Climate Disasters and Intergenerational Equity: A Fiscal Rule for Sustainable Development*

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March 3, 2026

Abstract

Climate disasters threaten intergenerational equity by exposing future generations to rising risks. We develop a model in which a government learns about disaster risk and enforces a sustainability criterion requiring expected social welfare to be non-decreasing over time. This criterion—similar to the principle underlying the UN Sustainable Development Goals—can be decentralized through state-contingent fiscal instruments: when perceived disaster risk is high, the constraint binds and government raises a consumption tax to finance investment subsidies for resilience. Such a fiscal rule leads to intergenerational-welfare smoothing and improves asset valuations despite adverse climate news due to commitments to future resilience. Compared with a government that adopts a social discount rate lower than households', the sustainability-constraint rule responds to disaster risk and is better aligned with observed consumption-based climate-resilience taxes, such as those implemented in Greece and Spain.

JEL Classification Numbers: H56; G01, E20

Keywords: Climate disasters, intergenerational equity, sustainable development, fiscal rules, consumption-style climate-resilience tax, ethical discount rate, stock markets

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*We thank Thomas Sargent, Geir Asheim, Patrick Bolton, Bard Harstad, Joel Hasbrouck, Yakov Amihud, Olivier Wang, Johannes Stroebel, Theresa Kuchler, Alan Moreira, Thomas Philippon, Adam Solomon, Ian Tonks, Chendi Zhang, and seminar participants at Norges Bank, University of Bristol, New York University, Columbia University, London School of Economics Environment Week, Exeter Sustainability Conference, and Lisbon Sustainability Conference for helpful comments. Hong acknowledges support from a Guggenheim Fellowship.

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1 Introduction

Future generations are expected to face substantially higher disaster risk—from heatwaves to tropical cyclones—than those living today (National Academies of Sciences, Engineering, and Medicine (2016)).[1] More frequent and severe climate disasters erode the capital stock and threaten standards of living, posing a challenge to intergenerational equity—the principle that development should "meet the needs of the present without compromising the ability of future generations to meet their own needs" (Brundtland, 1987). This principle underpins the United Nations Sustainable Development Goals (SDGs), which commit governments to ensuring that economic, social, and environmental welfare does not decline over time.

Survey evidence indicates that citizens support the sustainability objectives underlying the SDGs but expect governments—not individuals—to lead (OECD Development Communication Network, 2017).[2] Translating such sustainability commitments into fiscal design is at its infancy. Since 2016, about a dozen countries affected by climate disasters—including Greece and Spain—have introduced consumption-based sustainability taxes, earmarking the revenues for disaster resilience and climate adaptation.[3] While these instruments explicitly align with SDG 13 (Climate Action) and SDG 17 (Global Partnerships), the economics underlying their intergenerational rationale remains poorly understood.

We develop a continuous-time stochastic general-equilibrium model that formalizes this rationale. A social planner facing disaster risk imposes a sustainability criterion, requiring expected welfare to be non-decreasing over time. This criterion, rooted in the literature on sustainable development (Solow, 1993; Howarth, 1995; Heal, 1998; Asheim, 2003; Arrow et al., 2004; Pezzey, 2017), rationalizes the behavior of governments pursuing SDG-aligned fiscal

[1]According to a survey by Knutson et al. (2020), the median model projects a modest 13% increase relative to pre-industrial, while the most pessimistic climate model projects the frequency of tropical cyclones in a 2°C world to be 2.25 times greater than in the pre-industrial era. Perkins-Kirkpatrick and Lewis (2020) reviews evidence on increasing trends in regional heatwaves.

[2]See also various surveys from *Eurobarometer*, the polling instrument used by the European Commission, the European Parliament, and other EU institutions, regarding citizens' expectations of the role of government in pursuing SDGs.

[3]Similarly, France and several African nations have proposed a "premium flyers" levy to finance global loss-and-damage funds.

policy.[4] Consistent with survey evidence, we assume that households do not behave as if they face a sustainability constraint. Rather, governments act as the planner, employing fiscal instruments to enforce intergenerational equity.

