, and the planner's problem. The blue solid line in Figure 2 shows that the user base N_t in the tokenized economy is an S-shaped function of $\ln(A_t)$.[20] When the platform's productivity A_t is

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[20] The curve starts at $\ln(A_t) = -48.35$ ($A_t = 1e-21$), a number we choose because it is close to the left boundary, zero. The curve ends at $\ln(A_t) = 18.42$ ($A_t = 1e8$), the touching point between $P(A_t)$ and $\overline{P}(A_t)$.

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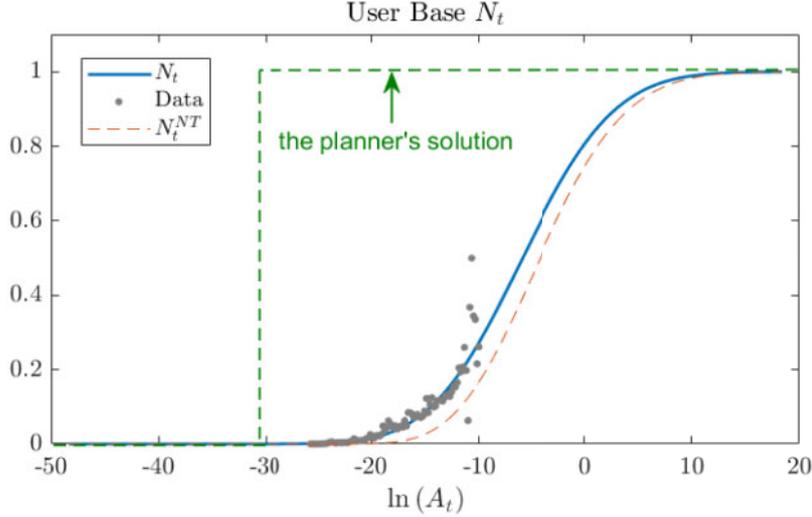
Figure 2
S-**shaped user adoption curve in tokenized and tokenless economies**
This graph shows that the use base, N_t, in the tokenized economy (blue solid curve) and tokenless economy (red dash line) as functions of the logarithmic productivity $\ln(A_t)$. The gray scatterplot is based on data of normalized active user addresses (details in Appendix C). The vertical green dash line marks the level of productivity, beyond which the planner chooses full adoption, and below which the planner chooses zero adoption.

low, the user base N_t barely responds to changes in A_t. In contrast, when A_t is moderately high, N_t responds much more to changes in A_t. The growth of the user base feeds onto itself: the more agents join the ecosystem, the higher transaction surplus each derives. User adoption eventually slows when the pool of newcomers is exhausted. We also plot the scattered data points of active user addresses, our proxy for N_t, that guide our choices of parameter values. In Appendix C, we detail the parameter choices and data sample construction.

By comparing the adoption dynamics in the tokenized and tokenless economies, we see that the tokenized economy has faster adoption than the tokenless economy in Figure 2. Introducing tokens effectively lowers the carry cost from rdt to $(r-\mu_t^P)dt$, because, under the current parameter values, $\mu_t^P > 0$. Tokens are of limited supply and are required for transactions, users expect tokens to appreciate when they expect adoption to grow.

We also plot in Figure 2 the planner's solution via the dash vertical line at $\ln\left(\underline{A}^{PL}\right)$, which is given by (41). Recall that the planner chooses full adoption if $A_t \geq \underline{A}^{PL}$ and zero adoption otherwise. Relative to the planner's solution, a tokenless economy features underadoption as its N_t is below that of the planner's solution (100%) when $A_t \geq \underline{A}^{PL}$.[21] This is because agents do not internalize the positive network externalities of adoption.

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[21] Proposition B2 in Appendix B shows that $\underline{A}^{NT}$, the lowest value of A_t, where $N_t^{NT} > 0$, is below $\underline{A}^{PL}$.

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Introducing tokens lifts the adoption curve relative to that of the tokenless economy but is still below the full adoption level for $A_t \geq \underline{A}^{PL}$. For $A_t < \underline{A}^{PL}$, though not sharply visible in the figure, the tokenized economy features a positive level of adoption while the planner chooses zero adoption. Here by introducing tokens, we change the payment technology that is accessible to platform users. Even for the planner, requiring agents to hold the means of payment incurs a carry cost of $r dt$ per numeraire value, but when tokens are available (and in the numerical solution, $\mu_t^P > 0$), the carry cost is reduced to $(r - \mu_t^P) dt$, and thus, the adoption level is higher.[22]

For tokens to accelerate adoption, there must be a market where tokens are traded and agents can form their expectations of token price change, μ_t^P. Therefore, tokens reduce the carry cost of payment by capitalizing the future growth of user base, and this mechanism does not exist in the planner's economy where agents cannot trade tokens and decide on $x_{i,t}$ voluntarily.

Among the purported reasons for this common practice of introducing tokens, entrepreneurs foremost believe that using tokens can "bootstrap" the community. Heuristically, practitioners have argued that tokens help grow the ecosystem and allow all participants to benefit from the growth prospect of platforms, although no formal analysis has been provided. Our paper exactly examines this argument formally.

4.3 User adoption volatility

In equilibrium, all endogenous variables are functions of A_t, the state variable, including $N_t = N(A_t)$. Therefore, the dynamics of N_t can be written as

$$dN_t = \mu_t^N dt + \sigma_t^N dZ_t^A, \tag{42}$$

where both the drift $\mu_t^N = \mu^N(A_t)$ and volatility $\sigma_t^N = \sigma^N(A_t)$ are functions of A_t.[23] We thus plot σ_t^N against N_t in panel A of Figure 3. Doing this allows us to compare σ_t^N of the two economies at the same stage of adoption N_t.[24] Both curves start and end at zero, consistent with the S-shaped adoption dynamics in Figure 2. A key result is that the tokenized has a lower σ_t^N. The intuition is as follows.

First recall that N_t increases in A_t (through the transactional benefit) and μ_t^P (through the carry cost reduction) as shown in Proposition 2. Therefore, to understand σ_t^N, that is, how N_t responds to shocks, we examine how A_t and μ_t^P respond to shocks. Consider a negative shock, $dZ_t^A < 0$. The platform

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[22] When comparing token-based and traditional payment systems, we focus on users' carry cost of holding means of payment. In reality, the traditional payment systems are operated by third parties, such as banks, and therefore feature other costs (e.g., the systemic risk generated by interbank networks in Denbee et al. (2016)).

[23] Proposition B3 in Appendix B solves σ_t^N for the tokenized and tokenless economies and analytically characterizes how tokens affect the user-base volatility.

[24] The adoption is either zero or full in the planner's solution, so its volatility is not economically interesting.

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A User-Base Volatility σ_t^N **B** Expected Token Return μ_t^P
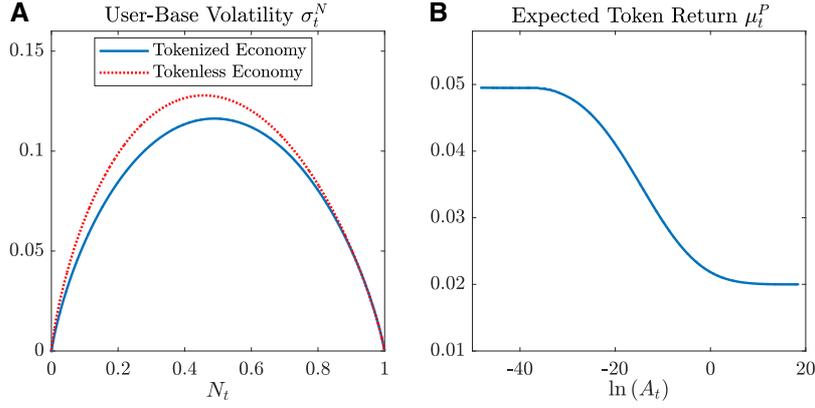
Figure 3
User adoption volatility reduction effect
Panel A plots the volatility of user base, σ_t^N, in the tokenized (blue solid curve) and tokenless (red dotted curve) economies as functions of N_t, respectively. Panel B shows that the expected token return under the risk-neutral measure, μ_t^M, as a function of logarithmic productivity, $\ln(A_t)$. The black dotted line represents the (risk-neutral) expected growth rate of A_t, μ^A.

productivity is thus lower, reducing the transactional benefit and hence N_t. How μ_t^P responds to $dZ_t^A < 0$ depends on the long-run prospect of adoption.

A smaller current user base implies a greater potential of future adoption (i.e., $100\% - N_t$) because under the current parameter choices for our numerical solution, $\mu^A > 0$ and N_t reaches 100% in the long run. Therefore, agents expect a stronger future token demand and a stronger token price appreciation (i.e., an increase of μ_t^P). In sum, while the decrease of A_t reduces N_t, the increase of μ_t^P dampens the reduction, making N_t less responsive to the shock. This buffering effect of μ_t^P is absent in the tokenless economy, so its user-base volatility is higher.

Panel B of Figure 3 shows that μ_t^P declines in $\ln(A_t)$, which generates the previously discussed user-base stabilizing effect of tokens. When A_t (and thus, N_t) is low, token price is expected to increase at a faster rate, reflecting the potential future adoption. As A_t and N_t grow, the pool of agents who have not adopted (i.e., $100\% - N_t$) shrinks, so the expected token appreciation declines.

We can also compare σ_t^N of the tokenized and tokenless economies over different values of A_t instead of N_t. For the same level of A_t, as the tokenized economy has a larger N_t, it is likely to have a larger variation in N_t (i.e., a larger σ_t^N) than the tokenless one. Therefore, instead of comparing σ_t^N, we compare σ_t^N/N_t, the volatilities of user-base growth *rate* (i.e., dN_t/N_t). Proposition B4 in Appendix B2 shows that when the two economies have the same productivity, the tokenized economy has a smaller volatility of user-base growth rate.

This user-base stabilizing effect also holds under $\mu^A < 0$, which implies a long-run adoption level of 0%. When N_t declines due to the decrease of A_t, μ_t^P increases and dampens the reduction of N_t because the potential of token

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demand (and price) to decline further (i.e., $N_t - 0\%$) is smaller. In sum, μ_t^P is a countercyclical force in the tokenized economy: it increases when $dZ_t^A < 0$ and decreases when $dZ^A > 0$, dampening N_t's to shocks.

So far, we have only considered a constant drift of A_t. The user-base stabilizing effect of tokens may not hold under an alternative specification of productivity dynamics. Consider a time-varying drift of A_t that follows a mean-reverting process and loads on a standard Brownian motion shock $dZ_t^{\mu^A}$,

$$d\mu_t^A = \psi \left(\bar{\mu}^A - \mu_t^A \right) dt + \sigma^{\mu^A} dZ_t^{\mu^A}, \tag{43}$$

where $\psi > 0$. Then the Markov equilibrium shall have two state variables, A_t and μ_t^A. Consider the scenario where the realized productivity shock is negative: $dZ_t^A < 0$. As long as $dZ_t^{\mu^A}$ and dZ_t^A are positively correlated, μ_t^A is expected to decline. Foreseeing a slower growth of productivity, user base, and token demand, agents expect a decline of μ_t^P, and thus, become more reluctant to participant. This amplifies the initial negative impact of $dZ_t^A < 0$ on N_t via A_t. Because of the lack of empirical studies on tokenized platforms' fundamentals, our modeling of A_t is guided by parsimony and how the model's adoption curve fits the data (see Appendix C). Our discussion of alternative μ_t^A specifications suggests that whether tokens amplify or dampen the response of N_t to variations of platform fundamentals sheds light on the underlying dynamics of platforms' fundamentals.

4.4 Token price dynamics under endogenous user adoption

In this section, we discuss how endogenous user adoption leads to nonlinear price dynamics that are broadly consistent with empirical observations.

The token price, $P(A_t)$, and user base, $N(A_t)$, are functions of platform productivity, A_t, the state variable. Figure 4 plots the joint dynamics of these two key observables. Token price sharply increases with adoption in the early stages, changes gradually in the intermediate stage, and speeds up again once the user base reaches a sufficiently high level. The two price run-ups in the early and final stages of adoption correspond to the slow user-base growth in these stages relative to token price changes. Consistent with our model's prediction, Liu and Tsyvinski (2018) find that the value of cryptocurrencies is significantly correlated with the growth of user networks.

This figure helps us understand the cross-sectional differences in token pricing. Consider blockchain platforms categorized in term of their adoption stages: early, intermediate, and late. For two blockchain platforms in the early stage, a small difference of N_t between them can generate a very large difference in the market capitalization of tokens ($P_t M$), as seen in Figure 4. Essentially the same result holds in the late stage. In contrast, in the intermediate stage, even a large difference of N_t between the two platforms only yields a small difference of $\ln(P_t)$. Shams (2019) documents that user network externality is a key factor driving the cross-sectional variation in cryptocurrency price dynamics.

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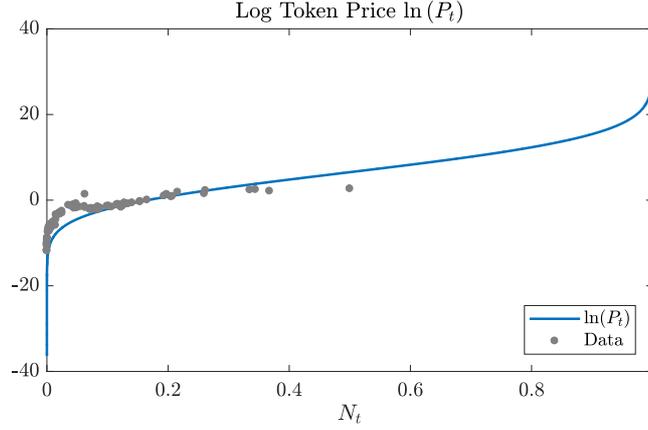
Figure 4
Token price dynamics over adoption stages
This graph plots the log token price over adoption stages, N_t (blue solid curve), and data as scattered dots.

