Consumer Credit and Asset Prices

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Abstract

Consumer credit introduces a wedge between consumption and payments. We incorporate this wedge into a standard consumption-based asset pricing framework and solve the equilibrium in closed form. When firms possess market power, the optimal credit contract endogenously embeds long-run consumption risk directly into the pricing kernel in a transparent and analytically tractable way. This mechanism eliminates the need to estimate the stochastic processes for consumption volatility and expected growth. The resulting pricing kernel accounts for a broad set of cross-sectional return patterns and successfully prices a wide range of stock market anomalies.

Keywords: asset pricing, yield curve, consumer credit, financial frictions, credit frictions
JEL: D11, D50, D51, D53, E21, E51, G12

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1 Introduction

Household borrowing has become a central feature of modern economies. Consumer credit allows households to smooth consumption by decoupling expenditures from income, thereby affecting aggregate demand and risk sharing. Yet its implications for asset prices remain poorly understood.

Most consumption-based asset pricing models implicitly assume that payments and consumption coincide, abstracting from the intertemporal frictions created by credit. This simplification overlooks a key mechanism through which credit markets influence the stochastic discount factor and, ultimately, the cross-section of asset returns.

We develop a general-equilibrium model in which payment delays generate a wedge between consumption and payments. Firms extend credit to consumers and possess market power in the credit market. The resulting equilibrium yields an analytically tractable pricing kernel that embeds long-run consumption risk directly and endogenously. Our analysis highlights how the structure of consumer credit can fundamentally alter the pricing of risk, offering a unified explanation for several persistent return anomalies.

Our key insight is that payment delays break the link between consumption and immediate expenditures. In the delay-adjusted Euler equation, the stochastic discount factor (SDF) is given by:

$$\text{SDF} = \frac{\text{Marginal Utility}}{\text{Price of Perpetuity (delay duration)}}. \tag{1}$$

Intuitively, under payment delays, optimal intertemporal decisions reflect expectations about future repayments. Increasing consumption today leads to a repayment stream whose duration equals the delay period, and the price of the perpetuity captures the present value of these repayments.

Analogously to recursive preference models (Epstein and Zin 1991; Duffie and Epstein

1992; Bansal and Yaron 2004), equation (1) incorporates asset returns directly into the equilibrium SDF. Building on theories of vendor financing (Brennan, Maksimovics, and Zechner 1988; Bertola, Hochguertel, and Koeniger 2005) and empirical evidence from Berg et al. (2025), we extend the model to allow firms with market power to endogenously choose contract terms—specifically, payment delay duration and interest rate—as a form of price discrimination. We derive a closed-form solution for the equilibrium contract and show that it generates an SDF featuring long-run consumption risk, consistent with the empirical findings of Parker and Julliard (2005). Remarkably, this long-run risk arises even though consumers have standard time-separable preferences.

The equilibrium contract results from a subtle tradeoff between demand and price discrimination by firms. Households' demand for current consumption is inversely related to (1) the expected utility of future consumption (via the standard intertemporal wealth channel) and (2) the price of borrowing through the present value of delayed payments, given by the price of perpetuity discussed above. The firm's problem thus reduces to choosing this price of perpetuity, taking the stochastic discount factor as given. In equilibrium, aggregate consumption equals output, which uniquely determines the price of perpetuity as an explicit function of expected future utility. This prediction holds under standard constant relative risk aversion (CRRA) preferences. Time separability and the law of iterated expectations then imply our central theoretical result: the realized utility of future consumption is a priced risk factor. Since realized utility is directly observable, testing our asset-pricing model is straightforward. This sharply contrasts with recursive preference models (Bansal and Yaron 2004), where only expected utility is priced and must be inferred from complex stochastic volatility dynamics (Campbell et al. 2018). We test our pricing kernel using a large dataset of stock market anomalies from Jensen, Kelly, and Pedersen (2023) and find that our model successfully explains many of them.

2 Literature Review

Our paper connects to several important strands of the literature. The first strand studies the impact of balance-sheet constraints on asset price dynamics. For instance, Bernanke and Gertler (1986), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999) examine how credit constraints affect the real (production) sector. Meanwhile, Adrian and Shin (2010), Adrian, Moench, and Shin (2010a), Adrian, Moench, and Shin (2010b), Brunnermeier, Eisenbach, and Sannikov (2012), Brunnermeier and Sannikov (2014), Brunnermeier and Sannikov (2016), He and Krishnamurthy (2013), Adrian, Etula, and Muir (2014), He, Kelly, and Manela (2017), Haddad and Muir (2021), and Baron and Muir (2022) analyze how balance-sheet constraints of financial intermediaries affect risk premia. See, also, Dou et al. (2023) for an excellent review.

A second related literature investigates constraints within the household sector, specifically consumer credit. Following the 2008–2009 financial crisis, extensive research has focused on mortgages—the largest component of household credit—because housing represents a major share of household wealth, serves as a durable good, and acts as collateral. The roles of housing and mortgages in amplifying macroeconomic shocks have been examined by Geanakoplos (2010), Mian and Sufi (2011), Schularick and Taylor (2012), Mian, Rao, and Sufi (2013), Mian and Sufi (2014), Guerrieri and Lorenzoni (2017), Di Maggio et al. (2017), Berger et al. (2018), Greenwald (2018), and Chen, Michaux, and Roussanov (2020). Although our model focuses on short-term, non-durable consumption credit, its main empirical asset-pricing implication—that the expected utility of consumption is priced—is conceptually broader and applies to models involving durable consumption such as housing (e.g., Yogo 2006).

A growing literature uses micro data to study the economics of consumer credit. Examples include Buchak et al. (2018), Fuster et al. (2019), Agarwal et al. (2020), Berg et al. (2020), Di Maggio, Williams, and Katz (2022), Boshoff et al. (2022), Bian, Cong, and Ji

(2023), Guttman-Kenney, Firth, and Gathergood (2023), DeHaan et al. (2024), Berg et al. (2025), and Alok et al. (2024). Many of these papers emphasize consumer heterogeneity, creditworthiness, and the effects of credit on overborrowing. Introducing heterogeneity and explicit credit frictions into our framework is a promising direction for future research.

Pistaferri (2016) extensively examines post-crisis consumption patterns of durables and non-durables, highlighting three channels through which financial frictions depress consumption: (1) wealth effects, (2) credit constraints, and (3) debt overhang or deleveraging. All three mechanisms operate in our model. Consumers who accumulate substantial debt (payments exceeding current consumption) experience reduced future wealth, generating deleveraging consistent with these channels. In our setup, relaxing credit constraints leads to household overconsumption and produces leverage cycles in line with the empirical evidence in Pistaferri (2016).

Our modeling of credit frictions is related to the classic framework of Leland (1998), which features exponentially decaying finite-maturity debt. However, in our frictionless and complete-market environment, default does not occur, and households optimally manage leverage while internalizing future budget constraints. Extending the model to include explicit leverage constraints—similar to Kehoe and Levine (1993) and Alvarez and Jermann (2000)—represents a fruitful avenue for future work.

Our key theoretical prediction-that the expected utility of future consumption is priced-links our paper to the long-run risk literature initiated by Bansal and Yaron (2004). That literature relies on time non-separable preferences, particularly a preference for early resolution of uncertainty, which complicates empirical estimation of the corresponding stochastic discount factor (see, e.g., Campbell et al. 2018). By contrast, our result arises in a simple CRRA setting. Time separability, combined with the law of iterated expectations, allows realized utility to serve directly as the pricing kernel. We find that this realized-utility-based risk successfully prices a large set of anomalies from Jensen, Kelly, and Pedersen (2023).

Our realized utility risk is closely related to the *ultimate consumption risk* introduced by Parker and Julliard (2005). Rather than focusing on contemporaneous covariance between returns and consumption growth, Parker and Julliard (2005) examine the covariance between returns and consumption growth over the quarter of the return and several subsequent quarters. They argue that this measure captures the delayed response of consumption to asset returns and show empirically that it explains a large share of the variation in returns on the Fama–French 25 portfolios. While their evidence is compelling, Parker and Julliard (2005) do not provide a theoretical foundation for this form of the pricing kernel. Our paper offers such a foundation, deriving a closely related measure from a distinct economic mechanism: although consumption itself adjusts immediately, payment obligations are staggered and thus exhibit inertia.

3 Model

In this section, we introduce the building blocks of our general-equilibrium model and outline how it is solved.

The economy consists of a continuum of identical consumers and a continuum of identical firms. Throughout, we focus on a representative consumer and a representative firm. The key assumption is that the representative consumer does not fully repay her time-t consumption immediately but instead borrows from the representative producer through a term debt contract with a geometrically decaying payment schedule.

At any time t, aggregate consumption equals aggregate output, as in Lucas (1978). However, aggregate payments to firms differ from the output they produce. Since the price of a financial asset equals the risk-adjusted present value of its cash flows, the value of an asset whose dividends are given by aggregate output may differ substantially from that in an otherwise identical pure-exchange economy without payment delays.

3.1 Representative Consumer's Problem

We begin by specifying the representative consumer's preferences, then describe her consumption financing and budget constraint, and finally present her optimization problem.

Preferences. We adopt the continuous-time version of Epstein–Weil–Zin (EWZ) preferences from Duffie and Epstein (1992), under which the representative consumer maximizes:[1]

$$V_t = E_t \left[\int_t^\infty f(C_s, V_s), ds \right], \tag{2}$$

where $f(C, V)$ is the normalized aggregator, defined as:

$$f(C, V) = \frac{\beta}{1 - \psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\omega}}{((1-\gamma)V)^{\omega-1}}, \tag{3}$$

Here, β denotes the rate of time preference, ψ the elasticity of intertemporal substitution (EIS), γ the coefficient of relative risk aversion, and $\omega = (1 - \psi^{-1})/(1 - \gamma)$.[2]

If $\gamma = \psi^{-1}$ so that $\omega = 1$, preferences reduce to the standard constant-relative-risk-aversion (CRRA) expected utility, represented by the additively separable aggregator:

$$f(C, V) \;=\; \beta \left(\frac{C^{1-\gamma}}{1-\gamma} - V \right). \tag{4}$$

Consumption, Payments, and Endowments (Dividends). The key assumption of our model is that the agent's consumption is not immediately settled but instead financed through an exponentially decaying stream of payments. While this friction is deliberately

[1] Epstein and Zin (1989) and Weil (1990) developed homothetic non-expected utility in discrete time, which separates the elasticity of intertemporal substitution from relative risk aversion.