Our framework can also be viewed as a novel class of fiscal rules. Unlike standard deficit or debt rules, which act as commitment devices that constrain time-inconsistent policymakers Halac and Yared (2014, 2018, 2022), our rule requires expected intergenerational welfare to be non-decreasing. This rule yields state-contingent consumption taxes and investment subsidies that rise with disaster risk—mirroring the 'golden rule' of government borrowing for public investment (Blanchard and Giavazzi, 2004), but adapted to a stochastic environment in which shocks endanger future welfare.

Our economy is exposed to weather disasters, which arrive as Poisson jumps that destroy a random fraction of productive capital, the sole factor of production.[5] Households have recursive Epstein–Zin preferences, and capital adjustment costs à la Lucas and Prescott (1971) and Hayashi (1982), which generate rents for installed capital so that Tobin's q exceeds one. A key driver of dynamics in our model is learning about the frequency of disasters (Hong, Wang, and Yang, 2023). Society updates beliefs about whether the world is in a "bad" high-risk state or a "good" low-risk state as extreme events occur. Extreme weather arrivals lead to higher perceived disaster risk and are associated with more future arrivals, and hence persistent damage to economic growth (Dell, Jones, and Olken, 2014).[6]

A new disaster both destroys capital and shifts beliefs toward the bad state, potentially activating the sustainability constraint. When a climate shock hits, capital falls and beliefs about disaster risk worsen. That is when the sustainability constraint may become binding—forcing resource reallocation from current consumption toward investment. The result is intergenerational smoothing: welfare remains stable, and asset valuations even rise, since the

[4] The sustainable development literature predominantly considers deterministic production economies. The constraint is first generalized to a stochastic endowment economy in Campbell and Martin (2025).

[5] The economy with rare disasters follows Rietz (1988), Barro (2006), and Pindyck and Wang (2013). We view capital as potentially encompassing not just manufactured capital but also human and natural capital following the literature on all inclusive wealth (Arrow et al., 2004).

[6] Many disasters follow a time-inhomogeneous Poisson process exhibiting volatility clustering that can be captured via a learning model (Hong, Ng, and Xu, 2025).

planner credibly commits to future resilience.

We show that a combination of state-contingent consumption taxes and investment subsidies can implement the planner's desired allocation subject to the sustainability constraint. When the economy faces low perceived disaster risk, the sustainability constraint is slack and fiscal wedges vanish. As beliefs tilt toward the high-risk state, the constraint binds and the government raises a consumption tax whose proceeds finance investment subsidies, maintaining the expected non-decline of welfare. A key object in our analysis is the Lagrange multiplier on the sustainability constraint. We derive closed-form expressions linking the optimal tax wedge to this Lagrange multiplier.

Recent fiscal innovations—such as Greece's climate-resilience tax and the proposed "premium flyers" solidarity levy led by France and African nations—illustrate governments' growing use of consumption-based instruments to internalize intergenerational risks. These policies implement the logic of our model: when disaster risk rises, a planner concerned with intergenerational welfare levies a state-contingent consumption tax whose proceeds fund resilience investment. The Greek tax enforces this sustainability constraint domestically, while the premium flyers tax extends it internationally, aligning fiscal design with SDG 13 (Climate Action) and SDG 17 (Partnerships for the Goals).

Quantitatively, we calibrate the model to match macro-finance moments for a representative tourism-dependent economy—such as aggregate investment, equity premium, real risk-free rate, and GDP per capita growth rate—and to empirical estimates of heatwave arrivals and their growth effects. The baseline economy, absent a sustainability rule, operates near the region where the constraint binds, i.e. at moderate disaster risk. When the constraint binds, the optimal policy then calls for a small consumption tax to subsidize investment; the tax then increases as perceived risk rises, consistent with observed climate resilience levies. Under our risk-sensitive fiscal rule, welfare becomes non-decreasing across generations, and asset valuations rise with worsening climate news—rationalizing the observed increase in Tobin's q and market valuations following adoption of resilience taxes in countries such as Spain and Greece.