Appendix E provides more asset pricing predictions. In Appendix E.1, we explore the implications of endogenous user adoption on the volatility of token price. The network effects amplify the transmission of A_t shocks to the transactional benefits of token and its price. The mechanism is related to the literature on strategic complementarity and fragility (e.g., Goldstein and Pauzner 2005). So far, our analysis has assumed that platform productivity shocks command a constant risk premium. In Appendix E.2, we follow Pástor and Veronesi (2009) by modeling the correlation between the SDF shock and A_t shock as an increasing function of N_t, which captures the rising systematic risk of widely adopted platforms. The resultant N_t-dependent risk premium induces a bubble-like behavior of the token price, namely, a gradual run-up followed by an eventual decline.

5. Extensions and Discussions

5.1 Subsidy as an alternative solution to user adoption
In this subsection, we compare tokens with a traditional alternative solution to network adoption—user subsidy (Rochet and Tirole 2006)—and discuss the associated technical and financial considerations that affect the implementation.

Let κ denote the lump-sum subsidy that the platform gives to a user for platform participation per unit of time. Given this subsidy, agent i solves the following problem at each t:

$$dy_{i,t} = \max\left\{0, \max_{x_{i,t}>0}\left(\kappa\,dt + dv_{i,t}\right)\right\}, \tag{44}$$

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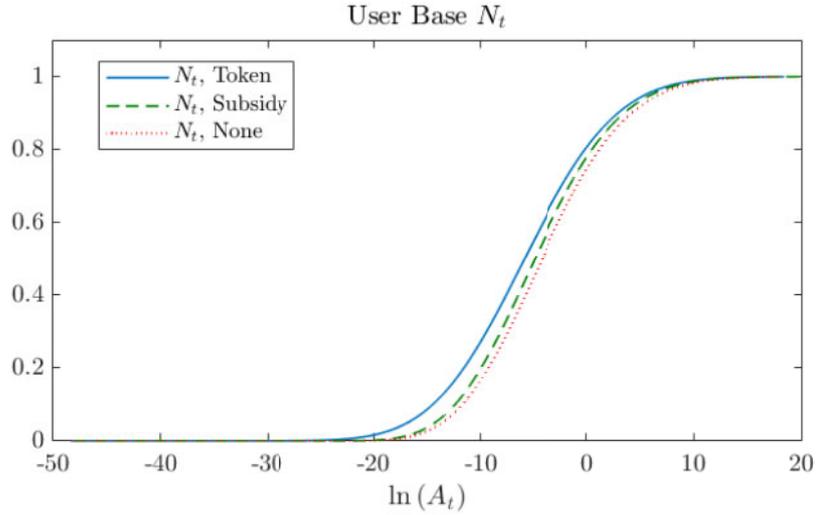
Figure 5
User subsidy versus using token
This figure plots N_t for the tokenized economy (blue solid curve), the tokenless economy (red dotted curve), and the tokenless economy with subsidy $\kappa = 0.5\phi$ (green dash curve).

where $dv_{i,t}$ is given in (2). Without this subsidy, this objective function is the same as (2) in the tokenless economy. By combining the subsidy κ with the participation cost ϕ, we can equivalently interpret introducing κ as reducing the participation cost from ϕ to $\phi - \kappa$.

Panel A of Figure 5 compares the adoption dynamics of token-based economy (the solid blue curve), the tokenless economy (the dotted red curve), and the tokenless economy with subsidy $\kappa = 0.5\phi$ (the dash green curve). Here we allow a sufficiently generous subsidy that effectively reduces agents' cost of participation by half. In this case, subsidy lifts the adoption curve, but it is still below what tokens can achieve.

5.1.1 Charging fees on profits. One question remains: How to finance the subsidies paid to users? Subsidies are often made possible by charging fees to users. One way to introduce fees in our analysis is via a standard proportional tax, τ_t, which can be time-varying and state-dependent. Given τ_t, agent i solves the following modified problem:

$$dy_{i,t} = \max\left\{0, (1-\tau_t)\max_{x_{i,t}}\left[(x_{i,t})^{1-\alpha}\left(N_t A_t e^{u_i}\right)^\alpha dt - (\phi-\kappa)dt - x_{i,t}rdt\right]\right\}.$$
(45)

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As the fee is charged on the maximized profit, introducing τ_t does not affect agent i's choice of $x_{i,t}$ and adoption decision.[25] The fee charged on users, $\tau_t > 0$, can be set to finance the subsidy:

$$\tau_t \int_{u_i \geq \underline{u}_t} \max_{x_{i,t} > 0} \left[\kappa dt + \left(x_{i,t} \right)^{1-\alpha} \left(N_t A_t e^{u_i} \right)^{\alpha} dt - \phi dt - x_{i,t} r dt \right] dG(u_i) = \kappa N_t dt,$$
(46)

where the left side gives the total fees collected from users and the right side is the total subsidy to users. For τ_t to be feasible, it has to be smaller than 100%. In the early stage where A_t is sufficiently low, the τ_t implied by the subsidy can be greater than 100% and thus infeasible. In contrast, tokens do not have this problem. The adoption-accelerating effect is active even when A_t is extremely low. In fact, as shown by panel B of Figure 3, the expected token return, μ_t^P, is larger when A_t and N_t are smaller, and thereby, the future potential of adoption, that is, $100\% - N_t$, is larger. This is one advantage of tokens over subsidies.

Second, implementing this tax subsidy scheme can be more difficult than tokens. Agents may not truthfully report their profits from platform activities because agents' type, u_i, can be private information. Incomplete information is a standard challenge recognized by studies on optimal taxation in the public finance literature (e.g., Mirrlees 1971). Moreover, our analysis assumes that agents receive subsidy only if they participate. If agents can obtain the subsidy and fake participation, the subsidy becomes a pure outflow of the platform and does not induce adoption. While by improving transparency, blockchain technology can potentially mitigate but cannot fully eliminate these frictions. Therefore, from an implementation perspective, token is a more operational mechanism to induce early adoption.

5.1.2 Charging fees on $x_{i,t}$. The platform may also directly charge fees on the value of users' holdings of means of payment:

$$dy_{i,t} = \max \left\{ 0, \max_{x_{i,t} > 0} \left[\left(x_{i,t} \right)^{1-\alpha} \left(N_t A_t e^{u_i} \right)^{\alpha} dt - (\phi - \kappa) dt - x_{i,t} r dt - x_{i,t} \tau_t dt \right] \right\}.$$
(47)

Here in addition to the subsidy term which effectively lowers the participation cost to $(\phi - \kappa) dt$, the fee, $\tau_t x_{i,t}$, effectively increases the carry cost from $r dt$ to $(r + \tau_t) dt$. In addition, the platform also faces a new budget constraint:

$$\tau_t dt \int_{u_i \geq \underline{u}_t} x_{i,t} dG(u_i) = \kappa N_t dt.$$
(48)

Here τ_t distorts agents' choice of $x_{i,t}$. Although easier to implement, this alternative fee subsidy is subject to the same problem we discussed earlier: it is only feasible when A_t is sufficiently large. In contrast, tokens accelerate adoption across all values of A_t.

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[25] If the maximized profit is positive when $\tau_t = 0$, it is still positive when $\tau_t \in (0, 1)$ and agent i still participates. This resembles the standard result that firms are neutral to taxes on profits.

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5.1.3 Intertemporal fee subsidy scheme. So far, we have only considered static fee subsidy scheme. Dynamic schemes are more flexible, but their applicability may be significantly compromised by financial constraints. An intertemporal transfer, subsidizing users in earlier stages (i.e., when A_t is low) and charging fees in later stages (i.e., when A_t is high), requires external financing to fund early-stage subsidy. Therefore, financial frictions may arise because of asymmetric information on the flow of fees and platform managers' moral hazard in administrating the fee-based payouts to investors.

5.1.4 Subsidies under incomplete information. Agents' adoption decisions feature strategic complementarity, but given A_t and μ_t^P, the assumption of monotonic hazard rate of $G(u)$ guarantees a unique $N_t > 0$ in equilibrium (see Proposition 2). Once we relax this assumption, there can exist multiple pairs of N_t and $\underline{u}_t$ that satisfy (30) and (31). To select the equilibrium user base, one may take a global game approach (e.g., Carlsson and van Damme 1993; Goldstein and Pauzner 2005; Morris and Shin 1998). For example, agents may receive noisy and correlated signals of platform productivity.[26] In this case, user subsidies, by effectively changing the participation cost, can potentially improve the coordination outcome.[27] A fruitful direction of future research is to investigate how tokens and subsidies can be jointly introduced to stimulate adoption and address coordination failure.

5.1.5 Subsidies and user-base volatility. Finally, we compare the impact of tokens and subsidies on the user-base volatility. In Section 5.3, we show that tokens reduce the user-base volatility. Alternatively, platforms may conduct countercyclical subsidies, that is, increasing subsidies following negative shocks, while decreasing subsidies following positive shocks. This is analogous to the countercyclical government expenditures as automatic stabilizers. However, to implement such state-contingent subsidy schemes, external funds are likely to be necessary because precisely when A_t and the aggregate profits (or tax base) on the platform are low, subsidies are high and have to be externally financed. Therefore, the effectiveness of this alternative mechanism depends on how severe the financing frictions are. In comparison, the stabilizing effect of tokens is always at work without requiring external financing.

5.2 Scalability and decentralization
So far, our analysis applies to both centralized and decentralized platform settings. Users in both settings eventually fully adopt as platform productivity

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[26] One may assume that productivity is immediately observed after agents make adoption and token holding decisions, so the model features a tractable, static form of incomplete information.

[27] This is analogous to government inventions that affect the parameters of individuals' decision-making (e.g., Bebchuk and Goldstein 2011).

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becomes sufficiently high. To provide a richer context for the adoption problem and blockchains' potential we next highlight a key trade-off between scalability and decentralization, which arises from a salient feature of blockchains: the (relatively) decentralized nature of record-keeping. Users value full decentralization, which is often costly for the platform. To ease our exposition of this trade-off, we focus on a tokenless economy.[28]

5.2.1 Generalizing agent's payoff. Specifically, we generalize the agent's flow payoff (conditional on participation) given in (8) to account for the effects of decentralization and scaling as follows:

$$\max_{x_{i,t}>0}\Big[dv_{i,t}+F(N_t,x_{i,t};\theta,\Delta,\check{N})dt\Big].\tag{49}$$

Here, the new term, $F(N_t,x_{i,t};\theta,\Delta,\check{N})$, captures the agent's preference captured by parameter Δ for decentralization captured by parameter Δ, as well as her disutility from congestion captured by parameter θ and from consensus delays (which is increasing in decentralization Δ).

We only require F to be weakly increasing in θ and $\check{N}$ and weakly decreasing in N_t, for a given level of decentralization Δ. For illustration, we consider:

$$F(N_t,x_{i,t};\theta,\Delta,\check{N})=\theta\Delta-c[N_t-\check{N}]^+-x_{i,t}h(\Delta),\tag{50}$$

where $h'(\Delta)>0$. The first term is the agent's additional utility flow from her preference for decentralization, for example, because of the reduction of single points of failure or intermediary rent (Chen, Cong, and Xiao 2019). The second term describes disutility from congestion when the user base is sufficiently large (e.g., Easley, O'Hara, and Basu 2019; Huberman, Leshno, and Moallemi 2019). This term captures the potential negative network effect for a relatively decentralized blockchain. The last term reflects that a more decentralized network is more costly for an agent with a larger choice of $x_{i,t}$. This is because the more decentralized a network is, the longer it takes to reach consensus (e.g., Hinzen, John, and Saleh 2020).

5.2.2 Adoption trajectory. Conditional on participating on the platform, the agent's optimal holdings of the medium of exchange in units of the numeraire good, $x_{i,t}$, satisfies the first-order condition:

$$(1-\alpha)(x_{i,t})^{-\alpha}\left(N_tA_te^{u_i}\right)^\alpha-r-h(\Delta)=0\quad\Rightarrow\quad x_{i,t}^*=N_tA_te^{u_{i,t}}\left(\frac{1-\alpha}{r+h(\Delta)}\right)^{1/\alpha}.\tag{51}$$

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[28] The tokenized economy features similar trade-offs, but the endogenous time variation of μ_t^P precludes a pure analytical characterization.

Therefore, an agent would participate if the maximized profit is nonnegative:

$$N_t A_t e^{u_i} \alpha \left(\frac{1-\alpha}{r+h(\Delta)} \right)^{\frac{1-\alpha}{\alpha}} - \phi + \theta\Delta - c[N_t - \check{N}]^+ \geq 0. \qquad (52)$$

By following essentially the same argument in Section 2, we obtain the following equilibrium condition for N_t:

$$1 - N_t = G\left(-\ln(N_t) + \ln\left(\frac{\phi - \theta\Delta + c[N_t - \check{N}]^+}{A_t \alpha} \right) - \left(\frac{1-\alpha}{\alpha} \right) \ln\left(\frac{1-\alpha}{r+h(\Delta)} \right) \right) \qquad (53)$$

Comparing (53) with (23), we notice the effects of θ, $\check{N}$, and $h(\Delta)$. An increase in θ or $\check{N}$ weakly reduces the right side of (53), which implies that the equilibrium N_t has to weakly increase. In other words, agents' preference for decentralization and efforts to improving scalability (such as increasing block size without reducing network security) would positively affect the adoption trajectory. The effect of decentralization Δ on adoption trajectory is ambiguous because on the one hand, agents prefer decentralization, but on the other hand, an increase in decentralization might create a large enough increase in consensus delay so that, in the last term within $G(\cdot)$, the effective carry cost increases and, thus, adoption drops.