[2] Unlike time-additive utility, recursive preferences as defined by equations (2)–(3) separate risk aversion from intertemporal substitution. The marginal benefit of consumption is given by $f_C = \beta C^{-\psi^{-1}}/[(1-\gamma)V]^{\omega-1}$, which depends not only on current consumption but also—through V—on the expected path of future consumption.

stylized and presented in reduced form, it captures a key feature of many real-world financial transactions, payment systems, and various forms of consumer credit.

On the production side, the economy consists of a continuum of identical firms. For simplicity, we abstract from modeling explicit production and investment decisions and assume that each firm owns a Lucas tree yielding a random number of "apples" that follow an exogenous stochastic process $\{X_t\}_{t\geq 0}$. The price of an apple is normalized to one.[3]

To model the payment delay, we assume that an agent consuming C_t units of apples produced by the representative firm at time $t \geq 0$ does not pay for the entire amount immediately. Instead, for each unit of consumption at time t, the agent continuously repays the firm from time t into the indefinite future. The repayment schedule is fully committed at all times.

Let $p_{t,s}$ denote the payment rate at time $s \geq t$ corresponding to consumption C_t at time t. We write

$$p_{t,s} = e^{(s-t)(n_t-\delta_t)}\delta_t C_t\,, \quad \forall\ s \geq t\,, \tag{5}$$

where δ_t is the instantaneous repayment rate at the consumption date t, and n_t captures the (interest payment) cost of delaying repayment. Together, δ_t and n_t define the contract associated with each unit of the agent's time-t consumption.[4]

At time t, the entire repayment stream financing time-t consumption, $\{p_{t,s}\}_{s\geq t}$, is known.

[3] Our model can be generalized to a production economy by incorporating a homogeneous production and capital accumulation technology, as in Hayashi (1982).

[4] Integrating all repayments for time-t consumption C_t from time t to ∞ without discounting yields:

$$\int_t^\infty p_{t,s}\,ds = \int_t^\infty e^{(s-t)(n_t-\delta_t)}\delta_t C_t\,ds = \frac{\delta_t}{\delta_t - n_t}\,C_t\,. \tag{6}$$

This provides intuition for the repayment schedule. When $n_t = 0$, the agent finances consumption without interest payments. Therefore, the time-t market value of consumption differs from C_t, and this wedge between the market value of the repayment stream and C_t captures the financial friction.

However, the repayment schedule that will finance a unit of future consumption at time $v > t$, given by $p_{v,s}/C_v = e^{(s-v)(n_v-\delta_v)}\delta_v$, is unknown at time t.[5]

Let P_t denote the agent's payment at time t for all past consumption $\{C_v; v \in (0,t)\}$. Starting at time 0 with $P_{0-} = 0$, the agent's total payment at time t, P_t, is given by:

$$P_t \;=\; \int_0^t p_{v,t}\, dv = \;\int_0^t e^{-(t-v)(\delta_v - n_v)}\delta_v C_v\, dv\,. \tag{7}$$

3.2 Effective Stochastic Discount Factor for Representative Consumer

We assume that markets are dynamically complete and there exists a unique (positive) stochastic discount factor (SDF) $\{M_t^*\}_{t\geq 0}$. A consumption plan $\{C_t\}_{t\geq 0}$ for the representative consumer is sustainable if and only if the payment plan $\{P_t\}_{t\geq 0}$ for $\{C_t\}_{t\geq 0}$ in (7) satisfies the following standard inter-temporal budget constraint (Cox and Huang 1989):

$$E\left[\int_0^\infty (P_t - D_t) M_t^* dt\right] \;=\; 0\,. \tag{8}$$

Unlike in standard asset pricing models (where $P_t = C_t$ for all t), in our model, the payment P_t is different from consumption C_t due to payment delays. The representative consumer's binding budget constraint can be written as

$$W_0 \;=\; E\left[\int_0^\infty D_t\, M_t^* dt\right] = E\left[\int_0^\infty P_t\, M_t^* dt\right]\,. \tag{9}$$

[5]For convergence, we require $\delta_t > n_t$ for all t. In real-world contracts, the interest rate n_t is typically positively related to the expected repayment duration $1/\delta_t$.

Substituting (7) into (9), changing the order of integration (using Fubini's Theorem), and using the law of iterated expectations, we can rewrite the budget constraint (9) as follows:

$$\begin{aligned} W_0 &= E\left[\int_0^\infty \int_0^t e^{-(t-v)(\delta_v - n_v)} \delta_v C_v \, dv \, M_t^* \, dt\right] \\ &= E\left[\int_0^\infty E_v\left[\int_v^\infty e^{-(t-v)(\delta_v - n_v)} M_t^* \, dt\right] \delta_v C_v \, dv\right]. \end{aligned} \tag{10}$$

Let

$$\widehat{M}_t \;=\; \delta_t E_t\left[\int_t^\infty e^{-(s-t)(\delta_t - n_t)} M_s^* \, ds\right]. \tag{11}$$

Then, (9) takes the form

$$E\left[\int_0^\infty C_t \widehat{M}_t \, dt\right] \;=\; W_0\,. \tag{12}$$

That is, the agent faces the same standard inter-temporal budget constraint, but with the "true" pricing kernel M_t^* replaced by the "effective pricing kernel" $\widehat{M}_t$.

Let

$$B(t,s) \;=\; E_t[M_s^*/M_t^*] \tag{13}$$

be the equilibrium price of zero-coupon bond at time t with time to maturity s, and

$$A(t,\delta,n) \;=\; \delta \int_t^\infty e^{-(s-t)(\delta - n)} B(t,s) ds \tag{14}$$

be the price of a perpetuity with duration $1/(\delta - n)$, rescaled by δ. The following theorem summarizes the relation between the true SDF M_t^* and the effective SDF $\widehat{M}_t$. (11)

Theorem 1 *In our economy, investors can price the equilibrium aggregate consumption*

(output) stream $C_t = X_t$ *as if in a pure-exchange endowment economy by using the effective SDF* $\widehat{M}_t$ *instead of the true SDF* M_t^**, where*

$$\widehat{M}_t \;=\; M_t^*\, A(t, \delta_t, n_t)\,. \tag{15}$$

The intuition behind (15) is as follows. When a unit of a good is consumed at time t, the agent repays over the interval $[t, \infty)$ in a geometrically decaying manner. A faster repayment schedule implies a smaller portion of the payment at times s contributes to financing consumption at time $t \leq s$. Additionally, the greater the time gap between the payment date s and the consumption date t (i.e., the larger $s-t$), the smaller the fraction of the payment made at s. By aggregating all anticipated future payments—appropriately weighted by their market value—we arrive at formula (15), which captures the present (discounted) market value of the economic cost of consuming one unit of the good at time t. As $\delta \to \infty$, the expected payment delay $1/\delta$ tends to zero, recovering the standard frictionless economy where $\widehat{M}_t = M_t^*$. Conversely, the longer the payment delay $1/\delta$, the greater the wedge between the two pricing kernels, $\widehat{M}_t$ and M_t^*.

Equation (12) allows us to apply standard first-order conditions and get the following result (see Duffie and Epstein 1992), linking consumption with the effective pricing kernel.

Theorem 2 *Suppose that there exists a constant* $\lambda > 0$ *(the Lagrange multiplier of the budget constraint) such that*

$$\lambda\, \widehat{M}_t \;=\; f_c(C_t, V_t)\, \exp\left(\int_0^t f_v(C_s, V_s) ds \right)\,, \tag{16}$$

where V_t *solves* (2) *and* (12) *holds. Then,* C_t *is an optimal consumption stream given the effective pricing kernel* $\widehat{M}_t$*. In particular, in the time-separable case with* $\gamma = \psi^{-1}$*, we get*

$$\lambda \widehat{M}_t \;=\; e^{-\beta t} C_t^{-\gamma}\,. \tag{17}$$

We assume that the firm operates in a simple, frictionless environment where it has no internal savings or investment opportunities. All output produced by the firm is immediately sold and consumed by the representative agent. Furthermore, any revenue generated by the firm is entirely distributed to its shareholders in the form of dividends—there are no retained earnings or reinvestment. In other words, the firm acts purely as a conduit: it transforms inputs into output and distributes the resulting economic value directly back to the consumer, who owns the firm. This setup ensures that the firm's financial activities are fully reflected in household income. Under these assumptions, the following equilibrium conditions hold:

$$\begin{aligned} C_t &= X_t \text{ (consumption = output)} \\ P_t &= D_t \text{ (payments = dividends).} \end{aligned} \tag{18}$$

We can now proceed to characterize Arrow-Debreu equilibria in our economy.

Proposition 3 *A tuple* $(C_t,\ M_t^*,\ \widehat{M}_t)$ *of adapted stochastic processes is an Arrow-Debreu equilibrium if and only if there exists a* $\lambda > 0$ *such that*

$$\begin{aligned} C_t &= X_t \\ \lambda\, \widehat{M}_t &= f_c(X_t, V_t)\, \exp\left(\int_0^t f_v(X_s, V_s) ds \right) \\ V_t &= E_t \left[\int_t^\infty f(X_s, V_s) ds \right] \\ \widehat{M}_t &= \delta_t E_t \left[\int_t^\infty e^{-(s-t)(\delta_t - n_t)} M_s^* ds \right]. \end{aligned} \tag{19}$$

Proposition 3 shows that characterizing an equilibrium in our economy ultimately boils down to solving the integral equation (15) for the fundamental or "true" pricing kernel M_t^*. This pricing kernel reflects the underlying value of consumption at each point in time, net of any frictions or delays in payment. Economically, this means that equilibrium prices must

adjust such that the present value of all future repayment obligations—arising from current consumption—is consistent with how the representative agent discounts future cash flows.

To build intuition, we begin by examining a simplified scenario in which the repayment intensity δ_t and the interest rate n_t are constant over time, i.e., $\delta_t = \delta,\ n_t = n$. This setting allows us to better isolate the role of delayed payments in shaping intertemporal pricing. To make progress on solving for equilibrium in our economy, everywhere in the sequel, we adopt a set of technical assumptions that shape the behavior of the pricing kernel in the frictionless benchmark.