We then contrast this policy with a planner who instead adopts a social rate of time preference that is lower than the private agent's discount rate—the Stern (2006) approach to intergenerational equity. Both tax and subsidy instruments appear similar to ours under the sustainability criterion, but their dynamics differ sharply in the two settings. The sustainability-based policy is risk-sensitive: it responds endogenously to evolving disaster beliefs and tightens only when equity is threatened. In contrast, the low-social-discount-rate policy turns out quantitatively to be *risk-insensitive* and achieves equity only by imposing uniformly and high consumption taxes, regardless of the realized climate state. Even in extremely low risk-states near zero, consumption taxes are high — at over 10% if the planner's time rate of preference is 4% as opposed to the households' 5% in our calibration. The taxes hardly move with disaster risk and are if anything decreasing as disaster risk rises. Without the sustainability criterion, bad climate news depresses asset prices. The sustainability approach thus achieves intergenerational equity that better aligns financial and social valuations than the low-social-discount-rate approach.

Policy implications. Our analysis offers several concrete policy insights. First, consumption-based sustainability taxes can serve as transparent, state-contingent fiscal instruments that automatically adjust to climate risk: as disaster probabilities increase, these levies should temporarily increase to finance investment in resilience, while remaining near zero in normal times. Second, governments can use asset-market indicators—such as Tobin's q, equity valuations, or the spread between sustainability bonds and standard bonds—as real-time signals of whether earmarked resilience investments are being credibly implemented. Third, the sustainability-constraint framework provides a practical benchmark for fiscal authorities and international institutions to assess whether national climate policies maintain or erode intergenerational welfare. More broadly, well-designed disaster-contingent fiscal policies can reconcile short-run efficiency with long-run equity, transforming climate resilience spending into a measurable component of macroeconomic policy.

Related literature. Our paper contributes to the literature on the sustainable development. The literature on the sustainable criterion is large and our results link the theory of sustainable development and inclusive wealth (Solow, 1993; Arrow et al., 2004) to modern macro-finance literature with rare disasters (Barro, 2009) and to analyses of the welfare consequences of disasters and adaptation (Pindyck and Wang, 2013; Martin and Pindyck, 2015, 2021; Fried, 2022; Hong, Wang, and Yang, 2023). Our paper provides a normative foundation for the emerging use of consumption-style climate-resilience taxes as instruments for equitable adaptation.

We also connect to recent formal treatments of the sustainability criterion under uncertainty. Pindyck (2024) study its cost under population growth. Campbell and Martin (2025) introduce the criterion into a consumption and portfolio choice problem (with risky capital). Because the investment opportunity is constant over time in Campbell and Martin (2025), the constraint always binds (or never binds) in their model. Pindyck (2024) and Campbell and Martin (2025) are the first to examine sustainability under uncertainty but abstract from disasters and do not analyze its implications for fiscal policy. In contrast, we analyze the interaction of disaster risk and the sustainability constraint. As the sustainability constraint in our model binds only only when the disaster is sufficiently likely, the optimal fiscal policies that ensure intergenerational equity are highly nonlinear and vary with the likelihood of the disaster arrival.

2 Model

In this section, we first introduce the consumption and production sectors building on Pindyck and Wang (2013), then turn to the social planner's objective with intertemporal sustainability considerations, and finally introduce how agents learn about disaster risks building on Hong, Wang, and Yang (2023).

2.1 Firms and Households

There is a continuum of identical firms and a continuum of identical households, each of measure one.

Firm's production. Each firm has an AK production technology:

$$Y_t = AK_t \,, \tag{1}$$

where constant $A > 0$ defines productivity and the capital stock K_t is the sole factor of production. We interpret K_t as the total stock of capital; it includes physical capital as traditionally measured, but also human capital and firm-based intangible capital.