5.2.3 Designer's problem and scale-decentralization trade-off. We recognize that it is technologically infeasible or economically very expensive to achieve high decentralization and large scale at the same time. For example, reaching consensus on a decentralized/permissionless blockchain (across the nodes of the record-keeper network through costly protocols, such as proof-of-work) takes time, a fact that inevitably constrains the transaction processing scale.

To model this trade-off, we let the designer choose a system with decentralization Δ and scalability $\check{N}$. To do so, the designer incurs a cost $C(\Delta, \check{N}; a)$, which is increasing in Δ and $\check{N}$. Here, a measures her attribute, For example, Visa. Inc is very efficient at conducting large-scale business transactions (large a), whereas Ethereum is known for well-designed consensus protocols that attract users who value decentralization (small a). Naturally, their different attributes lead to different choices of scalability and decentralization.

Specifically, the designer chooses the optimal $\check{N}^*$ and Δ^* which maximize the following:

$$\int_{t=0}^{\infty} e^{-rt} N_t \left[\alpha \left(\frac{1-\alpha}{h(\Delta)+r} \right)^{\frac{1-\alpha}{\alpha}} A_t \int_{i \in \mathcal{U}_t} e^{u_i} \, di - \phi + \theta\Delta - c[N_t - \check{N}]^+ \right] dt - C(\Delta, \check{N}; a), \qquad (54)$$

where the first term is the present value of aggregate transaction surplus (see (39) and (40)), and the last term reflects the designer's cost.

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In practice, the marginal cost of increasing the scalability/capacity $\check{N}$, $\frac{\partial C}{\partial \check{N}}$, is lower for a more reputable traditional designer (higher value of the attribute a in our convention, think about VISA). In other words, $\frac{\partial^2 C}{\partial a \partial \check{N}} < 0$. Similarly, the marginal cost of increasing the decentralization Δ, $\frac{\partial C}{\partial \Delta}$, is lower for an entrant with greater expertise at designing decentralized protocols (lower value of the attribute a in our convention, think about Ethereum). In other words, $\frac{\partial^2 C}{\partial a \partial \Delta} > 0$. Moreover, current technology dictates that it is more costly to scale business on a decentralized system than on a centralized system $\left(\frac{\partial^2 C}{\partial \check{N} \partial \Delta} \geq 0 \right)$.[29]

Concerning the ideal scale-decentralization choice, we first note that (54) exhibits negative cross-partial in a and Δ, positive cross-partial in a and $\check{N}$, and weakly negative cross partial in $\check{N}$ and Δ. Applying monotone comparative statics results, for example, theorem 4.1 in Athey, Milgrom, and Roberts (1998), we conclude that the designer's optimal Δ^* is decreasing in a, while the optimal $\check{N}^*$ is increasing in a. In other words, a more reputable traditional designer (bigger a) optimally implements a system that is bigger in scale but less decentralized.

We hope that our analysis of the trade-off between scalability and decentralization in this section offers a broader context of adoption problem and economic insights for the ongoing research in computer science on platform adoption and blockhain scalability.

6. Conclusion

We provide a tractable dynamic equilibrium model of token pricing and platform adoption. Platforms create value by supporting specific economic activities and platform tokens derive value by enabling transactions among heterogeneous users. As a result, token value reflects users' endogenous participation and the associated network externality effects. Endogenous user base also plays a critical role in explaining the cross-section variation of token pricing, the dynamics of token price volatility, and the run-up and crash of token prices.

By comparing platforms with and without tokens, we show that introducing tokens lowers the effective carry cost of conducting platform transactions and hence accelerates the adoption of productive platforms. The introduction of tokens also reduces the volatility of the user base, because agents' expectations about long-term growth in the token's value weaken the impact of temporary productivity shocks on the user base.

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[29] More generally, to capture these observations, we can assume that C has strictly increasing differences in a and Δ, strictly decreasing differences in a and $\check{N}$, and increasing differences in $\check{N}$ and Δ.

A key assumption in our model is that the token market is fully liquid.[30] However, the token market may be illiquid especially at early stages of platform adoption. We leave incorporating token market illiquidity into our model for future research. Another interesting extension of our model is to combine our analysis of user activities with the design of platform infrastructure (e.g., blockchain protocol design in Fanti, Kogan, and Viswanath 2019). Finally, tokens can play two roles. It serves as means of payment among platform users, and platforms can issue tokens to compensate resource contributors and reward their founders (Token2).

Appendix A. Transaction Surplus and Flow Utility

In this section, we provide theoretical foundations for our specification of platform transaction surplus in (2). We first present a general model based on transaction costs that applies to all platform tokens (not necessarily blockchain-based), which essentially captures a form of convenience yield (see, e.g., Cochrane, 2018), and then discuss a case that is specific to blockchain platform. Here, our goal is to illustrate practical settings that motivate our specification, not microfound every application scenario.

A.1 A Model of Convenience Yield. Agents have investment opportunities that occur at Poisson arrival times, $\{T_n\}_{n=1}^{+\infty}$, with time-varying and agent-specific intensity, $\lambda_{i,t}$. At a given Poisson time, T_n, agent i is endowed with a technology, $\omega_i F(\cdot)$, that transforms labor into goods, and is matched with another agent who can supply the required labor input. Agent-specific productivity is captured by ω_i. To simplify the exposition, we assume that the labor supply has a constant marginal cost of one, and the supplier breaks even, so the full trade surplus is enjoyed by agent i. This setup of uncertain lumpy transactions follows those of Alvarez and Lippi (2013) and Li (2018).

Agent i's labor demand, denoted by h, is not restricted by the real balance of token holdings, $P_t k_{i,T_n-}$, where k_{i,T_n-} denotes the units of tokens carried to T_n and we make token price explicit in the expression because we want to start with microfounding the utility flow in a tokenized economy. Since the focus of this paper is not on financial constraints, we allow the agent to borrow dollars (an instantaneous loan) at zero cost, so h may exceed agent i's wealth at the moment. The production is done immediately, and the loan is repaid immediately by the goods. So, given a competitive credit market, the loan rate is zero.

The lumpy payment for labor incurs a transaction cost that is proportional to the total payment value, δh ($\delta > 0$), but using tokens as means of payment save the transaction cost by $U\left(P_t k_{i,T_n-}\right)$ ($U' > 0$, $U'' < 0$) because agent i does not need to exchange dollars for tokens, the required means of payment on the platform.

Agent i maximizes the investment profit, which is a jump in wealth,

$$\max_h \omega_i F(h) - h - \left(\delta h - U\left(P_t k_{i,T_n-}\right)\right), \tag{A.1}$$

where the last term is the transaction cost. The optimal labor demand, h^*, is given by

$$\omega_i F'\left(h^*\right) = 1 + \delta, \tag{A.2}$$

so the marginal value of production is equal to the marginal cost of labor plus the transaction cost, δ. We can substitute the constant h^* into the investment profit to have

$$\omega_i F\left(h^*\right) - (1+\delta)h^* + U\left(P_t k_{i,T_n-}\right). \tag{A.3}$$

We assume that ω_i is sufficiently high so $h^* \geq P_t k_{i,T_n-}$. The conversion between the local currency (token) and other assets can be costly, especially when a lumpy transaction is required within a

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[30] Cryptocurrency exchanges help create liquidity, but may suffer from market manipulations such as wash trading, as discussed in Cong et al. (2020).

short period of time. By holding tokens, agents save such costs. Linking transaction costs to the monetary value of assets has a long tradition in economics (Baumol 1952; Tobin 1956; Duffie 1990).

Therefore, at time t, agent i has an expected gain of $\lambda_{i,t} U\left(P_t k_{i,t}\right) dt$ by holdings $k_{i,t}$ units of tokens for dt. To obtain a tighter analytical characterization of the equilibrium, we specify $\lambda_{i,t} = \left(N_t A_t e^{u_i}\right)^\alpha$, $\alpha \in (0,1)$. A larger community (N_t) makes it easier to find transaction counterparties. A higher platform quality (A_t) reflects a more efficient matching mechanism or the fact that the economic transactions supported by the platform are more popular. And u_i captures agent-specific transaction needs. We specify $U\left(P_t k_{i,t}\right) = \chi \left(P_t k_{i,t}\right)^{1-\alpha}$, so the expected transaction costs saved are

$$\lambda_{i,t} U\left(P_t k_{i,t}\right) dt = \left(N_t A_t e^{u_i}\right)^\alpha \left(P_t k_{i,t}\right)^{1-\alpha} \chi \, dt. \tag{A.4}$$

In the following we set $\chi = 1$ because its scaling effect can be subsumed by the level of A_t.

We may reinterpret h as goods or services other than labor, and the investment profit as a burst of consumption or utility value from transactions. The foundation of this specification of trade surplus is twofold: (1) the arrival of transaction opportunities depends on the user base, platform quality, and agent-specific factors, and (2) holding tokens on the tokenized platform save transaction costs for lumpy payments.

Note that the same setup can also generate the trade surplus for a tokenless economy where the transaction is settled using the numeraire good, so in the main text, we assume the same functional form of transaction surplus. In a tokenless economy, agents hold assets that yield a risk-adjusted return of r and dollars' worth of $x_{i,t}$. The transaction cost, δh, is thus the cost of immediately exchanging a lumpy chuck of assets for cash. Holding cash saves this cost by $U\left(x_{i,t}\right)$. If external financing is required, the per unit cost of transaction, δ, also captures the difficulty to raise funds in lumpy amounts. The concavity of $U(\cdot)$ can be motivated by models of cash holdings that recognize cash carry costs and external financial constraints (e.g., Bolton, Chen, and Wang 2011).

A.2 Staking Tokens. While the above transaction-cost based model applies to platform tokens that serve as means of payment, we next give another theoretical foundation to illustrate that (a) although we focus on tokens that explicitly serve as means of payment on platforms, our theory more generally applies to all tokens that provide users utilities specific to the underlying platform technology, and (b) blockchain-based platforms provide novel forms of transaction surplus from holding tokens on platforms and further motivate our specification of token flow utility.

Many blockchain-based platforms feature users providing service to peers to make a profit. For example, Filecoin, Golem, Storj, and Elastic all have "storage miners" who assist clients in storing digital files in exchange for native tokens. Oftentimes, storage miners have to "stake" native tokens (i.e., post it as collateral) in order to win the chance to service clients. In fact, staking is a common practice on blockchain platforms to encourage value-creating activities among their users. It goes beyond validating transactions and producing blocks within consensus mechanisms, such as Proof-of-Stake (POS) and Distributed Proof-of-Stake (DPOS). In general, holding/staking tokens may enable network participants to potentially receive access to exclusive features of the platform, partake of business activities, or receive status recognition. For example, OmiseGO (OMG), the first ERC20 tokens on Ethereum sold via an ICO to reach unicorn status (US$1 billion market cap) in August 2017 (coinmarketcap.com), has validators deposit OMGs in staking contracts to validate transactions. OmiseGo selects the validator based on who has staked the highest token to validate the transaction and it performs the task. Depending on the performance of the validator, the validator will receive either rewards or penalties. Filecoin, VentureFusion, Numerai, etc., all feature some forms of staking.

Now suppose a storage miner on the platform has a realized storage space $e^{u_i \alpha}$, and is waiting to be matched with clients demanding decentralized storage (similar to a labor-market search-and-matching scenario). Potential client demand (analogous to the number of job seekers) is proportional to N_t, the user base of the platform, whereas the matching effort of the storage miner is proportional to staking amount x_i (similar to job vacancies). Then if the platform has a matching efficiency of

A^α, a matching function with constant return to scale (e.g., Pissarides, 2000) would yield the storage miner a payoff of

$$e^{u_i \alpha} A^\alpha N_t^\alpha (P_t k_{i,t})^{1-\alpha} = (P_t k_{i,t})^{1-\alpha} (A N_t e^{u_i})^\alpha, \tag{A.5}$$

which exactly gives the surplus flow in (2). The storage miner then takes back the same number of tokens staked after providing the service for dt.

In the case above, it is crucial that a service provider (storage miner in the case of Filecoin) needs to stake/hold native tokens to have a chance of being matched with a customer. If she can match with a customer and instantaneously exchange tokens for the numeraire good, the velocity of native tokens can be infinite, resulting in price indeterminacy.

A.3 Other Tokens.   While our focus is on a majority of tokens whose value derives from the productivity of underlying platforms and user network effects, we acknowledge that in reality, there exists a variety of tokens that serve purposes other than facilitating platform-specific transactions. Digital currencies developed by central banks serve general payment purposes. They are typically not tied to specific user networks, and their adoption is driven by policy and legal decisions. There are also tokens that enable the sharing of future corporate revenues or the distribution of products and services. Such security tokens can be valued using traditional discounted cash flow models and are therefore not our focus.

Moreover, our model does not capture the more complex interdependence of platforms and their tokens. For example, Litecoin and Dogecoin are "altcoins," which are variants of the original open-sourced Bitcoin protocol to enable new features. Therefore, the productivity of their platforms significantly inherits from the Bitcoin blockchain. Other examples include "AppCoins," which entrepreneurs often sell through the initial coin offerings (ICOs). AppCoins are developed for specific applications (e.g., Gnosis and Golem) and are built on existing blockchain infrastructures (e.g., Ethereum or Waves).