Assumption 1 *We have* $\widehat{M}_t = \widehat{F}(\Theta_t)e^{-\int_0^t \beta(\Theta_s)ds}$, *where* Θ_t *is a Markov jump-diffusion process with an infinitesimal generator* $\mathcal{A}$. *Furthermore, the interest rate in the economy without frictions,*

$$\widehat{r}(\Theta_t) \;=\; (\beta(\Theta_t)\widehat{F}(\Theta) \;-\; \mathcal{A}\widehat{F}(\Theta))/\widehat{F}(\Theta_t)\,, \tag{20}$$

satisfies $1 + \delta^{-1}(\widehat{r}(\Theta_t) - n) > 0$ *almost surely, and*

$$E\left[\int_0^t e^{-2\int_s^r (\delta - n + \beta(\Theta_\theta))d\theta} \|\nabla \widehat{F}(\Theta_r)\sigma(\Theta_r)\|^2 dr\right] \;<\; \infty \tag{21}$$

for all t, *and that the transversality condition*

$$\lim_{t\to\infty} E\left[e^{-\int_0^t (\delta - n + \beta(\Theta_\theta))d\theta}\widehat{F}(\Theta_t)\right] \;=\; 0 \tag{22}$$

holds.

First, we model the frictionless pricing kernel $\widehat{M}_t$ as a function of an underlying state variable Θ_t, which captures the evolution of the economy over time. The pricing kernel $\widehat{M}_t$ then takes a familiar exponential form: it decays at a rate determined by the function $\beta(\Theta_t)$ and is scaled by a state-dependent factor $\widehat{F}(\Theta_t)$. The exponential term reflects the

agent's time preference and risk adjustment, while $\widehat{F}(\Theta_t)$ captures how the current state of the world affects marginal utility. The interest rate in this frictionless economy, $\widehat{r}(\Theta_t)$, is defined in terms of the drift and volatility of the pricing kernel. Economically, this interest rate represents the rate of return on a risk-free bond in a world without frictions. To ensure our model remains well-defined and does not produce nonsensical or explosive behavior, we impose the additional technical conditions of positivity of effective discounting ($1 + \delta^{-1}(\widehat{r}(\Theta_t) - n) > 0$), square integrability, and transversality. The following is true.

Theorem 4 (Constant payment delay duration) *Assume $\delta_t = \delta$ is constant. Then there exists a unique Markovian equilibrium stochastic discount factor (SDF), given by*

$$M_t^* \;=\; \left(1 + \delta^{-1}(\widehat{r}(\Theta_t) - n)\right) \widehat{M}_t \,. \tag{23}$$

This result reveals a clean relationship: the true pricing kernel M_t^* - which reflects the actual economic prices under payment frictions - is a simple scaling of the frictionless benchmark kernel $\widehat{M}_t$. The scaling factor depends on the frictionless interest rate $\widehat{r}(\Theta_t)$, the repayment interest rate n, and the expected repayment delay δ^{-1}.

Economically, formula (23) shows that payment frictions amplify the role of the interest rate in intertemporal pricing. The longer the expected repayment duration (that is, the smaller δ), the more pronounced this amplification becomes. When the effective interest rate is high, immediate consumption becomes more valuable to the representative agent—so the true pricing kernel M_t^* adjusts upward relative to $\widehat{M}_t$. Importantly, in the special case where $\widehat{r}(\Theta_t)$ is constant over time, the distortion of payment delays is purely proportional and does not alter the relative value of consumption between states:

$$M_t^* \;=\; \left(1 + \delta^{-1}(\widehat{r} - n)\right)\widehat{M}_t, \,. \tag{24}$$

3.3 Financial Wealth Dynamics

A large literature shows how credit distorts wealth dynamics, leads to leverage cycles and may serve as an amplification mechanism for macroeconomic shocks. See, for example, Geanakoplos (2010), Mian and Sufi (2011), Schularick and Taylor (2012), Mian, Rao, and Sufi (2013), Mian and Sufi (2014), Pistaferri (2016), Ramcharan, Verani, and Van den Heuvel (2016), and Benmelech, Meisenzahl, and Ramcharan (2017), Guerrieri and Lorenzoni (2017), Di Maggio et al. (2017), Berger et al. (2018), Greenwald (2018), and Chen, Michaux, and Roussanov (2020). Our tractable model directly links customer wealth dynamics and consumer credit. In this section, we investigate this link.

Under the transversality condition, the time$-t$ financial wealth of the representative consumer is given by the present value of future payments:

$$W_t^* \;=\; (M_t^*)^{-1} E_t \left[\int_t^\infty P_s M_s^* \, ds \right] . \tag{25}$$

After some algebraic manipulations, we arrive at the following result.

Proposition 5 *We have*

$$W_t^* \;=\; \int_0^t e^{-(\delta_v - n_v)(t-v)} A(t, \delta_v, n_v) C_v \, dv + A(t, \delta_t, n_t) \, \widehat{W}_t, \tag{26}$$

where

$$\widehat{W}_t \;=\; E_t \left[\int_t^\infty (\widehat{M}_v / \widehat{M}_t) C_v \, dv \right] \tag{27}$$

is the wealth in the frictionless economy.

Proposition 5 provides a closed-form expression connecting past and future (anticipated) consumer credit with actual consumption. The first term in (26) is the value of the total

accumulated consumer credit extended before t. It leads to a form of hysteresis through which past economic conditions affect future equilibrium dynamics. The second term is the present value of all the future consumer credit that is yet to be extended. It is given a simple, explicit formula: The "price (cost) of consumption" (the perpetuity $A(t, \delta_t, n_t)$) in our economy times the financial wealth $\widehat{W}_t$ in the frictionless economy. In the case of logarithmic preferences, $\widehat{M}_t \propto e^{-\beta t} C_t^{-1}$, and we get

$$W_t^* = \int_0^t e^{-(\delta_v - n_v)(t-v)} A(t, \delta_v, n_v) C_v \, dv + A(t, \delta_t, n_t) \frac{C_t}{\beta}, \tag{28}$$

implying a direct relationship between wealth and current and past consumption. In particular, the marginal propensity to consume (MPC) is given by

$$\frac{\partial C_t}{\partial W_t^*} = \frac{\beta}{A(t, \delta_t, n_t)}. \tag{29}$$

This simple, explicit expression allows us to shed new light on the transmission mechanism of monetary policy to consumption. Many macroeconomics models imply that monetary policy affects MPC (see, e.g., Auclert 2019). Formula (29) means that this link might be quite subtle: Lowering interest rates increases $A(t, \delta_t, n_t)$ and, hence, lowers the MPC. Understanding the implications of this mechanism within a New-Keynesian framework is an interesting direction for future research.

4 Endogenous Payment Contracts

In reality, contract terms are shaped by two main frictions: credit risk and price discrimination. Building on the insights of Berg et al. (2025), we now focus on the latter and extend our baseline model to allow firms to jointly choose contract duration and interest rate in order to extract consumer rents.[6]

[6] Incorporating credit risk into this framework remains an important direction for future research.

In this setting, the firm faces the same stochastic discount factor as consumers and evaluates the present value of future payments as

$$\mathcal{P}(\delta_t, n_t, C_t) \;=\; E_t\left[\int_t^\infty \frac{M_s^*}{M_t^*} p_{t,s}\, ds\right] = E_t\left[\int_t^\infty \frac{M_s^*}{M_t^*} e^{(s-t)(n_t-\delta_t)}\delta_t C_t ds\right] \;=\; C_t A_t^*\,, \quad (30)$$

where $p_{t,s} = e^{(s-t)(n_t-\delta_t)}\delta_t C_t$ (see (5)) and A_t^* is short for $A(t, \delta_t, n_t)$. Following the same reasoning as in (10), we obtain the corresponding version of the equilibrium equation (11) for the pricing kernel:

$$\widehat{M}_t \;=\; M_t^*\, A_t^*\,. \quad (31)$$

It is evident that $\mathcal{P}(\delta_t, n_t, C_t)$ increases monotonically with the interest rate n_t. When market interest rate r_t^* is positive, we have $\mathcal{P}(\delta_t, 0, C_t) < C_t$, while $\mathcal{P}(\delta_t, \delta_t, C_t) = +\infty$, indicating the existence of an intermediate interest rate $n_t^* \in (0, \delta_t)$ such that $\mathcal{P}(\delta_t, n_t^*, C_t) = C_t$. With this rate, the firm is indifferent between offering a payment contract or a conventional buy-now-pay-now contract, and equation (31) implies that $M_t^* = \widehat{M}_t$. This yields a "neutrality" result, similar to the Modigliani-Miller Theorem: when firms completely offset the effects of payment frictions, the equilibrium remains unaffected. In practice, numerous market frictions prevent the equality $\widehat{M}_t = M_t^*$ from holding. Such frictions include imperfect competition, heterogeneity in firms' capital costs, differences in discount rates between firms and households, and broader forms of market segmentation. Accounting for these factors would allow us to micro-found the joint dynamics of n_t and δ_t.

It is well documented that financial intermediaries possess significant market power over their clients and incorporate markups into the rates they quote (Drechsler, Savov, and Schnabl 2021; Wang et al. 2022). These frictions are particularly pronounced in over-the-counter markets. In this section, we adapt the local-monopoly framework of Malamud, Schrimpf, and Zhang (2025) and assume that, at time t, firms offer take-it-or-leave-it payment

contracts to households.[7] Firms optimally select the contract terms—the interest rate n_t and duration δ_t—to maximize expected discounted profits, taking into account consumers' downward-sloping demand (i.e., consumers purchase less when the interest rate is higher).

The local monopoly power of each firm arises from a simple search friction: if a consumer rejects a contract, they must wait one additional period before approaching another firm. The resulting downward-sloping demand curve can be derived directly from the consumer's intertemporal budget constraint. Given financial wealth W_t^* (see (25)), a consumer choosing consumption level C_t reduces their wealth by $C_t A_t^*$.

For simplicity, we assume that households maximize standard intertemporal CRRA utility, $U(c) = (c^{1-\gamma}-1)/(1-\gamma)$. The standard Euler equation for intertemporal optimization then implies the following characterization of households' demand function.