Let I denote the firm investment. We use $t-$ to denote the time immediately before t. The capital stock K evolves according to the controlled jump-diffusion process:

$$dK_t = \Phi(I_{t-}, K_{t-})dt + \sigma K_{t-} d\mathcal{B}_t - (1-Z)K_{t-} d\mathcal{J}_t \,, \quad \text{given } K_0 > 0 \,, \tag{2}$$

where $\{\mathcal{B}_t\}_{t\geq 0}$ is a standard Brownian motion and $\{\mathcal{J}_t\}_{t\geq 0}$ denotes a jump process governed by a constant arrival rate λ, modeled as a latent variable that captures the intensity of rare disaster shocks. The processes $\mathcal{B}_t$ and $\mathcal{J}_t$ are assumed to be independent. The agents form and update their beliefs about the unobservable disaster arrival rate λ from historical jumps. We will return to discuss the details for the arrival rate λ in subsection 2.3.

As in Lucas and Prescott (1971), Hayashi (1982), and Jermann (1998), we assume that $\Phi(I, K)$, the first term in (2), is homogeneous of degree one in I and K, and thus

$$\Phi(I, K) = \phi(i)K \,, \tag{3}$$

where $i = I/K$ is the investment-capital ratio and $\phi(\,\cdot\,)$ is increasing and concave. The second term in (2) captures continuous diffusion shocks to capital, where σ is the diffusion volatility. The jump part (the third term) in (2) models discrete disasters, which allows for sudden changes in K_t. Note that the increment $d\mathcal{J}_t$ only takes two possible values, namely $d\mathcal{J}_t = 0$ or $d\mathcal{J}_t = 1$. In normal time ($d\mathcal{J}_t = 0$), there is no jump arrival at t, and the third term in (2) vanishes. If a jump occurs at time t ($d\mathcal{J}_t = 1$), a stochastic fraction $(1 - Z) \in (0, 1)$ of capital stock K_{t-} is destroyed and hence the surviving capital stock is $K_t = ZK_{t-}$. Let $\Xi(\,\cdot\,)$ denote the cumulative distribution function (cdf) for the recovery fraction $Z \in (0, 1)$. We assume that Z is independent of $\mathcal{B}_t$ and $\mathcal{J}_t$.

Households' preferences. We work with the recursive utility developed by Epstein and Zin (1989) and Weil (1990), and subsequently formulated in continuous time by Duffie and Epstein (1992a,b). The lifetime utility of the representative household, denoted by V_t, is given by the following recursion:

$$V_t = \mathbb{E}_t \left[\int_t^\infty f(C_s, V_s) ds \right] \,, \tag{4}$$

where $\mathbb{E}_t[\,\cdot\,]$ denotes the conditional expectation given information available at time t, The normalized aggregator for consumption C and continuation utility V takes the form:

$$f(C,V)=\frac{\rho}{1-\psi^{-1}}\frac{C^{1-\psi^{-1}}-((1-\gamma)V)^{\omega}}{((1-\gamma)V)^{\omega-1}}\,, \tag{5}$$

where ρ is the rate of time preference, ψ is the elasticity of intertemporal substitution (EIS), and γ is the coefficient of relative risk aversion. We define $\omega=(1-\psi^{-1})/(1-\gamma)$. This formulation allows for a separation between the EIS and risk aversion. If $\gamma=\psi^{-1}$ so that $\omega=1$, we obtain the standard constant relative risk aversion expected utility, represented by the aggregator $f(C,V)=\rho U(C)-\rho V$, which is additively separable in C and V with

$$U(C)=\frac{C^{1-\gamma}}{1-\gamma}\,. \tag{6}$$

This recursive utility specification is widely used in asset pricing, e.g., Bansal and Yaron (2004) and the subsequent long-run risk literature.

2.2 Planner's Objective: Sustainability Constraint

Building on Solow (1993), Arrow et al. (2004), and Campbell and Martin (2025) (see also Howarth (1995), Pezzey (2017), Pindyck (2024), and Love and Phelan (2024)), we incorporate intertemporal welfare equity considerations by requiring that the representative household's value function is not expected to decline over time. We extend the sustainability constraint in expected-utility settings developed by Campbell and Martin (2025) to our recursive-utility framework. As we show later, this generalization is crucial: it enables us to match asset prices more closely by disentangling the roles of the risk-aversion coefficient and the EIS. This approach parallels Bansal and Yaron (2004) in the asset-pricing literature and Pindyck and Wang (2013) and Hong, Wang, and Yang (2023) in welfare-analysis contexts.