Appendix B - Proofs

Lemma B1. Given μ_t^P and a sufficiently high productivity, that is, $A_t > \underline{A}(\mu_t^P)$, and we have a unique nondegenerate solution, $N_t > 0$, and $\underline{u}_t > \underline{U}$ for (30) and (31) if $G(u)$ has an increasing hazard rate. The user base, N_t, increases in μ_t^P and A_t. Agent i participates when $u_i \geq \underline{u}_t$. Conditional on participating, the numeraire value of agent i's optimal token holding, $x_{i,t}^*$, is given by (28), and increases in A_t and μ_t^P.

Proof of Lemma B1.   (30) and (31) jointly determine the user base N_t given A_t and μ_t^P. First, we note that $N_t = 0$ is always a solution. Here we focus on the nondegenerate case, that is, $N_t > 0$. Fixing A_t and μ_t^P, we consider a *response* function $R(n; A_t, \mu_t^P)$ that maps a hypothetical value of N_t, say n, to the measure of agents who choose to participate after knowing $N_t = n$:

$$R(n; A, \mu_t^P) = 1 - G(\underline{u}(n; A, \mu_t^P))$$

$$= 1 - G\left(-\ln(n) + \ln\left(\frac{\phi}{A_t \alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r - \mu_t^P}\right)\right), \tag{B.6}$$

Before we start, for any $A_t = A > 0$, we define the value of its response function at $n = 0$: $R(0; A, \mu_t^P) = 0$. This is consistent with that given a zero user base, each agent derives zero transaction surplus from token holdings and chooses not to participate. Note that $\lim_{n \downarrow 0} R(n; A, \mu_t^P) = 0$, so the response function is continuous in n. As depicted in Figure B.1, the response curve originates from zero (the degenerate case).

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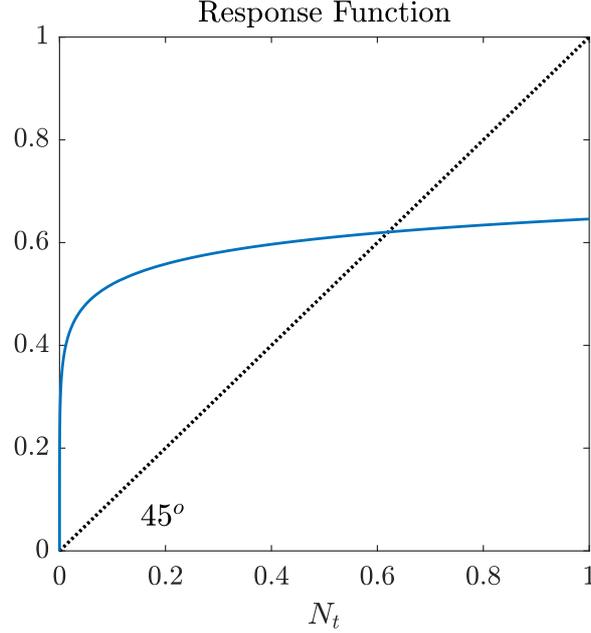
Figure B.1
Determining the user base
This graph shows the aggregate response of users' adoption decision, $R\left(n; A_t, \mu_t^P\right)$, to different levels of $N_t = n \in [0, 1]$, given A_t and μ_t^P.

First, we show that given μ_t^P, there exists $\underline{A}$ such that for $A_t = A > \underline{A}$, its corresponding response curve,

$$R\left(n; A, \mu_t^P\right) = 1 - G\left(\underline{u}\left(n; A, \mu_t^P\right)\right) \tag{B.7}$$

$$= 1 - G\left(-\ln(n) + \ln\left(\frac{\phi}{A_t \alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r - \mu_t^P}\right)\right), \tag{B.8}$$

crosses the 45° line at least once in $(0, 1]$, and for any value of $A_t = A < \underline{A}$, the response curve never crosses the 45° line in $(0, 1]$. Later in our numerical solution, we verify that this inequality holds at all values of A_t. Given μ_t^P, we define a mapping, $A(n)$, from any equilibrium, with nonzero value of user base, $n \in (0, 1]$, to the corresponding value of A_t, that is, the unique solution to

$$1 - G\left(-\ln(n) + \ln\left(\frac{\phi}{A_t \alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r - \mu_t^P}\right)\right) = n, \quad \forall n \in (0, 1]. \tag{B.9}$$

This mapping is a continuous mapping on a bounded domain $\subseteq (0, 1]$. Then by the Least-Upper-Bound-Property of real numbers, the image set of this mapping, $\{A(n), n \in (0, 1)\}$, has an infimum, which we denote by $\underline{A}$. Now, for $A_t = \underline{A}$, consider a $n\left(\underline{A}\right) \in (0, 1]$ such that (B.9) holds. For any $A > \underline{A}$, the left-hand side (LHS) of (B.9) is higher than the right-hand side, that is, $R\left(n\left(\underline{A}\right); A, \mu_t^P\right) > n\left(\underline{A}\right)$, so that the response curve of $A_t = A$ is above the 45° line at $n\left(\underline{A}\right)$. Next, because the response function $R\left(n; A, \mu_t^P\right)$ is continuous in n and $R\left(1; A, \mu_t^P\right) \leq 1$ by definition in (A8); that is, it eventually falls to or below the 45° line as n increases, there must exist a $n(A) \in (0, 1]$ such that when at $A_t = A$, (B.9) holds by the Intermediate Value theorem. Therefore, we have proved

that for any $A_t = A > \underline{A}$, there exists a nonzero user base. Throughout the proof, we fix μ_t^P, so $\underline{A}$ is a function of μ_t^P.

After proving the existence of $N_t > 0$ for $A_t \in [\underline{A}, +\infty)$, we prove the uniqueness given the increasing hazard rate of $g(u)$ (i.e., $\frac{g(u)}{1-G(u)}$ increase in u).[31] Specifically, we show that for $A_t \geq \underline{A}(\mu_t^P)$, the response curve crosses the 45° line exactly once (and from above) when $A_t \in [\underline{A}, +\infty)$. First note that $R(n; A_t, \mu_t^P) - n$ has either a positive derivative or a negative derivative at $n = 0$. If it has positive derivative (i.e., the response curve shoots over the 45° line), then at n', *the first time* the response curve crosses the 45° line again, the derivative of $R(n; A_t, \mu_t^P) - n$ must be weakly negative at n'; that is, the response curve crosses the 45° from above,

$$g\left(\underline{u}\left(n'; A_t, \mu_t^P\right)\right) \frac{1}{n'} - 1 \leq 0. \tag{B.10}$$

Now suppose the response curve crosses the 45° line for the second time from below at $n'' > n'$, so the derivative of $R(n; A_t, \mu_t^P) - n$ at n'' must be weakly positive, and is equal to

$$
\begin{aligned}
g\left(\underline{u}\left(n''; A_t, \mu_t^P\right)\right) \frac{1}{n''} - 1 \quad &= \quad \frac{g\left(\underline{u}\left(n''; A_t, \mu_t^P\right)\right)}{1 - G\left(\underline{u}\left(n''; A_t, \mu_t^P\right)\right)} - 1 \\
&\leq \quad \frac{g\left(\underline{u}\left(n'; A_t, \mu_t^P\right)\right)}{1 - G\left(\underline{u}\left(n'; A_t, \mu_t^P\right)\right)} - 1 \\
&= \quad \frac{g\left(\underline{u}\left(n'; A_t, \mu_t^P\right)\right)}{n'} - 1 \\
&< \quad 0, \tag{B.11}
\end{aligned}
$$

where the first inequality comes from the increasing hazard rate and the fact that $\underline{u}(n; A_t, \mu_t^P)$ is decreasing in n for $n \in (0, 1]$, and the second inequality follows from (B.10) and the fact that the response curve crosses the 45° line at n' (i.e., $n' = R(n'; A_t, \mu_t^P) = 1 - G(\underline{u}(n'; A_t, \mu_t^P))$). This contradicts the presumption that the response curve reaches the 45° line from below (and the derivative of $R(n; A_t, \mu_t^P) - n$ is weakly positive). Therefore, we conclude that for $A_t \in [\underline{A}, +\infty)$, there exists a unique adoption level n. Now if $R(n; A_t, \mu_t^P) - n$ has negative derivative at $n = 0$, then in the previous argument, we can replace n' with 0 and show that there does not exist another intersection between the response curve and the 45° line beyond $n = 0$. Therefore, only if $R(n; A_t, \mu_t^P) - n$ has positive derivative at $n = 0$, do we have a positive (nondegenerate) adoption level.

Finally, we prove that N_t increases in μ_t^P. Consider $\tilde{\mu}_t^P > \mu_t^P$. Suppose the contrary that their corresponding adoption levels satisfy $\tilde{N}_t \leq N_t$. Because we have proved that the response curve only crosses the 45° line only once and from above, given N_t, we have

$$1 - G\left(-\ln(n) + \ln\left(\frac{\phi}{A_t \alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right) \ln\left(\frac{1-\alpha}{r - \mu_t^P}\right)\right) \geq n, \quad \forall n \in (0, N_t]. \tag{B.12}$$

We know that, by definition,

$$
\begin{aligned}
\tilde{N}_t \quad &= \quad 1 - G\left(\underline{u}\left(\tilde{N}_t; A_t, \tilde{\mu}_t^P\right)\right) \\
&= \quad 1 - G\left(-\ln(\tilde{N}_t) + \ln\left(\frac{\phi}{A_t \alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right) \ln\left(\frac{1-\alpha}{r - \tilde{\mu}_t^P}\right)\right)
\end{aligned}
$$

---

[31] The hazard rate is increasing if and only if $1 - G(u)$ is log-concave. This assumption is common in the theory literature, for example, to avoid the complicated "ironing" of virtual values.

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$$> \quad 1 - G\left(-\ln\left(\tilde{N}_t\right) + \ln\left(\frac{\phi}{A_t\alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r-\mu_t^P}\right)\right)$$

$$\geq \quad \tilde{N}_t, \tag{B.13}$$

where the first inequality uses $\tilde{\mu}_t^P > \mu_t^P$ and the second inequality uses the fact that $\tilde{N}_t \in (0, N_t]$ and the inequality (B.12). This contradiction implies that the adoption level N_t has to be increasing in μ_t^P, because $\underline{u}_t$ decreases in both μ_t^P and A_t. The same method proves that the adoption level N_t increases in A_t.

Proof of Proposition 1. By setting μ_t^P to zero, we can apply Lemma B1 to the tokenless economy, and obtain the following the results: (a) (22) and (23) solve a unique pair of N_t^{NT} and $\underline{u}_t^{NT}$ as functions of A_t, and (b) the adoption level N_t^{NT} increases in A_t.

Proof of Proposition 2. We apply Lemma B1 to the tokenized economy, and obtain the following the results: (i) (30) and (31) solve a unique pair of N_t and $\underline{u}_t$ as functions of A_t and μ_t^P; (ii) the adoption level N_t (the participation threshold, $\underline{u}_t$) increases (decreases) in A_t and μ_t^P. Next, we show that (32) implies a second-order ordinary differential equation for the token price, $P_t = P(A_t)$.

Because $\underline{u}_t$ decreases in μ_t^P, the right side of (32) is monotonic (increasing) in μ_t^P, so, given A_t and P_t, (32) uniquely solves μ_t^P. This means μ_t^P can be expressed as a function of P_t and A_t, denoted by $\mu_t^P = H(A_t, P_t)$. Note that by applying Itô's lemma, we have $\mu_t^P = \frac{P'(A_t)}{P(A_t)} A_t \mu^A + \frac{1}{2} \frac{P''(A_t)}{P(A_t)}\left(A_t\sigma^A\right)^2$. Therefore, (32) implies an ordinary differential equation:

$$\frac{P'(A_t)}{P(A_t)} A_t \mu^A + \frac{1}{2}\frac{P''(A_t)}{P(A_t)}\left(A_t\sigma^A\right)^2 = H(A_t, P_t). \tag{B.14}$$

Given the boundary conditions (35) and (38), this ODE satisfies the regularity conditions in theorems 4.17 and 4.18 in Jackson (1968), so the solution exists and is unique.

In the following, we summarize the steps of solving the token pricing ODE. First, the user-type cutoff threshold $\underline{u}_t$ and the user base N_t jointly solve (30) and (31). Moreover, we define the aggregate of agents' type in (33), which only depends on $\underline{u}_t$. All three variables, $\underline{u}_t$, N_t, and S_t, can be expressed as functions of A_t and μ_t^P.

Second, the token market-clearing condition, (32), only contains P_t, A_t, and μ_t^P. Because N_t and S_t increase in A_t and μ_t^P (as previously discussed), the right side of (32) is monotonic in μ_t^P, so (32) implies a unique value of μ_t^P given the values of A_t and P_t.

Third, because in equilibrium, the token price is a function of A_t, we can apply Itô's lemma: $\mu_t^P = \frac{P'(A_t)}{P(A_t)} A_t \mu^A + \frac{1}{2} \frac{P''(A_t)}{P(A_t)}\left(A_t\sigma^A\right)^2$. (32) then implies a mapping from A_t, $P(A_t)$, and $P'(A_t)$ to $P''(A_t)$, that is, a second-order ODE of $P(A_t)$. Augmenting the ODE with the boundary conditions (35) and (38), we can solve the token price function $P(A_t)$.

Finally, once we have solved $P(A_t)$, we can apply Itô's lemma again to solve μ_t^P as a function of A_t, and then using (30) and (31) to solve N_t and $\underline{u}_t$ (and thus S_t) as functions of A_t. Figure B.2 plots the complete solution of the model that includes all the endogenous variables as functions of A_t, the state variable of the Markov equilibrium.