Lemma 1 *Let*

$$\Xi_t \;=\; E_t\left[\int_0^\infty e^{-\beta s/\gamma}\left(\frac{\widehat{M}_{t+s}}{M_t^*}\right)^{1-1/\gamma} ds\right]. \tag{32}$$

Given the contract (n_t, δ_t) quoted by the firm, households optimally select

$$C_t(A_t^*) \;=\; \frac{W_t^*}{A_t^* + (A_t^*)^{1/\gamma}\,\Xi_t}. \tag{33}$$

Lemma 1 implies that, instead of solving for the optimal contract (n_t, δ_t), the firm can directly target the level of A_t^* (the present value of future payments per unit of consumption). The size of intermediation rents is captured by the deviation of A_t^* from the frictionless level, $A_t^* - 1$. Following the ideas of Gabaix and Maggiori (2015), we assume that the company faces a monetary cost of introducing these frictions, controlled by a constant Γ.[8] We assume

[7] It would be interesting to consider other bargaining protocols used in modern Over-The-Counter-Markets. See, e.g., Hugonnier, Lester, and Weill (2025) for an excellent overview.

[8] For example, Γ may capture the unit cost of maintaining the contract (monitoring payments, controlling and measuring creditworthiness, etc.)

that the actual cost is proportional to $\frac{W_t^*}{A_t^*}(A_t^*-1)^2$, so that the profit maximization problem for the firm is scale invariant and takes the form

$$\max_{A_t^*} \ \left\{ A_t^* C_t(A_t^*) - C_t(A_t^*) - \Gamma \frac{W_t^*}{A_t^*}(A_t^*-1)^2 \right\}. \tag{34}$$

Solving this problem, we arrive at the following result.

Lemma 2 *The optimum $A_t^* > 1$ is the unique solution to the fixed point equation*

$$\frac{1+(1-\gamma^{-1})(A_t^*)^{1/\gamma}\Xi_t + \gamma^{-1}(A_t^*)^{\gamma^{-1}-1}\Xi_t}{\left(1+(A_t^*)^{\gamma^{-1}-1}\Xi_t\right)^2} - \Gamma\left((A_t^*)^2-1\right) \ = \ 0\,. \tag{35}$$

To complete the characterization of equilibrium, we notice that, substituting (31) into (32), we can rewrite it as

$$\Xi_t \ = \ (A_t^*)^{1-1/\gamma}\Xi_t^*, \quad \Xi_t^* \ = \ E_t\left[\int_0^\infty e^{-\beta s/\gamma}\left(\frac{\widehat{M}_{t+s}}{\widehat{M}_t}\right)^{1-1/\gamma} ds\right], \tag{36}$$

where Ξ_t^* is an exogenous quantity that depends only on aggregate consumption. In equilibrium, consumption market clearing uniquely pins down the dynamics of $\widehat{M}_t$ and Ξ_t^*. Substituting (36) into (35), we obtain a quadratic equation for A_t^* :

$$\frac{1+(1-\gamma^{-1})A_t^*\Xi_t^* + \gamma^{-1}\Xi_t^*}{\left(1+\Xi_t^*\right)^2} - \Gamma\left((A_t^*)^2-1\right) \ = \ 0\,. \tag{37}$$

Solving for A_t^* and noting that there is only one positive root above 1, we obtain the following result.

Proposition 6 *Suppose that households have CRRA preferences and let*

$$\widehat{M}_t \ = \ e^{-\beta t}(X_t/X_0)^{-\gamma}, \tag{38}$$

where $X_t = C_t$ is aggregate consumption. Then, the unique equilibrium SDF is given by

$$M_t^* = \underbrace{e^{-\beta t}(X_t/X_0)^{-\gamma}}_{marginal\ utility} \cdot \underbrace{(A_t^*)^{-1}}_{long\ run\ risk}, \tag{39}$$

with

$$\begin{aligned} A_t^* &= f(\Xi_t^*) \\ f(x) &= \sqrt{1 + \frac{1}{\Gamma}\frac{1+\gamma^{-1}x}{(1+x)^2} + \Big(\frac{1-\gamma^{-1}}{2\Gamma}\frac{x}{(1+x)^2}\Big)^2} + \frac{1-\gamma^{-1}}{2\Gamma}\frac{x}{(1+x)^2}. \end{aligned} \tag{40}$$

The equilibrium contract arises from a delicate balance between households' demand behavior and firms' ability to engage in price discrimination. The demand for present consumption declines with two factors: (1) the expected utility derived from future consumption, reflecting the standard intertemporal wealth effect, and (2) the borrowing cost, captured by the present value of deferred payments—namely, the price of the perpetuity defined earlier. Consequently, the firm's optimization problem reduces to choosing this perpetuity price while taking the stochastic discount factor as given. In equilibrium, aggregate consumption equals aggregate output, which uniquely determines the perpetuity price as an explicit function of expected future utility.

Formula (40) provides a key insight into the economic drivers of the rent $A_t^* - 1$ extracted by the firm. It is governed by two forces: the elasticity of intertemporal substitution (EIS) and expectations about future consumption growth. When $\gamma > 1$, the EIS $= 1/\gamma$ is below one, implying that agents place a strong emphasis on consumption smoothing and assign high value to future low-consumption states.

Proposition 6 then demonstrates that these closed-form, optimal contracts yield an equilibrium SDF (39) that is *independent of δ_t and n_t*, depending solely on a form of long-run

risk captured by

$$\Xi_t^* \;=\; E_t\left[\int_0^\infty e^{-\beta\tau}(X_{t+\tau}/X_t)^{1-\gamma}d\tau\right]. \tag{41}$$

Modern measures of long-run risk typically involve modeling long-run consumption volatility (Bansal and Yaron 2004), which is difficult to estimate empirically. Instead, Parker and Julliard (2005) propose a direct measure, showing that an SDF of the form

$$M_{t+1} \;=\; E_{t+1}[(X_{t+S}/X_t)^{-\alpha}],\ \alpha > 0 \tag{42}$$

is able to price the cross-section of stock returns.[9]

Using a simple first-order Taylor approximation for the function f around the steady-state value $\bar{\Xi}^*$ in (40), we obtain

$$M_t^* \;\approx\; e^{-\beta t}(X_t/X_0)^{-\gamma}(a + b\Xi_t^*)), \tag{43}$$

with $a = 1/f(\bar{\Xi}^*) - b\bar{\Xi}^*$ and $b = -f'(\bar{\Xi}^*)/f(\bar{\Xi}^*)^2$. By direct calculation, $f(x)$ is monotone decreasing for $x > 1$, and $\hat{\Xi}^*$ exceeds one by a wide margin under typical calibrations. Consequently, M_t^* in (43) depends positively on $E_t[(X_{t+k}/X_t)^{-\alpha}]$, $\alpha = \gamma - 1$, aligning with the empirical observations of Parker and Julliard (2005).[10] However, the underlying mechanism here is fundamentally different and does not rely on time-inseparable preferences. Exploring the nuanced links between long-run consumption risk and consumer credit offers a promising avenue for future research.

[9] Parker and Julliard (2005) use realized *future* growth, $(X_{t+S}/X_t)^{-\alpha}$. However, a simple argument based on the law of iterated expectations implies that it can be replaced with $E_{t+1}[(X_{t+S}/X_t)^{-\alpha}]$. Indeed, $E_t[R_{t+1}(X_{t+S}/X_t)^{-\alpha}] = E_t[R_{t+1}E_{t+1}[(X_{t+S}/X_t)^{-\alpha}]]$.

[10] Di Maggio, Kermani, and Majlesi (2020) offer a different channel relating stock returns to future consumption, whereby households treat capital gains and dividends as separate sources of income.

5 Empirical Results

In this section, we evaluate the ability of the pricing kernel (43) to explain the cross-section of well-known stock market anomalies. For this purpose, we employ the dataset of Jensen, Kelly, and Pedersen (2023), which contains 153 anomaly portfolios (factors) constructed from a wide range of stock characteristics. Let $R^e_{k,t+1}$ denote the excess return on anomaly portfolio k over one period (a month or a quarter in our analysis below).

In our framework, under the approximation (43), these excess returns satisfy the standard asset-pricing condition

$$E_t[M^*_{t+1}R^e_{k,t+1}] \;=\; 0 \tag{44}$$

for all k. The following result then follows directly from the law of iterated expectations.

Corollary 7 *Under the approximation* (43)*, excess returns satisfy*

$$E[\widetilde{M}_{t+1}R^e_{k,t+1}] \;=\; 0, \tag{45}$$

where

$$\widetilde{M}_{t+1} \;=\; (X_{t+1}/X_t)^{-\gamma}\Big(a + b\int_0^\infty e^{-\beta\tau}(X_{t+1+\tau}/X_{t+1})^{1-\gamma}\,d\tau\Big)\,. \tag{46}$$

Owing to the time additivity of preferences, Corollary 7 allows us to test the asset pricing model *without estimating the conditional expectation* (41). Estimating this expectation is a highly nontrivial econometric task that typically requires strong and often untestable assumptions about the underlying data-generating process (Bansal and Yaron, 2004; Campbell et al., 2018; Chen, Dou, and Kogan, 2024); researchers continue to debate the reliability of proposed proxies for long-run risk (see e.g. Liu and Matthies, 2022, 2024; Maio, 2024). In

contrast, Corollary 7 provides a simple and directly observable measure of this risk that relies solely on realized consumption data rather than on model-dependent forecasts.