Specifically, we impose a sustainability constraint that requires the household's utility V_t not to decline in expectation over time:

$$\frac{\mathbb{E}_{t-}[dV_t]}{dt}=-f(C_{t-},V_{t-})\geq 0\,. \tag{7}$$

Note that (7) is equivalent to requiring that the normalized aggregator defined in (5), $f(C,V)$, be non-positive to satisfy the sustainability constraint. For Duffie-Epstein-Zin recursive utility, (7) is equivalent to the following constraint:

$$U(C_{t-}) \leq V_{t-}\,. \tag{8}$$

Intuitively, the sustainability constraint (8) requires that the current generation's utility flow, $\rho U(C_{t-})$, cannot exceed the (annualized) continuation utility ρV_{t-}, which is a measure of welfare for future generations.[7] See Appendix A.1 for a derivation of (7) and (8).

2.3 Learning about the Disaster Arrival Rate

Information structure. As in Hong, Wang, and Yang (2023), households and firms cannot directly observe the latent arrival rate λ. What they know is that the true value of λ is either λ^G or λ^B with $0 < \lambda^G < \lambda^B$, where G and B stand for *Good* and *Bad* states, respectively.[8]

Belief updating. We assume that at time 0, all agents share a common prior belief π_0 that the true value of λ is λ^B. As time evolves, at each $t > 0$, agents update their belief π_t—the posterior probability that $\lambda = \lambda^B$—based on the observed history of disaster events up to time t. Formally,

$$\pi_t = \mathbb{P}_t\left(\lambda = \lambda^B\right)\,, \tag{9}$$

where $\mathbb{P}_t\left(\,\cdot\,\right)$ denotes the conditional probability given the information available at time t. The time-t expected arrival rate of disaster is then

$$\lambda_t = \pi_t \lambda^B + (1-\pi_t)\lambda^G\,, \tag{10}$$

which we write as a function of beliefs: $\lambda_t = \lambda(\pi_t)\,.$

Starting from the prior π_0, agents observe realized jump events and update their beliefs using Bayes' rule (Liptser and Shiryaev, 2001). The belief dynamics are governed by

$$d\pi_t = \sigma_\pi(\pi_{t-})\left(d\mathcal{J}_t - \lambda(\pi_{t-})dt\right)\,, \tag{11}$$

[7]To illustrate this intuition, consider the special case of expected power utility where the normalized aggregator is $f(C,V) = \rho U(C) - \rho V$.

[8]The learning environment is similar to that used in macro-finance models such as Collin-Dufresne, Johannes, and Lochstoer (2016) and Wachter and Zhu (2025).

where

$$\sigma_\pi(\pi) = \frac{\pi(1-\pi)\left(\lambda^B - \lambda^G\right)}{\lambda(\pi)} \geq 0\,. \tag{12}$$

Noe that $\sigma_\pi(0) = \sigma_\pi(1) = 0$, and hence $\pi = 0$ and $\pi = 1$ are absorbing states implied by (11): once agents are fully convinced that the true arrival rate is either λ^G or λ^B, their belief remains fixed thereafter. It allows us to characterize our model's boundary conditions.

If no disaster occurs over an increment dt, i.e., $d\mathcal{J}_t = 0$, agents become more optimistic. In this case, (11) simplifies to

$$d\pi_t = \mu_\pi(\pi_{t-})dt\,, \tag{13}$$

where

$$\mu_\pi(\pi_{t-}) = -\sigma_\pi(\pi_{t-})\lambda(\pi_{t-}) = \pi_{t-}\left(1-\pi_{t-}\right)\left(\lambda^G - \lambda^B\right) \leq 0\,. \tag{14}$$

Intuitively, the longer the time since the last disaster, the more likely agents are to believe that the true arrival rate is the lower value λ^G. In this sense, the absence of jumps is interpreted as good news.