Proposition B1. The token price is positive in the Markov equilibrium in Propsition 2.

Proof of Proposition B1. We prove this proposition by contradiction. The token market-clearing condition implies the aggregate token demand, $M = \int_{u_i \geq \underline{u}_t} k_{i,t} di > 0$. Consider a value of A_t such that $P(A_t) = 0$. Agent i chooses the units of tokens

$$\max_{k_{i,t}} \left\{ \left(P_t k_{i,t}\right)^{1-\alpha}\left(N_t A_t e^{u_i}\right)^\alpha dt - \phi dt - P_t k_{i,t} r dt + k_{i,t}\mathbb{E}_t\left[dP_t\right] \right\}. \tag{B.15}$$

Given that $P_t = 0$, the first term of transactional benefit is zero, so for $k_{i,t}^*$ to be a finite number, agent i must be indifferent between the marginal benefit from the expected token price change,

A Log Token Price $\ln(P_t)$

B Expected Token Return μ_t^P

C Token Volatility σ_t^P

D User Base N_t

E User Threshold $\underline{u}_t$

F Transaction Needs S_t
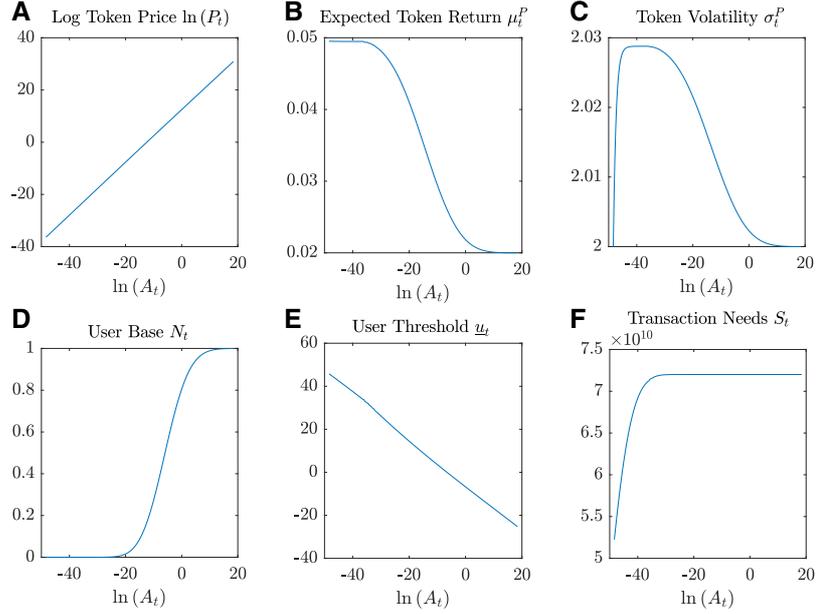
Figure B.2
Endogenous variables as functions of the state variable

$\mathbb{E}_t[dP_t]$, and the marginal cost of losing interests $P_t r\,dt$. Since the latter is zero, the former must be zero $\mathbb{E}_t[dP_t]=0$. As a result, the maximized profit is $-\phi<0$ for all agents, and no agent chooses to participate, resulting in a zero aggregate token demand that violates the token market-clearing condition.

Proposition B2. Define $\underline{A}^{NT}\equiv\min\{A_t:N^{NT}(A_t)>0\}$. We have $\underline{A}^{PL}<\underline{A}^{NT}$.

Proof of Proposition B2. To prove this inequality, consider the agent whose type is $\underline{u}^{NT}$, that is, the type whose flow profit is equal to zero when $A_t=\underline{A}^{NT}$ in the tokenless economy. Therefore, we have the following

$$0=N^{NT}\underline{A}^{NT}e^{\underline{u}^{NT}}\alpha\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}}-\phi<\alpha\underline{A}^{NT}\overline{S}\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}}-\phi,\qquad(\text{B.16})$$

where we use

$$N^{NT}e^{\underline{u}^{NT}}=\left[1-G\left(\underline{u}^{NT}\right)\right]e^{\underline{u}^{NT}}<\int_{\underline{u}^{NT}}^{\overline{U}}e^{\underline{u}^{NT}}\,dG(u)+\int_{\underline{U}}^{\underline{u}^{NT}}e^u\,dG(u)$$

$$<\int_{\underline{u}^{NT}}^{\overline{U}}e^u\,dG(u)+\int_{\underline{U}}^{\underline{u}^{NT}}e^u\,dG(u)\equiv\overline{S}.\qquad(\text{B.17})$$

Recall that in the planner's solution, we have

$$0=\alpha\underline{A}^{PL}\overline{S}\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}}-\phi.\qquad(\text{B.18})$$

By comparing the right sides in the two preceding inequalities, we conclude $\underline{A}^{NT}>\underline{A}^{PL}$.

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Proposition B3 (Comparing User-Base Volatilities over Adoption Stages). The user-base volatility of the tokenless economy is

$$\sigma_t^N = \left(\frac{g\left(\underline{u}_t^{NT}\right)}{1 - g\left(\underline{u}_t^{NT}\right)/N_t^{NT}} \right) \sigma^A. \tag{B.19}$$

The user-base volatility of the tokenized economy is

$$\sigma_t^N = \left(\frac{g\left(\underline{u}_t\right)}{1 - g\left(\underline{u}_t\right)/N_t} \right) \left[\sigma^A + \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{\sigma_t^{\mu^P}}{r - \mu_t^P} \right) \right], \tag{B.20}$$

where $\sigma_t^{\mu^P}$ is the diffusion of μ_t^P as defined below:

$$d\mu_t^P = \mu_t^{\mu^P} dt + \sigma_t^{\mu^P} dZ_t^A. \tag{B.21}$$

Under the same level of adoption, that is, $N_t = N_t^{NT}$, the user-base volatility of the tokenized economy is smaller than that of the tokenless economy if and only if μ^P decreases in A_t.

Proof of Proposition B3.   Without loss of generality, we write the dynamics of N_t as:

$$dN_t = \mu_t^N dt + \tilde{\sigma}_t^N N_t dZ_t^A, \tag{B.22}$$

where $\tilde{\sigma}_t^N N_t = \sigma_t^N$ is a more convenient notation to work with for the derivations.

First, we solve the user-base volatility without token. Using Itô's lemma, we can differentiate (23) and then, by matching coefficients with (42), derive μ_t^N and σ_t^N:

$$dN_t = -g\left(\underline{u}_t^{NT}\right) d\underline{u}_t^{NT} - \frac{1}{2} g'\left(\underline{u}_t^{NT}\right) \langle d\underline{u}_t^{NT}, d\underline{u}_t^{NT} \rangle, \tag{B.23}$$

where $\langle d\underline{u}_t^{NT}, d\underline{u}_t^{NT} \rangle$ is the quadratic variation of $d\underline{u}_t^{NT}$. Using Itô's lemma, we differentiate (22)

$$d\underline{u}_t^{NT} = -\frac{1}{N_t} dN_t + \frac{1}{2N_t^2} \langle dN_t, dN_t \rangle - \frac{1}{A_t} dA_t + \frac{1}{2A_t^2} \langle dA_t, dA_t \rangle$$

$$= -\left(\frac{\mu_t^N}{N_t} - \left(\tilde{\sigma}_t^N\right)^2 + \mu^A - \frac{\left(\sigma^A\right)^2}{2} \right) dt - \left(\tilde{\sigma}_t^N + \sigma^A\right) dZ_t^A. \tag{B.24}$$

Substituting this dynamics into (B.23), we have

$$dN_t = \left[g\left(\underline{u}_t^{NT}\right) \left(\frac{\mu_t^N}{N_t} - \left(\tilde{\sigma}_t^N\right)^2 + \mu^A - \frac{\left(\sigma^A\right)^2}{2} \right) - \frac{1}{2} g'\left(\underline{u}_t^{NT}\right) \left(\tilde{\sigma}_t^N + \sigma^A\right)^2 \right] dt$$

$$+ g\left(\underline{u}_t^{NT}\right) \left(\sigma_t^N + \sigma^A\right) dZ_t^A, \tag{B.25}$$

By matching coefficients on dZ_t^A with (42), we can solve for σ_t^N.

Next, we solve the user-base volatility in the tokenized economy where N_t depends on the expected token price appreciation μ_t^P that is a univariate function of A_t. In equilibrium, its law of motion is given by a diffusion process

$$d\mu_t^P = \mu_t^{\mu^P} dt + \sigma_t^{\mu^P} dZ_t^A. \tag{B.26}$$

Now, the dynamics of $\underline{u}_t$ becomes

$$d\underline{u}_t = -\frac{1}{N_t} dN_t + \frac{1}{2N_t^2} \langle dN_t, dN_t \rangle - \frac{1}{A_t} dA_t + \frac{1}{2A_t^2} \langle dA_t, dA_t \rangle$$

$$-\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{r-\mu_t^P}\right)d\mu_t^P - \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{2\left(r-\mu_t^P\right)^2}\right)\langle d\mu_t^P, d\mu_t^P\rangle \tag{B.27}$$

Let $\sigma_t^{\underline{u}}$ denote the diffusion of $\underline{u}_t$. By collecting the coefficients on dZ_t^A in (B.27), we have

$$\sigma_t^u = -\tilde{\sigma}_t^N - \sigma^A - \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\sigma_t^{\mu^P}}{r-\mu_t^P}\right), \tag{B.28}$$

which, in comparison with (B.24), contains an extra term that reflects the volatility of expected token price change. Note that, similar to (B.23), we have

$$dN_t = -g\left(\underline{u}_t\right)d\underline{u}_t - \frac{1}{2}g'\left(\underline{u}_t\right)\langle d\underline{u}_t, d\underline{u}_t\rangle, \tag{B.29}$$

so the diffusion of N_t is $-g\left(\underline{u}_t\right)\sigma_t^{\underline{u}}$. Matching it with the conjectured diffusion coefficient σ_t^N gives σ_t^N. Under the same level of adoption, that is, $N_t = N_t^{NT}$, we also have $\underline{u}_t = \underline{u}_t^{NT}$, so the first brackets in (B.19) and (B.20) have the same value. Therefore, the only difference between the two curves of user-base volatility arises from $\sigma_t^{\mu^P}$, the volatility of expected token price change. By Itô's lemma, $\sigma_t^{\mu^P} = \frac{d\mu_t^P}{dA_t}\sigma^A A_t$, so the sign of $\sigma_t^{\mu^P}$ depends on whether μ_t^P increases or decreases in A_t. When μ_t^P decrease in A_t, $\sigma_t^{\mu^P} < 0$, and thereby, the tokenized economy has a smaller user-base volatility than the tokenless economy.

Proposition B4 (Comparing the Volatilities of User-Base Growth Rates). *For any given A_t, the volatility of user-base growth in the tokenized economy is smaller than that of the tokenless economy if $\mu_t^P > 0$ and μ_t^P decreases in A_t.*

Proof of Proposition B4. From the proof of Proposition B3, we know that in the tokenless economy, the volatility of user-base growth rate, dN_t/N_t, is

$$\tilde{\sigma}_t^N = \left(\frac{g\left(\underline{u}_t^{NT}\right)}{N_t^{NT} - g\left(\underline{u}_t^{NT}\right)}\right)\sigma^A = \left(\frac{g\left(\underline{u}_t^{NT}\right)}{1-G\left(\underline{u}_t^{NT}\right) - g\left(\underline{u}_t^{NT}\right)}\right)\sigma^A, \tag{B.30}$$

and similarly, in the tokenized economy,

$$\tilde{\sigma}_t^N = \left(\frac{g\left(\underline{u}_t\right)}{1-G\left(\underline{u}_t\right) - g\left(\underline{u}_t\right)}\right)\left[\sigma^A + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\sigma_t^{\mu^P}}{r-\mu_t^P}\right)\right]. \tag{B.31}$$

Under the assumption of a weakly monotone hazard rate (introduced in Proposition 2), $\frac{g(\cdot)}{1-G(\cdot)}$ is weakly increasing in $\underline{u}_t$, so is $\frac{g(\cdot)}{1-G(\cdot)-g(\cdot)}$. If $\mu_t^P > 0$, a tokenized platform has $\underline{u}_t < \underline{u}_t^{NT}$ (Proposition 2). Therefore,

$$\left(\frac{g\left(\underline{u}_t^{NT}\right)}{N_t^{NT} - g\left(\underline{u}_t^{NT}\right)}\right) \geq \left(\frac{g\left(\underline{u}_t\right)}{1-G\left(\underline{u}_t\right) - g\left(\underline{u}_t\right)}\right). \tag{B.32}$$

Moreover, if μ_t^P decreases in A_t, by Itô's lemma, $\sigma_t^{\mu^P} = \frac{d\mu_t^P}{dA_t}\sigma^A A_t < 0$, and thereby,

$$\left[\sigma^A + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\sigma_t^{\mu^P}}{r-\mu_t^P}\right)\right] < \sigma^A. \tag{B.33}$$

Consequently, the volatility of user-base growth rate of the tokenized platform is smaller than that of the tokenless economy for any given A_t.

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Appendix C - Parameter Choices

C1. Parameter Values

We choose the model parameters under the physical measure so that the model generates patterns that are broadly consistent with user adoption and token price dynamics. Recall from Section 2.1 that under the widely used SDF,

$$\frac{d\Lambda_t}{\Lambda_t} = -r\,dt - \eta d\widehat{Z}_t^\Lambda, \tag{C.34}$$

A_t under the physical measure follows a GBM process, where the drift coefficient, $\widehat{\mu}^A$, is equal to $\mu^A + \eta\rho\sigma^A$ and the volatility coefficient is σ^A.