To operationalize this test, we require an empirical counterpart to the model-implied measure of future utility. Given a horizon H, we approximate the future realized utility term $\int_0^\infty e^{-\beta\tau}(X_{t+1+\tau}/X_{t+1})^{1-\gamma}d\tau$ in (46) as follows:

$$\tilde{Y}_{t+1,t+1+H} = \sum_{\tau=1}^{H} e^{-\beta\tau}\left(\frac{C_{t+1+\tau}}{C_{t+1}}\right)^{1-\gamma} + \frac{e^{(H+1)(-\beta+(1-\gamma)\bar{g}_{0,T})}}{1-e^{-\beta+(1-\gamma)\bar{g}_{0,T}}}, \tag{47}$$

where $\bar{g}_{0,T} = \frac{1}{T}\sum_{t=1}^{T}\ln\frac{C_t}{C_{t-1}}$ denotes the full-sample average consumption growth rate, β is the subjective discount rate, and C_t is realized aggregate consumption in the U.S. economy. Our approximate SDF takes the form

$$\widetilde{M}_{t+1} \;\approx\; (C_{t+1}/C_t)^{-\gamma}\left(a \;+\; b\,\tilde{Y}_{t+1,t+1+H}\right). \tag{48}$$

For benchmarking, we compare our SDF's performance to that of Parker and Julliard (2005), whose pricing kernel takes the form

$$M^{PJ}_{t+1} = R^{f}_{t+1,t+1+H}\left(\frac{C_{t+1+H}}{C_t}\right)^{-\gamma}. \tag{49}$$

Given the structural similarity between the two specifications, their empirical performance is expected to be comparable; however, only (46) possesses the advantage of being derived from a fully micro-founded equilibrium model.

5.1 Data

Consumption Data. We measure consumption using real per capita expenditures on non-durable goods and services. The data are obtained from the Federal Reserve Economic

Data (FRED) database and cover the period 1959:01–2025:07 at a monthly frequency and 1947:Q1–2025:Q2 at a quarterly frequency.

The real per capita consumption series is constructed using three components: (i) nominal personal consumption expenditures on services and non-durable goods, (ii) the personal consumption expenditures chain-type price index, and (iii) total population.

All series are sourced from the National Income and Product Accounts (NIPA) Tables—[2.8.5, 2.8.4, 2.6] for the monthly data and [2.3.5, 2.3.4, 2.1] for the quarterly data, respectively.

Equity Portfolios Used for Parameter Estimation. For each horizon H, the stochastic discount factor (SDF) in (48) involves parameters that need to be estimated. Estimation requires selecting an appropriate set of test assets. Our primary test assets are the 153 characteristic-sorted monthly portfolio returns compiled by Jensen, Kelly, and Pedersen (2023)[11]. This comprehensive dataset spans the period 1926:01–2024:12. Following the authors' recommendation, we employ the capped value-weighted portfolio returns for our baseline analysis.

For greater granularity, we further adopt the thematic clustering introduced by Jensen, Kelly, and Pedersen (2023), which groups the 153 portfolios into 13 economically meaningful themes. This structure allows us to evaluate the SDF's ability to price different clusters of anomalies and to assess whether its performance varies across distinct economic dimensions.

Out-of-Sample Bond and Equity Portfolios. To evaluate the model's out-of-sample performance on assets not used in estimation, we employ a broad set of standard test portfolios from Kenneth French's Data Library and other established sources. Specifically:

- **Equity Portfolios:** 25 portfolios sorted by size and value; 25 by size and momentum; 25 by size and short-term reversal; and 25 by size and long-term reversal. We also

[11] To access the stock-level characteristic data and accompanying documentation, see `jkpfactors.com`.

include 10 portfolios sorted by accruals, beta, cash-flow-to-price, dividend-price ratio, investment, net issuance, operating profitability, residual variance, and total variance, along with industry portfolios comprising 5, 10, 12, 17, and 30 sectors. For all of these, we test both value-weighted and equal-weighted returns.

- **Bond Portfolios:** Ten zero-coupon nominal government bond portfolios with maturities ranging from one to ten years, constructed using the dataset of Filipović, Pelger, and Ye (2024).

Sample Period. Our main empirical analysis spans from 1972:03 to 2018:06. The start date is determined by the broad availability of returns for the characteristic-sorted portfolios, particularly those within the value theme. The end date is constrained by the construction of our SDF, which requires up to 84 months of future realized consumption data to compute the longest-horizon proxy for consumption expectation.

Performance Evaluation. Following Hansen and Jagannathan (1997), we use the Hansen-Jagannathan distance (HJD) as a statistic for comparing the efficacy of SDF models in terms of their pricing error magnitudes. For a candidate SDF $\widetilde{M}_t$, the Hansen-Jagannathan distance (HJD) is a weighted sum of squared pricing errors for the set of test assets R_t:

$$\mathcal{D}^{HJ} = E[\widetilde{M}_t R_t]' E[R_t R_t']^{-1} E[\widetilde{M}_t R_t]. \tag{50}$$

The most attractive feature of the HJD is its invariance with respect to linear transformations: replacing R_t with a set of portfolios in R_t that preserves their span keeps the HJD unchanged. Our tests compute the HJD for various horizons H from 12 to 84 months to assess the performance of each SDF specification across different time scales.

Stochastic Discount Factor Specifications We empirically evaluate two distinct specifications of the stochastic discount factor (SDF).Our proposed SDF, $M_t(\theta)$, takes the form:

$$M_t(\theta) = \left(\frac{C_t}{C_{t-1}}\right)^{-\gamma} \left(\alpha + \tilde{Y}_{t,t+H}(\gamma; H, \beta)\right). \tag{51}$$

This specification is parameterized by the coefficient of relative risk aversion, γ, and a transformed parameter, $\alpha = a/b - \bar{\Xi}$. Throughout the analysis, we set the coefficient of relative risk aversion to $\gamma = 10$.[12]

The state variable $\tilde{Y}_{t,t+H}$ captures realized future consumption utility and is a function of γ as well as two pre-specified hyperparameters: the consumption horizon, H, and the time discount factor, β. As a benchmark, we consider the SDF proposed by Parker and Julliard (2005):

$$M_t(\theta) = R_{f,t,t+H} \left(\frac{C_{t+H}}{C_{t-1}}\right)^{-\gamma}, \tag{52}$$

where $R_{f,t,t+H}$ is the realized cumulative risk-free return from time t to $t+H$. This model depends on a single parameter, γ.

The asset pricing performance of each SDF is evaluated based on its ability to price a vector of test asset excess returns, $\mathbf{R}_t^e$. The theoretical moment condition is $E[M_t(\theta)\mathbf{R}_t^e] = \mathbf{0}$. We test this using its sample analogue:

$$\mathbf{g}_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} M_t(\theta)\mathbf{R}_t^e. \tag{53}$$

In our empirical implementation, we normalize the SDF by its sample mean. This is an

[12] Values of $\gamma \approx 10$ are standard in consumption-based asset pricing models. Empirically, we also find that $\gamma \approx 10$ maximizes the performance of the Parker and Julliard (2005) specification. Although our model attains slightly better results for $\gamma > 10$, we maintain $\gamma = 10$ to avoid invoking implausibly high levels of risk aversion.

innocuous transformation, as the pricing condition for excess returns is invariant to the scaling of the SDF.

5.2 Discussion of Results

The comparative analysis of the PJ-SDF (49) and our SDF (51) (see Figure 1 for cross-theme results and Figure 2 for aggregate performance) highlights important differences in their asset pricing properties, particularly regarding stability and horizon dependence.

Our SDF (51), which jointly incorporates log consumption growth and realized future consumption utility, delivers markedly more stable pricing performance across horizons ranging from $H = 12$ to $H = 84$ months. The corresponding Hansen–Jagannathan (HJ) distance remains relatively flat and even exhibits a modest improvement as the horizon increases. Notably, our M specification (depicted in blue in Figure 2) consistently attains the lowest average HJ distance.

These findings suggest that incorporating long-term consumption expectations plays a key role in mitigating model misspecification and improving intertemporal pricing accuracy. Moreover, the model successfully prices roughly half of the 13 thematic portfolios at the monthly frequency. Overall, its performance is comparable to that of the PJ-SDF, with the crucial distinction that our measure is derived from a fully micro-founded equilibrium model.

Out-of-Sample Asset Pricing on Other Portfolios We evaluate the robustness of our measure through an out-of-sample (OOS) analysis, applying the previously estimated optimal parameters to an extended set of test assets. Detailed results for value-weighted and equal-weighted portfolios are presented in Figures 3 and 4, respectively. The empirical evidence indicates that our measure maintains strong performance across a broad range of equity and bond portfolios, on par with the PJ benchmark. The improvement is particularly pronounced for value-weighted portfolios, consistent with the notion that small-stock returns

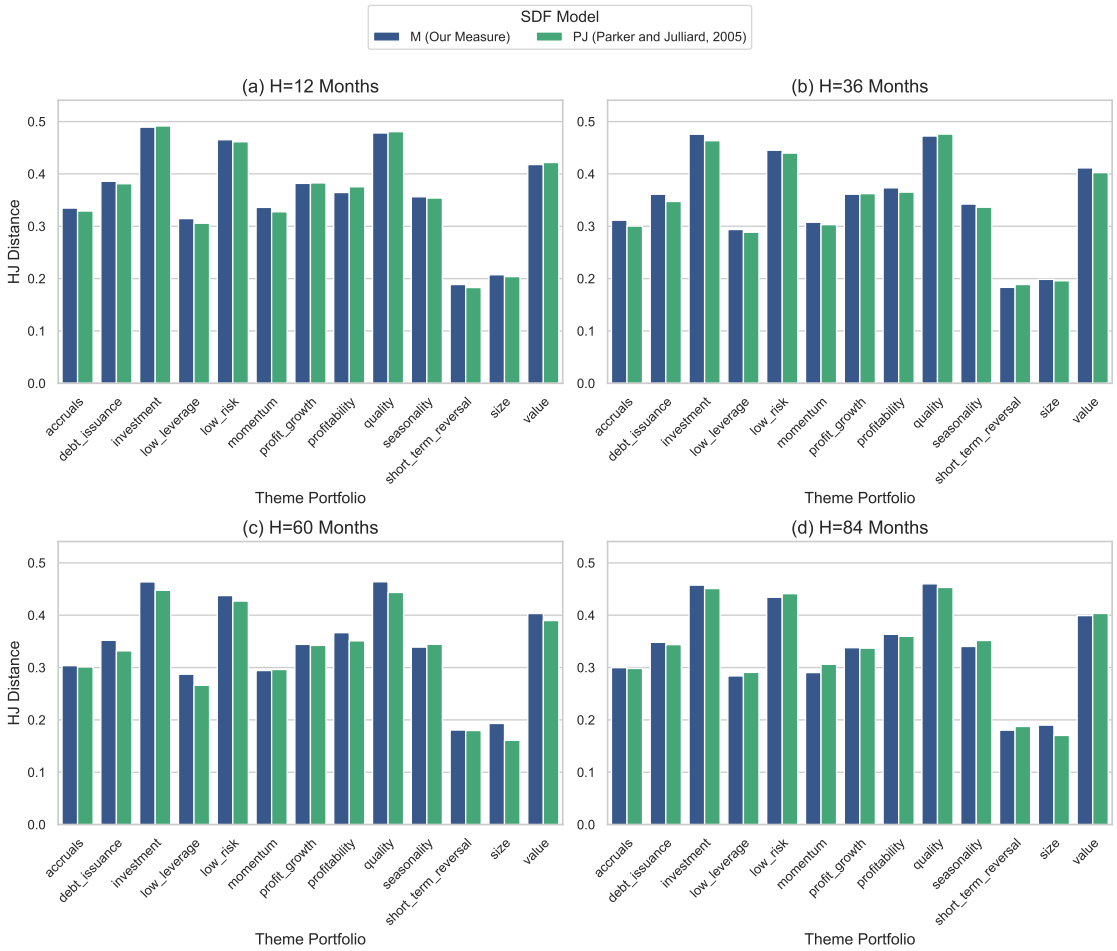

Figure 1: Hansen-Jagannathan Distance Comparison of Our Measure (M) and Parker and Julliard (PJ) SDFs.