3 Planner's Solution under the Sustainability Constraint

Throughout the paper, we use **boldfaced** letters for aggregate variables to distinguish them from their micro-level counterparts. The planner chooses consumption $\mathbf{C}$ and investment $\mathbf{I}$ to maximize the representative household's utility given in (4)–(5) subject to the sustainability constraint (7) and the resource constraint:

$$A\mathbf{K} = \mathbf{C} + \mathbf{I}\,. \tag{15}$$

Hereafter, we use the corresponding lowercase letters to denote the variables scaled by the contemporaneous capital stock $\mathbf{K}$; for example $\mathbf{c} = \mathbf{C}/\mathbf{K}$ and $\mathbf{i} = \mathbf{I}/\mathbf{K}$.

Let $V(\mathbf{K}, \pi)$ denote the representative household's value function, where capital stock $\mathbf{K}$ and posterior belief π are two state variables: $V(\mathbf{K}_t, \pi_t) = V_t$. Using dynamic programming, we obtain the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} 0 \;=\; \max_{\mathbf{C},\mathbf{I}}\; & f(\mathbf{C}, V(\mathbf{K},\pi)) + \Phi(\mathbf{I},\mathbf{K})V_{\mathbf{K}}(\mathbf{K},\pi) + \mu_\pi(\pi)V_\pi(\mathbf{K},\pi) + \frac{1}{2}\sigma^2\mathbf{K}^2V_{\mathbf{KK}}(\mathbf{K},\pi) \\ & + \lambda(\pi)\mathbb{E}\left[V\left(\mathbf{K}^{\mathcal{J}}, \pi^{\mathcal{J}}\right) - V(\mathbf{K},\pi)\right]\,, \end{aligned} \tag{16}$$

subject to the sustainability constraint (7), where $\mathbf{K}^{\mathcal{J}}$ is the post-jump capital stock given by

$$\mathbf{K}^{\mathcal{J}} = Z\mathbf{K} < \mathbf{K}\,, \tag{17}$$

and $\pi^{\mathcal{J}}$ is the post-jump belief given by

$$\pi^{\mathcal{J}} = \pi + \sigma_{\pi}(\pi) \geq \pi\,. \tag{18}$$

Again, it is worth emphasizing that (17) and (18) indicate that the disaster not only destroys capital stock, but also makes people pessimistic. The expectation on the second line of (16) is taken with respect to the recovery fraction Z.

We solve the optimization problem characterized by the HJB equation (16) using the Lagrangian method. We use superscript $*$ to denote the solution under the sustainability constraint. Let α_t^* denote the time-varying Lagrangian multiplier for the sustainability constraint (7). Because of our model's homogeneity property, we can show that the Lagrangian multiplier α_t^* is a function of π_t, so that we write $\alpha_t^* = \alpha^*(\pi_t)$.

The (simplified) Lagrangian for an embedded constrained optimization problem given in (16) is as follows:

$$\mathcal{L}(\mathbf{C}^*, \alpha^*) = \big(1 - \alpha^*(\pi)\big) f(\mathbf{C}^*, V^*(\mathbf{K}, \pi)) + \Phi(A\mathbf{K} - \mathbf{C}^*, \mathbf{K}) V^*_{\mathbf{K}}(\mathbf{K}, \pi)\,. \tag{19}$$

The planner's first-order condition for consumption associated with (19) is:[9]

$$(1 - \alpha^*(\pi)) f_{\mathbf{C}}(\mathbf{C}^*, V^*(\mathbf{K}, \pi)) = \Phi_{\mathbf{I}}(\mathbf{I}^*, \mathbf{K}) V^*_{\mathbf{K}}(\mathbf{K}, \pi)\,. \tag{20}$$

The left side is the marginal benefit of consumption and the right side is the product of $\Phi_{\mathbf{I}}(\mathbf{I}^*, \mathbf{K})$ and marginal value of capital $V^*_{\mathbf{K}}(\mathbf{K}, \pi)$. While the planner cares about intertemporal (intergenerational) equity, it still cares about the current generation and has a positive discount rate so that $\alpha^*(\pi) < 1$ under optimality.[10]

[9]Additionally, we have the standard dual feasibility condition, $\alpha^* \geq 0$, and the complementary slackness condition, $\alpha^* f(\mathbf{C}^*, V^*(\mathbf{K}, \pi)) = 0$. The latter implies that $\alpha^* = 0$ whenever $f(\mathbf{C}^*, V^*(\mathbf{K}, \pi)) < 0$.