We use token price and blockchain user-base dynamics from July 2010 and April 2018. We normalize one unit of time in the model to be 1 year. Since we fix the token supply at M, the token price P_t completely drives the market capitalization ($P_t M$). We map P_t to the *aggregate* market capitalization of major cryptocurrencies.[32] Since we study a representative token economy, focusing on the aggregate market averages out idiosyncratic movements due to specificalities of token protocols.

We collect the number of active user addresses for these cryptocurrencies and map the aggregate number to N_t. We map the data to early stage of adoption in the model (i.e., $N_t \leq 0.5$). We normalize the maximum number of active addresses (in December 2017) to $N_t = 0.5$ and scale the number of addresses in other months by that of December 2017. For each month, we also need a value of $\ln(A_t)$. Since we cannot observe the platform quality, we assign December 2017 the value of $\ln(A_t)$ in our model that corresponds to $N_t = 0.5$. With December 2017 as a reference point, we calculate the values of $\ln(A_t)$ for other months by applying forward and backward the expected growth rate of A_t under the physical measure. As a result, we focus on the stage of adoption, that is, $N_t \in [\underline{N}, 0.5]$, where $\underline{N} = 0.0001$.

Next, we choose parameter values such that the model generates data patterns in Figures 2 and 4. We set the annual risk-free rate, r, to 5% and choose $\mu^A = 2\% < r$ to satisfy the no-arbitrage restriction. As we have previously discussed, we interpret A_t as a process that broadly captures technological advances, regulatory changes, and the variety of activities feasible on the platform, all of which suggest a fast and volatile growth of A_t. This consideration motivates us to choose $\sigma^A = 200\%$. In Appendix C2, we conduct comparative statics for μ^A and σ^A. As shown in Figure E.6, the model does generate a close link between the technology volatility and that of token returns, likely to due the fact that we focus on fundamental aspects of adoption and valuation while do not fully capture the behavioral and, in general, speculative factors in the model. That said, our choice of $\sigma^A = 200\%$ leads to a token return volatility that is close to the median cryptocurrency's return volatility in Hu, Parlour, and Rajan (2018). They document that the median cryptocurrency's daily return volatility is 14.6%, which is annualized to 232%.

This choice of $\sigma^A = 200\%$ gives us not only a high volatility for A_t but also much of the growth for A_t under the physical measure, as the physical measure drift of A_t is $\widehat{\mu}^A = \mu^A + \eta\rho\sigma^A$ (Girsanov's theorem). To match the growth of N_t in the data, we set $\eta\rho = 1$, so that $\widehat{\mu}^A = 202\%$ using the preceding equation. As a result, the user base N_t grows from $\underline{N} = 0.0001$ to 0.5 during the 8-year period of our data sample and the growth rate for the model-implied N_t matches that in data. One way to generate $\eta\rho = 1$ is to set η to 1.5, which is roughly the Sharpe ratio of ex-post efficient portfolio in the U.S. stock market (combining various factors) and ρ to 0.67, a sensible choice of betas for the technology sector (Pástor and Veronesi, 2009).

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[32] We include all sixteen cryptocurrencies with complete market cap and active address information on bitinfocharts.com: AUR (Auroracoin), BCH (Bitcoin Cash), BLK (BlackCoin), BTC (Bitcoin), BTG (Bitcoin Gold), DASH (Dashcoin), DOGE (DOGEcoin), ETC (Ethereum Classic), ETH (Ethereum), FTC (Feathercoin), LTC (Litecoin), NMC (Namecoin), NVC (Novacoin), PPC (Peercoin), RDD (Reddcoin), VTC (Vertcoin). They represent more than 2/3 of the entire crypto market.

By no arbitrage, the drift of A_t, μ^A, is smaller than r under the risk-neutral measure, because after full adoption, $\mu_t^P = \mu^A$ as implied by the boundary condition. Therefore, for the model to generate the high growth of user base in data, we need the drift of A_t to be high under the physical measure, which requires, first, a high volatility of A_t and, second, a high enough $\eta\rho$. Setting ρ to 0.67 seems at odds with the existing studies on the returns of cryptocurrencies that show their correlations with the returns of traditional assets and macroeconomic factors are low (Hu, Parlour, and Rajan 2018; Liu and Tsyvinski 2018). However, we argue that in our model, dZ_t^A captures the shocks to the underlying technology or platform quality instead of direct return shocks. Moreover, the returns of cryptocurrencies can be driven by factors outside of our model, and such factors can add noise orthogonal to the SDF and reduce the correlation between cryptocurrency returns and the SDF.

We use the normalized distribution for u_i by truncating the Normal density function $g(u) = \sqrt{\frac{1}{2\pi\theta^2}} e^{-\frac{u^2}{2\theta^2}}$ within six-sigma on both sides. As the dispersion of u_i determines how responsive N_t is to the change of A_t, we match the curvature of N_t with respect to A_t by setting $\theta = 10\sqrt{2}$, which implies that thee cross-section variance of u_i is 50. Note that what drives agents' adoption and token holding decisions is e^{u_i}. A normal distribution of u_i implies a log-normal distribution of e^{u_i} that features a concentrated mass in the range of low u_i and a heavy right tail. Our choice of this distribution is guided by data. In Appendix C2, we compare the adoption curve implied by the normal distribution of u_i and that from a uniform distribution, and show that the normal distribution generates an adoption curve that fits data better.

We set α to 0.3 so that the senstivity of $\ln(P_t)$ with recpect to N_t matches the data in the region where $N_t \in [\underline{N}, 0.5]$ as we show in Figure 4. The remaining parameters quantitatively do not affect much the equilibrium dynamics. We set the participation cost, ϕ, to one and normalize M to 10 billion. As our model features monetary neutrality, P_t is halved when M is doubled but importantly the equilibrium dynamics is invariant.

C2. Comparative Statics

We conduct comparative static analysis for μ^A, σ^A, and $G(u)$. We report in the following that the qualitative results are robust to the choices of μ^A and σ. The normal distribution of agents' type, u_i, is guided by data. Agents' transaction need, that is, what drives token demand, is e^{u_i}, which follows a log-normal distribution featuring heavy tail. To demonstrate the desirability of using a normal distribution of u_i, we also report the model-implied adoption curve from a uniform distribution of u_i, and show that it fails to match data.

Risk-adjusted productivity growth μ^A.   As shown in our revised draft, the dynamics of A_t under the physical measure is

$$\frac{dA_t}{A_t} = \left(\mu^A + \eta\rho\sigma^A\right) dt + \sigma^A dZ_t. \tag{C.35}$$

Here we do not necessarily require the growth rate of A_t under the physical measure to be small, and in particular, below r.

The risk-adjusted drift of A_t, μ^A, must be below r following the standard no-arbitrage condition. Under the risk-neutral measure, the total return of any asset is equal to r. Upon full adoption, the expected token return under the risk-neutral measure, μ_t^P, converges to μ^A (see the upper boundary condition for the token pricing ODE), so we must have $\mu_t^P = \mu^A < r$ once A_t increases beyond the upper boundary.

Figure C.3 reports the ratio of token price to productivity, P_t/A_t, over the stages of adoption, N_t in panel A and the user base, N_t, against the logarithmic productivity $\ln(A_t)$ under different values of μ^A. The figures show that the qualitative dynamics of P_t and N_t, the two key endogenous variables in the model, are robust to the choices of μ^A. While the level of token price increases significantly in the drift of platform productivity, the adoption curve is largely stable across

38

A

B
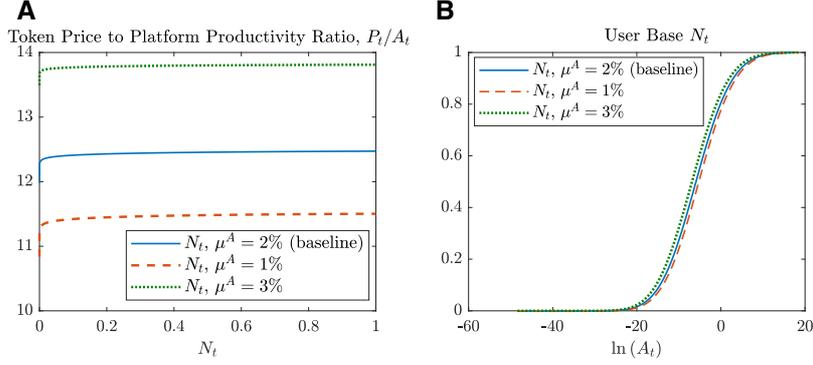
Figure C.3
Comparative statics: Risk-adjusted productivity growth μ^A
Panel A plots the ratio of token price to productivity, P_t/A_t, against the user base, N_t, under different values of μ^A. Panel B plots the user base, N_t, against the logarithmic productivity, $\ln(A_t)$, under different values of σ^A. The values of the other parameters are set according to Appendix C1.

A **B** **C**
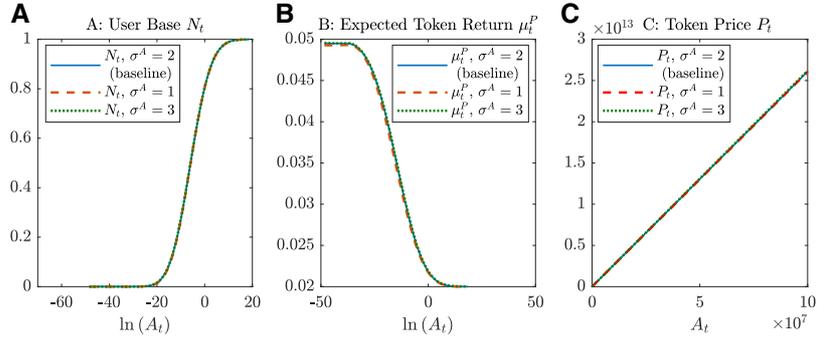
Figure C.4
Comparative statics: Productivity volatility σ^A
Panel A plots the user base, N_t, against the logarithmic productivity, $\ln(A_t)$, under different values of σ^A. Panel B plots the expected token return under the risk-neutral measure, μ_t^P, against the logarithmic productivity, $\ln(A_t)$, under different values of σ^A. Panel C plots the token price, P_t, against the productivity, A_t, under different values of σ^A. The values of the other parameters are set according to Appendix C1.

different values of parameters with the case of the highest μ^A (=0.03) featuring the fastest adoption.

Productivity volatility σ^A. The volatility of platform productivity, A_t, certainly drives the volatility of token price. While the endogenous adoption amplifies the volatility, the token price volatility, σ_t^P, is largely in the same magnitude of A_t's volatility as shown in Appendix E. The focus of this paper is not on generating excessive volatility. Here we model agents' adoption and token demand decisions that are purely based on utility maximization under rational expectation, while in reality, a great number of other factors affect token demand (e.g., Griffin and Shams 2020; Liu, Tsyvinski, and Wu 2019). In fact, the upper boundary condition at full adoption shows that the
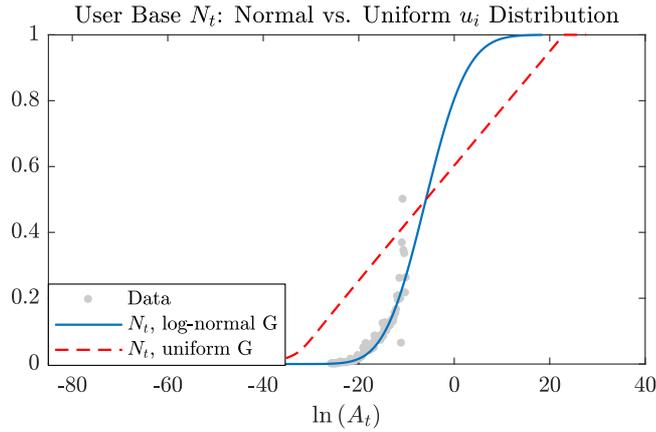
Figure C.5
Comparative statics: The distribution of u_i
This figure plots the user base, N_t, against the logarithmic productivity, $\ln(A_t)$, under the normal distribution of u_i in Appendix C1 (blue solid curve) and the uniform distribution of u_i (red dash line). The support of the uniform distribution is $[-29, 29]$, chosen so that the productivity at full adoption matches that of the baseline model.

token price evolves in lockstep with the platform productivity after N_t reaches 100%. Therefore, this paper does not investigate whether token volatility deviates from that of fundamentals or why.

The aim is thus to develop a rational benchmark of token pricing and endogenous adoption. Accordingly, the model has the desirable feature that the key results are largely not affected by the choices of the volatility parameter, σ^A. Panel A of Figure C.4 plots the user base, N_t, for different values of σ^A. The curves are almost identical. As shown by (30) and (31), what determines N_t are A_t and μ_t^P. Therefore, in Panel A of Figure C.4, given A_t, the adoption curves are similar across different values of σ^A because the values of μ_t^P are similar as shown in panel B. In our model, agents' decisions on adoption and token demand depends on A_t and μ_t^P, and the parameter σ^A only affects agents' decision through μ_t^P. Using Itô's lemma, we have

$$\mu_t^P = \frac{P'(A_t)}{P(A_t)} A_t \mu^A + \frac{1}{2} \frac{P''(A_t)}{P(A_t)} \left(A_t \sigma^A\right)^2. \tag{C.36}$$

The choices of σ^A does not strongly affect the equilibrium outcome because the second-order derivative of P_t to A_t is close to zero. Panel C reports the token price, which is almost linear in A_t (i.e., has a second-order derivative close to zero).