This figure plots the Hansen-Jagannathan (HJ) distance for 153 test portfolios (capped value weighted returns) grouped into 13 themes (see Jensen, Kelly, and Pedersen 2023) against the horizon for SDF construction. Our Measure (M) corresponds to (51), while the PJ measure corresponds to (49) as in Parker and Julliard (2005). Consumption is measured by the per capita real non-durable goods and services. The sample period spans 1972-03-31 to 2018-06-30 (monthly frequency).

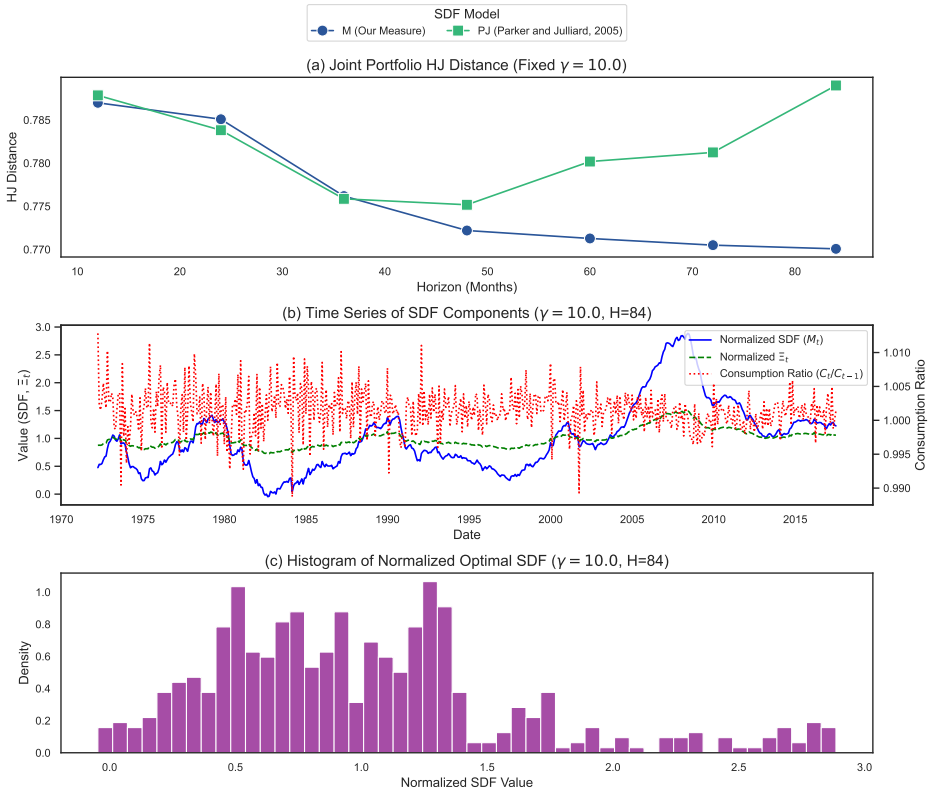

Figure 2: Hansen-Jagannathan Distance for the 153 Testing Portfolios across Horizons with $\gamma = 10$. SDF Distribution and SDF Components for Horizon $= 84$.

are more affected by idiosyncratic and liquidity shocks, whereas large-stock returns are primarily driven by macroeconomic forces such as expected consumption growth.

The model attains its highest performance on industry portfolios, consistent with the intuition that industry-level returns are more strongly linked to aggregate macroeconomic conditions. By contrast, performance is weakest for portfolios sorted (1) book-to-market (`25_ME_BEME`); (2) momentum (`25_ME_Prior_12_2`); and (3) short-term reversal (`25_ME_Prior_1_0`). The book-to-market anomaly is notoriously difficult to reconcile with a pure consumption-based CAPM and typically requires more sophisticated risk-based explanations—such as the investment-based CAPM of Zhang (2020) or the cash-flow news channel of Campbell and Vuolteenaho (2004)—or behavioral mechanisms such as those proposed by Lakonishok, Shleifer, and Vishny (1994). Likewise, momentum appears largely unrelated to consumption risk and is widely attributed to investors' underreaction and sentiment dynamics (Barberis, Shleifer, and Vishny, 1998; Daniel, Hirshleifer, and Subrahmanyam, 1998; Hong and Stein, 1999; Hong, Lim, and Stein, 2000), as well as to limits-to-arbitrage effects (Stambaugh, Yu, and Yuan, 2012). Finally, short-term reversal patterns are often linked to investors' overreaction and to liquidity-driven frictions (Nagel, 2012).

6 Conclusions

This paper examines how consumer credit and delayed payments shape asset prices. We develop a tractable general-equilibrium model in which households finance consumption through debt with a geometrically decaying repayment structure. This delay between consumption and payments creates a persistent wedge that alters the stochastic discount factor and, consequently, equilibrium asset valuations.

Our main empirical contribution is to show that the model delivers a simple, observable pricing kernel based on the realized utility of future consumption. Using a large cross-section of stock market anomalies, we find that this kernel explains return patterns that

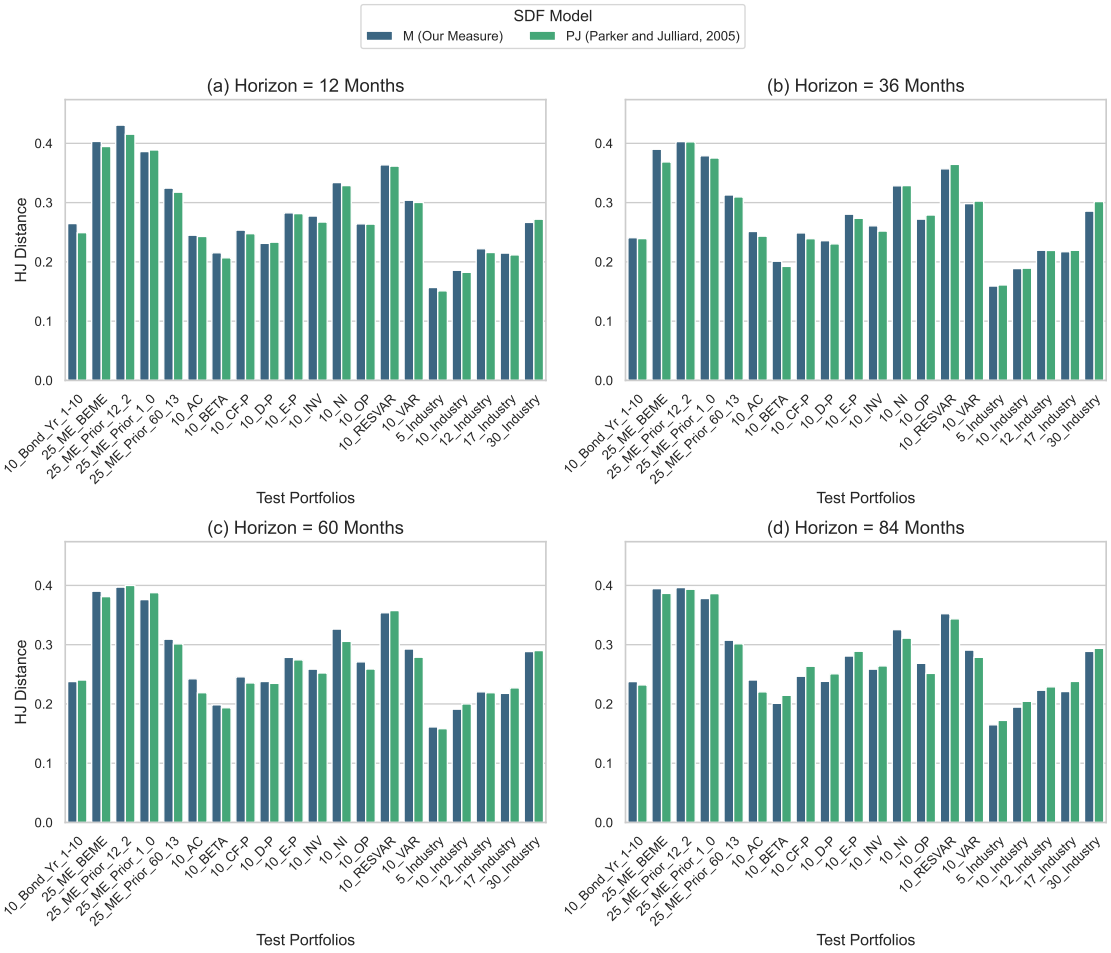

Figure 3: Hansen-Jagannathan Distance Comparison for Out-of-Sample Testing Portfolios.

This figure plots the out-of-sample Hansen-Jagannathan (HJ) distance for bond and equity portfolios against the horizon for SDF construction. Equity portfolio returns are value weighted. Our Measure (M) corresponds to (51), while the PJ measure corresponds to (49) as in Parker and Julliard (2005). Consumption is measured by the per capita real non-durable goods and services. The sample period spans 1972-03-31 to 2018-06-30 (monthly frequency).