Agent type distribution $G(u)$. We choose a normal distribution for $G(u)$ and set the parameter in order to generate an adoption curve the matches well the data. Note that even though the participation threshold, $\underline{u}_t$ in (30), has $\ln(A_t)$ in its expression, we cannot draw the conclusion that the adoption curve, $N_t = 1 - G\left(\underline{u}_t\right)$, is S-shaped as the cumulative distribution function of normal distribution. The threshold also contains other endogenous variables, such as the current user base, N_t, and the expected token return, μ_t^P. Therefore, the adoption curve is S-shaped as a result of multiple endogenous forces in equilibrium with the parameter choice of $G(u)$. Many blockchain platforms cater to a small group of enthusiasts and thus a distribution of agents' type should feature a fat tail. Indeed, this is what a normal distribution of u_i captures. Agents' token holdings and participation profits are given by (28) and (29) respectively. Both are proportional to e^{u_i}. Therefore, in our model, what drives agents' activities and profits on the platform is e^{u_i}, which follows a log-normal distribution when u_i follows a normal distribution. The log-normal

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distribution has its mass concentrated in the small values (i.e., the majority who have low transaction needs) and a heavy tail that extends to very large values (i.e., the minority of very active users).

For robustness, we plot in Figure C.5 the user base, N_t, against the logarithmic productivity, $\ln(A_t)$, under the normal distribution of u_i in Appendix C1 (blue solid curve) and the uniform distribution of u_i (red dash line). The support of the uniform distribution is $[-29, 29]$, chosen so that the productivity at full adoption matches that of the baseline model. It is clear that a uniform distribution would not fit the data and our distributional assumption is guided by the need to fit empirical data.

Appendix D. Endogenous Choice of Platform Currency

In the main text, we analyze the tokenized and tokenless economies and compare their adoption dynamics. The numerical solution shows that μ_t^P is positive across the values of A_t with $\mu^A > 0$, and therefore, tokens accelerate adoption by reducing the carry cost by μ_t^P. A potential concern is that the conclusion is only valid under the specific set of parameter values. We address this issue by extending the model to incorporate platform users' voluntary choice between the numeraire good and token as means of payment on the platform. For this general setup, we can prove that $\mu_t^P > 0$ in equilibrium. Every instant, agents can choose either the numeraire good or tokens as means of payment. The maximized profit from using the numeraire good is given by (21):

$$\left[N_t A_t e^{u_i} \alpha \left(\frac{1-\alpha}{r} \right)^{\frac{1-\alpha}{\alpha}} - \phi \right] dt. \tag{D.37}$$

The maximized profit from using token is given by (29):

$$\left[N_t A_t e^{u_i} \alpha \left(\frac{1-\alpha}{r - \mu^P(A_t)} \right)^{\frac{1-\alpha}{\alpha}} - \phi \right] dt. \tag{D.38}$$

Considering the choice of currency, a participating agent's maximized profit is

$$\max \left\{ \left[N_t A_t e^{u_i} \alpha \left(\frac{1-\alpha}{r} \right)^{\frac{1-\alpha}{\alpha}} - \phi \right] dt, \left[N_t A_t e^{u_i} \alpha \left(\frac{1-\alpha}{r - \mu^P(A_t)} \right)^{\frac{1-\alpha}{\alpha}} - \phi \right] dt \right\}$$

$$\geq \left[N_t A_t e^{u_i} \alpha \left(\frac{1-\alpha}{r} \right)^{\frac{1-\alpha}{\alpha}} - \phi \right] dt. \tag{D.39}$$

In the states where $\mu^P(A_t) > 0$, agents choose token; otherwise, agents can always opt for the numeraire good as means of payment. The adoption-accelerating effect does not require $\mu_t^P > 0$ for every value of A_t. As long as there exists values of A_t such that $\mu^P(A_t) > 0$, in those states, participating agents choose to use tokens and derive more profits, and as a result, the user base is higher than that in a tokenless economy.

Proposition D1 (Endogenous Currency Choice). If token is available, the user base, N_t, is equal to or larger than the user base in the tokenless economy for any given A_t.

In this more general setting, agents only use tokens when $\mu_t^P > 0$, and thus, by introducing tokens, platforms accelerate adoption by simply expanding agents' choice set, allowing them to pick the currency with the lowest carry cost.

Next, we show that in equilibrium, agents either always choose to use tokens (and $\mu_t^P > 0$) or always choose to use the numeraire goods as means of payment. We prove this statement by contradiction. Consider an equilibrium where at time t, agents choose to use the numeraire goods, but there exists a positive probability that agents choose to use tokens in the future. To clear the

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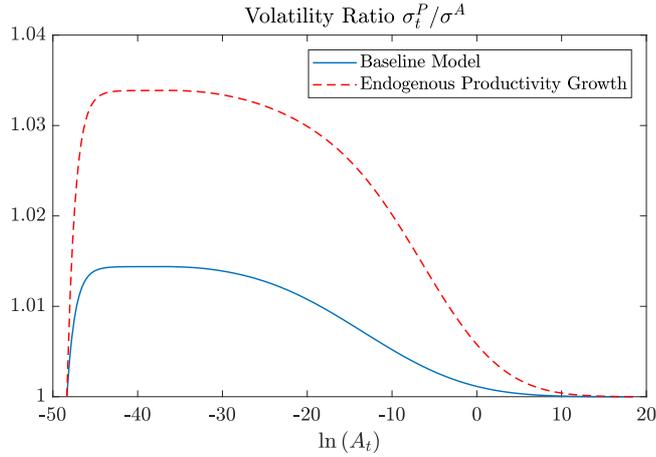
Figure E.6
The persistent impact of shocks and volatility amplification
This graph shows the ratio of token price volatility, σ_t^P, to productivity volatility, σ^A, which quantifies the strength of volatility amplification by the endogenous user adoption. The solid line represents the baseline model. The dashed line represents the model with endogenous platform productivity.

token market, the aggregate token demand must be positive and equal to M. Since agents do not use tokens as means of payment and $\mu_t^P \leq 0$, for the aggregate demand to be equal to M, the token price must be zero because the transactional benefit is zero and the expected financial return, $\mu_t^P \leq 0 < r$. However, there is a positive probability that in the future, agents use tokens, derive transactional benefits, and thus $P_t > 0$. The current $P_t = 0$ then implies a positive infinite expected return, contradicting $\mu_t^P \leq 0$.

Proposition D2 (Consistent Choice of Currency). In equilibrium, agents either always use tokens and $\mu_t^P > 0$ or always use the numeraire goods as means of payment.

Proposition D2 states that our choice to examine the tokenized and tokenless economics separately in the main text does not lose generality, because even when agents are allowed to choose the platform currency every instant, the equilibria that emerge will either feature complete token usage or complete usage of the numeraire goods. Moreover, in the equilibrium where agents use tokens, $\mu_t^P > 0$ holds in every state of the world.

Appendix E: Additional Results on Token Price

E1: Token Volatility and Endogenous Adoption
The shocks to platform productivity are transmitted to token price through users' decision on adoption and token holdings. In fact, token-price volatility σ_t^P is generally larger than σ^A, the productivity volatility, as Figure E.6 illustrates. To see the intuition, consider a positive shock to A_t that directly increases the utility flow of token holdings. User adoption increases as a consequence, which leads to an even higher utility flow (as N_t enters into the utility flow). This feedback effect amplifies the shock's impact on token price, which implies that endogenous user adoption amplifies volatility.

Importantly, our model features a new form of endogenous risk that is unique to platform economics. The volatility of token price is larger than the productivity volatility (exogenous risk) in equilibrium due to endogenous adoption. The amplification constitutes an endogenous asset-price risk that is distinct from the fire-sale risk triggered by the balance sheet channel in the macro-finance

literature (e.g., Brunnermeier and Sannikov 2014; He and Krishnamurthy 2011, 2013). Note that under full adoption $N_t = 1$, (36) reveals that the ratio of P_t to A_t is a constant, so $\sigma_t^P = \sigma^A$ and the endogenous risk disappears. The network effect induces strategic complementarity in agents' adoption decision, and thereby, amplifies the impact of fundamental shock. This mechanism is related to the literature on strategic complementarity and fragility (e.g., Goldstein and Pauzner 2005). Here, the endogenous risk from strategic complementarity manifests itself in the equilibrium asset (token) price.

Figure E.6 also shows that the token price volatility exhibits nonmonotonic dynamics as the platform productivity grows: σ_t^P shoots up in the early stage, gradually declines, and eventually converges to σ^A as N_t approaches one. This dynamic is broadly consistent with the following observations: token price volatility for a nascent platform is large and the cross-sectional variation of volatility is also greater for nascent platforms.

Our analysis thus far has taken the platform productivity process as exogenous. In reality, many token and cryptocurrency applications feature an endogenous dependence of platform productivity on the user base. A defining feature of blockchain technology is the provision of consensus on decentralized ledgers.[33] In general, the more users on the platform, the more economic activities taking place (i.e., higher A_t). Moreover, a greater user base attracts more resources and research onto the platform, accelerating the technological progress on the platform and creating a positive feedback loop.

To capture this positive feedback feature between productivity and user base, we generalize the process of A_t as follows:

$$\frac{dA_t}{A_t} = \omega\left(N_t - \underline{N}\right)dt + \sigma^A dZ_t^A, \tag{E.40}$$

where $\omega > 0$ and $\underline{N}$ is the minimal user base to achieve nonnegative drift. The lack of early adoption reduces the likelihood of the platform achieving a high level of adoption. Because when A_t and N_t, are low, μ_t^A is negative, and thus, the only way for A_t to grow, that is, $dA_t > 0$, is to have a long sequence of positive shocks, that is, $dZ_t^A > 0$. In contrast, when A_t and N_t are high, A_t grows from both a positive drift and positive shocks.

For our numerical solution, we set ω to 0.04 and $\underline{N}$ to 0.5, so when $N_t < 0.5$, $\mu_t^A < 0$, and when $N_t = 1$, μ_t^A is equal to 0.02, which is the value of μ^A in the numerical solution of the main model. Solving the Markov equilibrium under endogenous productivity only requires to replace μ^A in (6) with $\omega\left(N_t - \underline{N}\right)$. The growth rate of A_t is no longer i.i.d. and the shock's impact on A_t becomes more persistent. Consider a positive shock. Not only the current A_t increases, but through N_t, the growth *rate* of A_t increases, propagating the shock's impact into the future. This amplifies the volatility of token price. Consider a negative shock. It not only drags down the current productivity and adoption but also, through $\mu_t^A = \omega\left(N_t - \underline{N}\right)$, reduces the likelihood of higher future productivity and adoption. The direct impact on the current adoption decreases the token price via a decline of the current token demand. Moreover, the decline of expected future adoption implies a weaker token demand in the future, and as a result, a decrease of μ_t^P that further discourages adoption and depresses the current token demand, resulting in a further decline of token price. Therefore, we expect the current specification to deliver a stronger shock amplification effect than the case of constant (and positive) μ^A. Figure E.6 shows that the ratio of σ_t^P to σ^A is higher with endogenous productivity than with exogenous productivity.

Our discussion on the token price volatility relates to the current debate on sources of cryptocurrency volatilities and the possibility of creating stable coins for means of payment. The endogenous adoption is a natural mechanism that transmits and amplifies the shocks to platform fundamentals (A_t) to the token price. However, as shown in Figure E.6, this amplification

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[33] In a "proof-of-stake" system, the consensus is more robust when the user base is large and dispersed because no single party is likely to hold a majority stake; in a "proof-of-work" system, more miners deliver faster and more reliable confirmation of transactions, and miners' participation in turn depends on the size of user base (e.g., through transaction fees).

mechanism is not strong under the current parameter choices for our numerical solution. Therefore, if one has in mind a token-based ecosystem as described in our paper, the concern over token price stability, and the associated benefits of having a stable token, should be directed towards the volatility of platform fundamentals rather than active exit/entry of users. To look for stronger volatility amplification mechanism, one may consider the economic forces outside of our model, such as behavioral factors and token market structure.

E2: Technology Adoption and Bubbly Token Price

The prices of several prominent cryptocurrencies experienced significant runup followed by a crash and subsequent stabilization. We show that such "bubbly" dynamics of token price dynamics can arise in a rational model with endogenous adoption driving the correlation between the SDF and token return. So far, we have set up and analyzed the model under the risk-neutral measure. Next, we explicitly model the risk premium as a function of user base N_t. Let ρ_t denote the instantaneous correlation coefficient between the productivity shock, $d\widehat{Z}_t^A$ in (1), and the SDF shock, $d\widehat{Z}_t^\Lambda$ in (4). To model the endogenous beta of platform productivity, we allow ρ_t to depend on N_t. Suppose that $d\rho(N_t)/dN_t > 0$, which means that the productivity beta increases as the user base grows. As the technology becomes more "mainstream," shocks to it become increasingly systematic. This assumption is inspired by the adoption-dependent beta of new technologies in Pástor and Veronesi (2009). By using Girsanov's theorem, we obtain the following productivity process under the risk-neutral measure,

$$\frac{dA_t}{A_t} = \left[\widehat{\mu}^A - \eta\rho(N_t)\sigma^A\right]dt + \sigma^A dZ_t^A. \tag{E.41}$$

When the productivity shock becomes more correlated with the SDF shock, investors demand a higher risk premium, which lowers the risk-adjusted growth of productivity. In other words, the risk-neutral expected growth rate of A_t is $\widehat{\mu}^A - \eta\rho(N_t)\sigma^A$, is lower. To solve the Markov equilibrium, we simply replace μ^A in (6) with this risk-adjusted drift of A_t.

Thus, two opposing forces drive P_t as A_t grows. On the one hand, the mechanism that increases P_t is still present: when A_t directly increases the flow utility of token or indirectly through its positive impact on N_t, token price increases. On the other hand, the risk premium increases as N_t increases, so the risk-adjusted growth of A_t declines, which in turn decreases P_t. The former channel could dominate in the early stage of adoption, whereas the latter channel dominates in the late stage of adoption. Therefore, P_t first rises with N_t and then declines as N_t reaches a sufficiently high level, which resembles a bubble, as shown in the left panel of Figure E.7.[34]

Appendix F: Classical Models and Tokenomics

F1: Tokenomics and Monetary Economics

By trading in the token market, users profit from platform growth and effectively reduce the carry cost of conducting transacting. Therefore, for payment platforms, the prominent adoption problem in platform economics (e.g., Rochet and Tirole 2006) is naturally connected with the carry cost in the classical models of money as a transaction medium (e.g., Baumol 1952; Tobin 1956). In the following, we compare our model with the standard monetary models (i.e., the money-in-utility and cash-in-advance models).

The token price, P_t is the price of tokens denominated in the numeraire consumption goods. It is how many units of goods that one unit of token can buy. Therefore, $x_{i,t} = P_t k_{i,t}$ is the *real* value of token holdings, and $k_{i,t}$, the units of tokens, is the *nominal* value of token holdings. An agent cares about her *real* token value, that is, how many units of goods the $k_{i,t}$ units of tokens can be exchanged for, rather than the *nominal* token value, that is, the units of tokens, as the agent derives utilities from the real value of transactions in the units of numeraire consumption goods.

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[34] In the numerical solution, we set $\widehat{\mu}^A = 4\%$ and $\rho(N_t) = \widehat{\mu}^A N_t/3$ for illustrative purposes.

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A

Token Price to Long-run Value Ratio $P_t/\overline{P}$

B

Token Price to Long-run Value Ratio $P_t/\overline{P}$
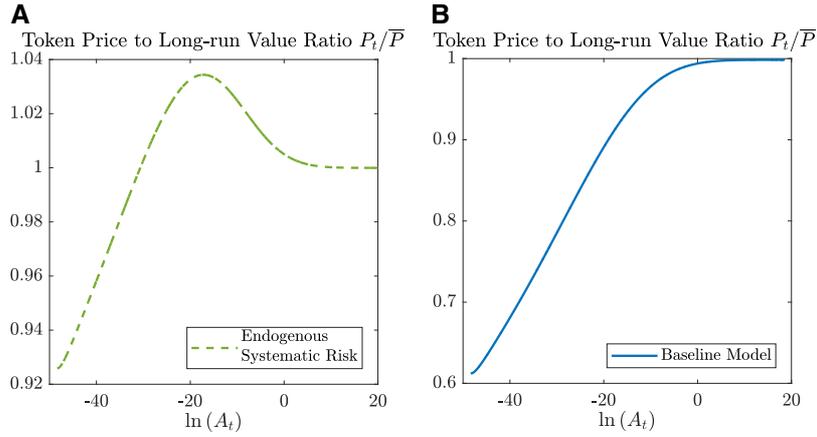
Figure E.7
The "bubbly" behavior of token price This graph plots the ratio of token price, $P(A_t)$, to the long-run (full-adoption) value of token, $\overline{P}(A_t)$, which shows how endogenous adoption shapes the token price dynamics. The solid line shows the baseline model. The dash line shows the model with endogenous systematic risk.

We follow the existing literature on money as means of payment. In theoretical models of monetary economics (e.g., Baumol 1952; Tobin 1956; Feenstra 1986; Freeman and Kydland 2000) and empirical studies (e.g., Poterba and Rotemberg 1986; Lucas and Nicolini 2015; Nagel 2016), agents derive utility from the real value of money holdings.[35] Table F.1 provides a side-by-side mapping between our model of tokens as means of payment and the standard models in the monetary economics literature (e.g., money-in-utility and cash-in-advance models). We refer readers to the classical textbook treatments regarding monetary economics (e.g., Galí 2015; Ljungqvist and Sargent 2004; Walsh 2003).

Note that our notations differ from those in standard monetary economics literature. We use P_t as the value of token denominated in the numeraire consumption good. This is different from the convention that money serves as the numeraire and the prices of goods are denominated in money.

The micro-foundation of our flow utility in Appendix A also sheds light on why it is the real (goods) value of token holdings, instead of the nominal value, that matters. Agent i holds $k_{i,t}$ units of tokens in preparation for a Poisson-arriving transaction opportunity because carrying tokens provide payment convenience. When agent i conducts transactions, the transaction counterparty demands payments in the units of numeraire consumption goods instead of in the units of tokens, because eventually, the transaction counterparty cares about the amount of goods that tokens can buy, not tokens themselves.[36]

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[35] For the nominal value of means of payment to affects agents' decisions, additional frictions, such as nominal illusion (e.g., Shafir, Diamond, and Tversky 1997) or sticky prices (e.g., Christiano, Eichenbaum, and Evans 2005), have to be introduced into the model.

[36] For example, Opensea is an DApp marketplace on Ethereum that allows users to buy and sell through smart contracts the crypto collectibles, gaming items, and other items backed by the Ethereum blockchain. A seller Koryue listed to sell MCH Hero: #40320018 Lv.78 at 6:17:37 a.m. on January 4, 2020, for 2.1 ETH, with Ether price then being about US$133. The same person relists the item at 5:54:29 p.m. on January 16, 2020, for 1.5 ETH, when Ether price was about US$166. The item is sold about an hour later. To the extent that during the short span, agents' valuation of the item did not change dramatically, we can see that they care about not the number of tokens (2.1 vs. 1.5) but the actual dollar value that they can use for offline consumption (279 vs. 249). The latter only has slight drop (possibly because of the seller lowering the price slightly), but it differs a lot in ETH.

Table F.1
Comparing our model and standard monetary models

	Our model	Monetary economics
One unit of token/money buys:	P units of numeraire goods	$1/\mathbb{P}$ units of goods
Token/money units (nominal value):	$k_{i,t}$ units of token	$\mathbb{M}$ units of money
Real token/money balance:	$Pk_{i,t}$	$\mathbb{M}/\mathbb{P}$
Effective carry cost:	$r - \mu_t^P$	r

This table provides a side-by-side mapping between our tokenized model economy and the standard monetary economic models, such as money-in-utility and cash-in-advance models.

F2: Token Pricing and the Black-Scholes Differential Equation

Our token pricing equation (32) shares some common features with the Black-Scholes derivative pricing equation. Both are second-order ordinary differential equation of an asset price that evolves with an underlying state variable. However, the economic mechanisms that lead to these equations are fundamentally different. The Black-Scholes equation is derived from the no-arbitrage condition. Our token pricing equation (32) is obtained from aggregating platform users' token demand and clearing the token market. In the following, we list the steps that we use to obtain (32) and then discuss its differences and similarities with the Black-Scholes option derivative equation.

First note that the state variable for our equilibrium solution is the exogenous productivity A_t, and thus, all endogenous variables are functions of A_t in equilibrium. For example, the equilibrium token price can be written as $P_t = P(A_t)$, a univariate function of A_t. Similarly, $N_t = N(A_t)$ and $S_t = S(A_t)$. Figure B.2 in Appendix B plots the solved endogenous variables that are all functions of A_t from our model solution. By rewriting the token market-clearing condition (32), we obtain the following equilibrium condition for the drift μ_t^P:

$$\mu_t^P = r - (1-\alpha)\left(\frac{N_t S_t A_t}{P_t M}\right)^\alpha. \tag{F.42}$$

By applying Itô's lemma to $P_t = P(A_t)$, we may express the drift of P_t, μ_t^P, as follows:

$$\mu_t^P = \frac{P'(A_t)}{P(A_t)} A_t \mu^A + \frac{1}{2}\frac{P''(A_t)}{P(A_t)}\left(A_t \sigma^A\right)^2. \tag{F.43}$$

Finally, (F-42) with (F-43) together give us (34), which we state below for the ease of reference:

$$r P(A_t) = P'(A_t) A_t \mu^A + \frac{1}{2}P''(A_t)\left(A_t \sigma^A\right)^2 + (1-\alpha)\left(\frac{N_t S_t A_t}{P_t M}\right)^\alpha P(A_t). \tag{F.44}$$

Next, we compare our token pricing equation with the Black-Scholes derivative pricing equation that values a derivative on a dividend-paying stock. Let S_t denote the ex-dividend price of stock, which follows a geometric Brownian motion under the physical measure:

$$\frac{dS_t}{S_t} = (\mu^S - \delta)dt + \sigma^S dZ_t, \tag{F.45}$$

where μ^S, σ^S, and δ are all constant and Z_t is a standard Brownian motion. Note that this stock continuously pays dividends at a constant dividend yield δ. Let V_t denote the price of a derivative which delivers a contingent payment $G(S_T)$ at maturity T, where $G(\cdot)$ is a function of S_T, the underlying stock price at maturity T. (See, e.g., the Black-Scholes European call option pricing formula corresponds to the case where $G(S_T) = \max\{S_T - K, 0\}$.)

Many similar cases show that people think in terms of numeraire (the actual consumption value of those Ethers), not in terms of the number of Ethers. When the Ether price fluctuates, the agents simply adjust the number of Ethers when listing prices.

By the standard no-arbitrage argument (e.g., Duffie, 2001), the value of this derivative, V_t, is given by

$$V_t = \mathbb{E}_t\left[e^{-r(T-t)} G(S_T) \right], \tag{F.46}$$

where $\mathbb{E}_t$ is the conditional expectation under the risk-neutral measure. We know that the value of the derivative V_t is Markovian, which we express as $V(S_t,t)$, The standard no-arbitrage argument implies that $V(S_t,t)$ satisfies the following PDE

$$r V(S_t,t) = \frac{\partial V(S_t,t)}{\partial S_t} S_t(r-\delta) + \frac{1}{2}\frac{\partial^2 V(S_t,t)}{\partial S_t^2}\left(S_t\sigma^S\right)^2 + \frac{\partial V(S_t,t)}{\partial t}. \tag{F.47}$$

Comparing our equilibrium token pricing ODE (34) with (F-47), the PDE for derivative pricing, we make the following observations:

1. **Difference: Price determination mechanism.** The derivative pricing equation is derived from the absence of arbitrage between the derivatives and their underlying assets (and risk-free bonds). Economic profits from trading do not exist in the standard derivative pricing framework. Our token pricing equation is derived from the token market-clearing condition, which implies that the marginal participant (with $u_i = \underline{u}_t$) breaks even but all other participants collect positive profits. Agents with $u_i > \underline{u}_t$ break even *at the margin* as required by their first-order conditions, but their *average* revenues are larger than their *costs* due to the concavity (or decreasing return to scale) of transactional benefits, $\left(x_{i,t}\right)^{1-\alpha}(N_t A_t e^{u_i})^\alpha dt$, in the numeraire value of token holdings, $x_{i,t}$.

2. **Difference: Token's "flow" term.** Another difference between the two pricing equations is that token has an additional "flow" term, $(1-\alpha)\left(\frac{N_t S_t A_t}{P_t M}\right)^\alpha P(A_t)$, while the derivatives pricing model does not.[37] This "flow" term appears in our model because of the aggregation of heterogeneous users' demand for tokens as a transaction medium.

3. **Similarity: the same risk-adjusted expected return.** Our token pricing equation and the Black-Scholes pricing equation have the value of asset multiplied by r on the left side, meaning that both feature a marginal risk-adjusted return equal to r, the risk-free rate. Note that the total risk-adjusted return of tokens is the sum of transactional benefits and the financial return (see (27) in our revision).

4. **Similarity: The coefficients of $\frac{\partial V(S_t,t)}{\partial S_t}$ and $\frac{dP(A_t)}{dA_t}$.** The state variable of derivatives is the underlying stock's price. Since the stock is traded, so it has an expected cum-dividend return equal to r, and thus, its ex-dividend stock price has a drift that is equal to $S_t(r-\delta)$ under the risk-neutral measure, which is the coefficient on $\frac{\partial V(S_t,t)}{\partial S_t}$. The underlying state variable of tokens is the platform productivity A_t. The coefficient on $\frac{dP(A_t)}{dA_t}$ is $A_t\mu^A$, also the state variable's drift. Since A_t is not traded, the risk-neutral drift, $A_t\mu^A$, is not equal to $A_t r$, resulting in a shortfall similar to δ in the $(r-\delta)$ in the coefficient.

5. **Similarity: The quadratic variation terms.** Both our token pricing equation and the standard derivative pricing equation have the second-order derivative term that depends on the volatility of the underlying state variable (A_t for tokens and S_t for derivatives). As a result, we have the same similar quadratic variation terms ($\left(A_t\sigma^A\right)^2$ for tokens, and $\left(S_t\sigma^S\right)^2$ for derivatives) that is, the coefficients on $\frac{\partial^2 V(S_t,t)}{\partial S_t^2}$ and $\frac{d^2 P(A_t)}{dA_t^2}$ respectively, from Itô's calculus.

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[37] The absence of a flow term in the derivative pricing formula (F-47) is the result of this derivative not having any flow payment before maturity T. If we had allowed this derivative to pay holders prior to maturity, the derivative pricing equation (F-47) would have a "flow" term as well.

6. **Nonessential differences: finite maturity versus perpetuity.** Tokens in our model are perpetual with no maturity. Standard derivatives often have maturity, and hence, have the additional term $\frac{\partial V(S_t,t)}{\partial t}$ in the pricing equation.

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