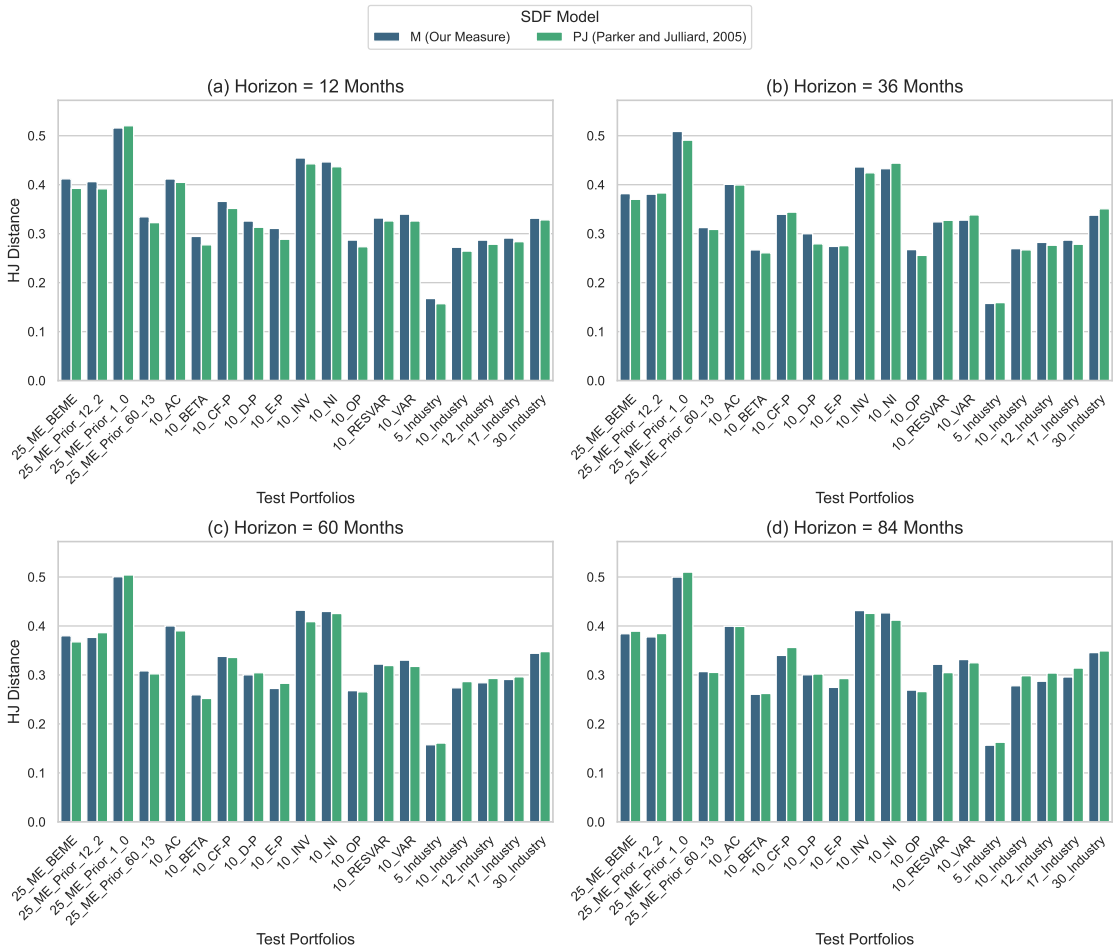

Figure 4: Hansen-Jagannathan Distance Comparison for Out-of-Sample Testing Portfolios.

This figure plots the out-of-sample Hansen-Jagannathan (HJ) distance for equity portfolios (equal-weighted returns) against the horizon for SDF construction. Our Measure (M) corresponds to the two-factor model (43), while the PJ measure corresponds to the one-factor model (49) as in Parker and Julliard (2005). Consumption is measured by the per capita real non-durable goods and services. The sample period spans 1972-03-31 to 2018-06-30 (monthly frequency).

standard consumption-based models fail to capture. The results suggest that the timing of payments—rather than the timing of consumption itself—plays a central role in the propagation of macroeconomic and financial risks.

The model's mechanism provides a new interpretation of long-horizon return predictability: when households borrow to smooth consumption, repayment inertia naturally generates exposure to long-run risks. This insight complements existing work on intermediary and long-run risk channels by highlighting a household-level source of systematic variation in risk premia.

Beyond its asset-pricing implications, our framework offers testable predictions for credit-market dynamics and consumer behavior. For instance, shifts in repayment duration or credit-market power should translate into predictable changes in expected returns. Future research could extend these ideas by incorporating heterogeneity in credit access, explicit default, and institutional features of consumer lending to better understand how credit frictions transmit to financial markets.

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A Proofs

Proof of Theorem 1. See the main text above the theorem statement. □

Proof of Theorem 2. This follows directly the argument in Duffie and Epstein (1992). □

Proof of Proposition 3. See the main text above the proposition. □

Proof of Theorem 4. We first verify that if M_t^* is the proposed solution, it satisfies the integral equation defined by (23). Let us define a discount process $\mathcal{D}_t \equiv \exp\left(-\int_0^t (\delta - n + \beta(\Theta_u))du\right)$ and an auxiliary process Y_t as follows:

$$Y_t \equiv \mathcal{D}_t \widehat{F}(\Theta_t) + \int_0^t \mathcal{D}_s \delta F(\Theta_s) ds, \tag{54}$$

where $F(\Theta_s)$ is given by

$$F(\Theta_s) = \left(1 + \frac{\widehat{r}(\Theta_s) - n}{\delta}\right) \widehat{F}(\Theta_s). \tag{55}$$

Our objective is to show that Y_t is a martingale. By applying Itô's product rule to $\mathcal{D}_t \widehat{F}(\Theta_t)$, we have

$$\begin{aligned} d(\mathcal{D}_t \widehat{F}(\Theta_t)) &= \widehat{F}(\Theta_t) d\mathcal{D}_t + \mathcal{D}_t d\widehat{F}(\Theta_t) \\ &= -\left(\delta - n + \beta(\Theta_t)\right) \mathcal{D}_t \widehat{F}(\Theta_t) dt + \mathcal{D}_t \left(\mathcal{A}\widehat{F}(\Theta_t) dt + \nabla \widehat{F}(\Theta_t) \cdot \sigma(\Theta_t) dB_t\right) \\ &= \mathcal{D}_t \left[\left(-\delta + n - \beta(\Theta_t) + \mathcal{A}\right) \widehat{F}(\Theta_t)\right] dt + \mathcal{D}_t \nabla \widehat{F}(\Theta_t) \cdot \sigma(\Theta_t) dB_t. \end{aligned}$$

The differential of Y_t is then

$$\begin{aligned} dY_t &= d(\mathcal{D}_t \widehat{F}(\Theta_t)) + \mathcal{D}_t \delta F(\Theta_t) dt \\ &= \mathcal{D}_t \left[(-\delta + n - \beta(\Theta_t) + \mathcal{A}) \widehat{F}(\Theta_t) + \delta F(\Theta_t) \right] dt + \mathcal{D}_t \nabla \widehat{F}(\Theta_t) \cdot \sigma(\Theta_t) dB_t. \end{aligned}$$

Substituting the definition of $F(\Theta_t)$ from (55) into the drift term yields:

$$\begin{aligned} & (-\delta + n - \beta(\Theta_t) + \mathcal{A}) \widehat{F}(\Theta_t) + (\delta - n + \widehat{r}(\Theta_t)) \widehat{F}(\Theta_t) \\ = \; & (-\beta(\Theta_t) + \mathcal{A} + \widehat{r}(\Theta_t)) \widehat{F}(\Theta_t). \end{aligned}$$

From the definition of the short rate, $\widehat{r}(\Theta_t)\widehat{F}(\Theta_t) = \beta(\Theta_t)\widehat{F}(\Theta_t) - \mathcal{A}\widehat{F}(\Theta_t)$, which implies that the drift term is identically zero. Thus,

$$dY_t = \mathcal{D}_t \nabla \widehat{F}(\Theta_t) \cdot \sigma(\Theta_t) dB_t. \tag{56}$$

The process Y_t is a local martingale. The square-integrability condition (21) ensures that the stochastic integral is a true martingale, so $E_t[Y_s] = Y_t$ for any $s > t$. Writing this out, we have:

$$E_t \left[\mathcal{D}_s \widehat{F}(\Theta_s) + \int_0^s \mathcal{D}_v \delta F(\Theta_v) dv \right] = \mathcal{D}_t \widehat{F}(\Theta_t) + \int_0^t \mathcal{D}_v \delta F(\Theta_v) dv.$$

Rearranging and dividing by $\mathcal{D}_t$ gives:

$$\widehat{F}(\Theta_t) = E_t \left[\frac{\mathcal{D}_s}{\mathcal{D}_t} \widehat{F}(\Theta_s) + \int_t^s \frac{\mathcal{D}_v}{\mathcal{D}_t} \delta F(\Theta_v) dv \right].$$

Taking the limit as $s \to \infty$ and applying the transversality condition (22), which ensures that $\lim_{s\to\infty} E_t[(\mathcal{D}_s/\mathcal{D}_t)\widehat{F}(\Theta_s)] = 0$, we obtain the desired integral representation:

$$\widehat{F}(\Theta_t) = E_t \left[\int_t^\infty e^{-\int_t^s (\delta - n + \beta(\Theta_u)) du} \delta F(\Theta_s) ds \right].$$

Conversely, assume that $\widehat{F}(\Theta_t)$ satisfies the integral equation above. This implies that the process Y_t, defined with the integral from t to ∞, is a martingale under the specified measure. For a process to be a martingale, the drift of its differential must be zero. From our earlier calculation, the drift of Y_t is proportional to

$$\Big[(-\delta + n - \beta(\Theta_t) + \mathcal{A})\,\widehat{F}(\Theta_t) + \delta F(\Theta_t)\Big]\,.$$

Setting this to zero and solving for $F(\Theta_t)$ gives

$$\delta F(\Theta_t) = (\delta - n + \beta(\Theta_t) - \mathcal{A})\widehat{F}(\Theta_t).$$

Using the definition $\widehat{r}(\Theta_t) = \beta(\Theta_t) - \mathcal{A}\widehat{F}(\Theta_t)/\widehat{F}(\Theta_t)$, we recover the relationship $F(\Theta_t) = \delta^{-1}(\delta - n + \widehat{r}(\Theta_t))\widehat{F}(\Theta_t)$, which completes the proof. □

Proof of Proposition 5. By direct calculation, we obtain

$$\begin{aligned}
W_t &= (M_t^*)^{-1}E_t\Big[\int_t^\infty \int_0^s e^{-(\delta_v - n_v)(s-v)}\delta_v C_v\,dv\;M_s^*\,ds\Big] \\
&= (M_t^*)^{-1}E_t\Big[\int_t^\infty \int_0^s e^{-(\delta_v - n_v)(s-v)}\delta_v C_v\,dv\;M_s^*\,ds\Big] \\
&= (M_t^*)^{-1}E_t\Big[\int_t^\infty \int_0^t e^{-(\delta_v - n_v)(s-v)}\delta_v C_v\,dv\;M_s^*\,ds \;+\; \int_t^\infty \int_t^s e^{-(\delta_v - n_v)(s-v)}\delta_v C_v\,dv\;M_s^*\,ds\Big] \\
&= E_t\Big[\int_0^t e^{-(\delta_v - n_v)(t-v)}\Big(\int_t^\infty e^{-(\delta_v - n_v)(s-t)}\delta_v \frac{M_s^*}{M_t^*}\,ds\Big)\,C_v\,dv\Big] \\
&\quad + \;E_t\Big[\int_t^\infty \Big(\int_v^\infty e^{-(\delta_v - n_v)(s-v)}\delta_v \frac{M_s^*}{M_t^*}\,ds\Big)\,C_v\,dv\Big] \\
&= \int_0^t e^{-(\delta_v - n_v)(t-v)}A(t,\delta_v,n_v)\,C_v\,dv \;+\; E_t\Big[\int_t^\infty A(v,\delta_v,n_v)\frac{M_v^*}{M_t^*}\,C_v\,dv\Big] \\
&= \int_0^t e^{-(\delta_v - n_v)(t-v)}A(t,\delta_v,n_v)\,C_v\,dv \;+\; A(t,\delta_t,n_t)\,E_t\Big[\int_t^\infty \frac{\widehat{M}_v}{\widehat{M}_t}\,C_v\,dv\Big]\,,
\end{aligned} \tag{57}$$

as claimed in the main text. □

Proof of Lemma 1. We prove the lemma by first considering the household's inter-temporal problem in discrete time, solving it, and then taking the continuous time limit. Let δ be a small time step.

The household time-t problem is to solve

$$\max_{C_t, C_{t+\delta}, \ldots} \quad \frac{C_t^{1-\gamma}}{1-\gamma} + e^{-\beta\delta} E_t\Big[\frac{C_{t+\delta}^{1-\gamma}}{1-\gamma}\Big]\delta + \ldots\,, \tag{58}$$

subject to the budget constraint

$$E_t\left[\sum_{s=1}^{\infty} \frac{\widehat{M}_{t+s\delta}}{M_t^*} C_{t+s\delta}\delta\right] \;=\; W_t^* - C_t \frac{\widehat{M}_t}{M_t^*}\,. \tag{59}$$

First-order conditions with respect to consumption then imply,

$$\frac{\widehat{M}_{t+s\delta}}{M_t^*}\Lambda_t \;=\; e^{-\beta s\delta} C_{t+s\delta}^{-\gamma}\,, \quad \forall s \geq 0\,, \tag{60}$$

with Λ_t being the Lagrange multiplier for the budget constraint. Substitute $C_{t+s\delta}$ for all s into the budget constraint to obtain,

$$W_t^* \;=\; \Lambda_t^{-1/\gamma} \underbrace{\sum_{s=0}^{\infty} e^{-\beta s\delta/\gamma}\delta E_t\left[\left(\frac{\widehat{M}_{t+s\delta}}{M_t^*}\right)^{1-1/\gamma}\right]}_{=\,\Xi_t}\,. \tag{61}$$

Then, substitute $\Lambda_t^{-1/\gamma} = C_t(\widehat{M}_t/M_t^*)^{1/\gamma}$, and solve for C_t to get

$$C_t \;=\; \frac{W_t^*}{\frac{\widehat{M}_t}{M_t^*} + \left(\frac{\widehat{M}_t}{M_t^*}\right)^{1/\gamma}\Xi_t}\,. \tag{62}$$

Recall that $A_t^* = \frac{\widehat{M}_t}{M_t^*}$. In the limit as $\delta \to 0$, Ξ_t converges to the continuous time expression in the main text. □

Proof of Lemma 2. Let

$$g(x) \;=\; \Gamma(x^2 - 1)\,. \tag{63}$$

$g(x)$ is strictly increasing in $x \geq 0$, and $g(0) = -1$, $g(1) = 0$.

Let

$$f(x) \;=\; \frac{1 + (1-\gamma^{-1})x^{1/\gamma}\Xi_t + \gamma^{-1}x^{\gamma^{-1}-1}\Xi_t}{(1 + x^{\gamma^{-1}-1}\Xi_t)^2}\,. \tag{64}$$

Then

$$f'(x) \;=\; \frac{(x-1)\Xi_t(1-\gamma^{-1})x^{\gamma^{-1}-2}}{(1 + x^{\gamma^{-1}-1}\Xi_t)^2} - 2\frac{\Xi_t(\gamma^{-1}-1)x^{\gamma^{-1}-2}}{(1 + x^{\gamma^{-1}-1}\Xi_t)}f(x)\,. \tag{65}$$

If $\gamma \geq 1$, $f(x)$ is strictly increasing in x, and $\lim_{x\to 0} f(x) = 0$. Meanwhile, $0 < f(1) = (1+\Xi_t)^{-1} < 1$, and $f'(1) = 2\Xi_t(1-\gamma^{-1})(1+\Xi_t)^{-2} < 2 = g'(1)$. Then, there exists a unique positive x^* such that $g(x^*) = f(x^*)$ and $x^* > 1$, as $g(1) < f(1)$ and $g'(1) > f'(1)$.

If $0 < \gamma < 1$, $f(x)$ first increases with x for $x < 1$, then decreases with x for $x > 1$. Then, there exists a unique positive x^* such that $g(x^*) = f(x^*)$ and $x^* > 1$. □

Proof of Proposition 6. The equilibrium condition in (37) can be rearranged into the following quadratic equation for A_t^*:

$$(A_t^*)^2 - \left(\frac{1-\gamma^{-1}}{\Gamma}\frac{\Xi_t^*}{(1+\Xi_t^*)^2}\right)A_t^* - \left(1 + \frac{1}{\Gamma}\frac{1+\gamma^{-1}\Xi_t^*}{(1+\Xi_t^*)^2}\right) = 0. \tag{66}$$

Let us denote this equation as $f(A_t^*) \equiv (A_t^*)^2 - bA_t^* - c = 0$, where the coefficients are defined

as:

$$b \equiv \frac{1-\gamma^{-1}}{\Gamma}\frac{\Xi_t^*}{(1+\Xi_t^*)^2} \quad \text{and} \quad c \equiv 1 + \frac{1}{\Gamma}\frac{1+\gamma^{-1}\Xi_t^*}{(1+\Xi_t^*)^2}.$$

Under the standard assumptions that the risk-aversion coefficient $\gamma > 0$, the market power cost parameter $\Gamma > 0$, and the state variable $\Xi_t^* > 0$, the constant term c is strictly greater than 1. The discriminant of the quadratic, $\Delta = b^2 + 4c$, is therefore strictly positive, which ensures the existence of two distinct real roots.

By Vieta's formulas, the product of these roots is equal to $-c$. Since $c > 1$, the product is negative, implying that one root is positive and the other is negative. As A_t^* represents an asset price, its value must be non-negative. We thus select the unique positive root as the sole economically meaningful equilibrium solution.

To characterize this positive root, we evaluate the function $f(\cdot)$ at $A_t^* = 1$:

$$\begin{aligned} f(1) &= 1 - b - c \\ &= 1 - \frac{1-\gamma^{-1}}{\Gamma}\frac{\Xi_t^*}{(1+\Xi_t^*)^2} - \left(1 + \frac{1}{\Gamma}\frac{1+\gamma^{-1}\Xi_t^*}{(1+\Xi_t^*)^2}\right) \\ &= -\frac{1}{\Gamma(1+\Xi_t^*)^2}\left[(1-\gamma^{-1})\Xi_t^* + (1+\gamma^{-1}\Xi_t^*)\right] \\ &= -\frac{1+\Xi_t^*}{\Gamma(1+\Xi_t^*)^2} = -\frac{1}{\Gamma(1+\Xi_t^*)}. \end{aligned}$$

Given that $\Gamma > 0$ and $\Xi_t^* > 0$, it follows that $f(1) < 0$. Since $f(A_t^*)$ is an upward-opening parabola, its positive root must be greater than 1 for any finite $\Gamma > 0$ and for all $\gamma > 0$.

Finally, consider the case where the cost of exerting market power becomes prohibitively large, i.e., $\Gamma \to \infty$. In this limit, the coefficients $b \to 0$ and $c \to 1$. Equation (66) converges to $(A_t^*)^2 - 1 = 0$, which has roots $A_t^* = \pm 1$. The relevant positive solution is $A_t^* = 1$, which corresponds to the competitive market outcome. □

Proof of Corollary 7. The asset pricing equation is given by

$$\begin{aligned}
0 &= E_t[M^*_{t+1} R^e_{k,t+1}] \\
&= E_t\Big[\frac{M^*_{t+1}}{C_t^{-\gamma} e^{-\beta t}} R^e_{k,t+1}\Big] \\
&\approx E_t\left[e^{-\beta}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(a + b\Xi^*_{t+1}) R^e_{k,t+1}\right] \\
&= E_t\left[E_{t+1}\left[e^{-\beta}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(a + b\sum_{\tau=1}^{\infty} e^{-\beta\tau}\left(\frac{C_{t+1+\tau}}{C_{t+1}}\right)^{1-\gamma}\right) R^e_{k,t+1}\right]\right] \\
&= E_t\big[E_{t+1}[e^{-\beta}\widetilde{M}_{t+1} R^e_{k,t+1}]\big]\,.
\end{aligned} \tag{67}$$

Then, we take the unconditional expectation on both sides to arrive at the claim. □