[10]Note that $f_{\mathbf{C}} = \rho(\mathbf{C}^*)^{-\psi^{-1}}/((1-\gamma)V^*)^{\omega-1} > 0$, $\Phi_{\mathbf{I}} = \phi'(\mathbf{i}^*) > 0$, and $V^*_{\mathbf{K}} = (b^*(\pi))^{1-\gamma}\mathbf{K}^{-\gamma} > 0$, implying $\alpha^*(\pi) < 1$.

Using the homogeneity property, we write the value function as follows:

$$V^*(\mathbf{K},\pi) = \frac{1}{1-\gamma}\left(b^*(\pi)\mathbf{K}\right)^{1-\gamma}\,, \tag{21}$$

where $b^*(\pi)$ is a welfare measure (proportional to certainty equivalent wealth) to be determined.[11] Using (21) to re-express (8), we obtain the following simplified sustainability constraint:

$$\mathbf{c}^*(\pi) \le b^*(\pi)\,. \tag{22}$$

That is, the sustainability constraint requires that current (scaled) consumption $\mathbf{c}^*(\pi)$ to be lower than the welfare measure $b^*(\pi)$ (in a flow term).

Substituting (21) into the planner's FOC (20) and simplifying the expression, we can show that the aggregate investment-capital ratio $\mathbf{i}^*(\pi)$ solves the following implicit equation:

$$\mathbf{c}^*(\pi) = A - \mathbf{i}^*(\pi) = \left(\frac{(1-\alpha^*(\pi))\,\rho}{\phi'\left(\mathbf{i}^*(\pi)\right)}\right)^{\psi} b^*(\pi)^{1-\psi}\,. \tag{23}$$

Importantly, the Lagrange multiplier $\alpha^*(\pi)$ appears in (23). Using (23) to simplify the HJB equation (16), we obtain:

$$\begin{aligned} 0 \ =\ & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{\mathbf{c}^*(\pi)}{b^*(\pi)}\right)^{1-\psi^{-1}} - 1\right] + \phi\left(\mathbf{i}^*(\pi)\right) + \mu_\pi(\pi)\frac{(b^*)'(\pi)}{b^*(\pi)} - \frac{\gamma\sigma^2}{2} \\ & + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b^*\left(\pi^{\mathcal{J}}\right)}{b^*(\pi)}\right)^{1-\gamma}\mathbb{E}\left[Z^{1-\gamma}\right] - 1\right]. \end{aligned} \tag{24}$$

Next, we summarize the solution for the planner under the sustainability constraint (22).

Proposition 1. *If (22) binds,* $\mathbf{c}^*(\pi) = b^*(\pi)$ *and the Lagrangian multiplier is given by:*

$$\alpha^*(\pi) = 1 - \frac{1}{\rho}\phi'\left(\mathbf{i}^*(\pi)\right) b^*(\pi)\,. \tag{25}$$

If (22) does not bind, i.e., $\alpha^*(\pi) = 0$, $\mathbf{c}^*(\pi) = \left(\frac{\rho}{\phi'(\mathbf{i}^*(\pi))}\right)^{\psi} b^*(\pi)^{1-\psi}$.

See Appendix O.1.2 for the algorithm used to solve the planner's problem.

[11] The value function is homogeneous in $\mathbf{K}$. Intuitively, if the capital stock is doubled, it is optimal for the planner to simultaneously double consumption and investment. Formally, since the aggregator is homogeneous in the sense that $f(a\mathbf{C}, a^{1-\gamma}V) = a^{1-\gamma}f(\mathbf{C}, V)$ for any $a > 0$, it follows that $V(a\mathbf{K},\pi) = a^{1-\gamma}V(\mathbf{K},\pi)$, which motivates (21).

4 Market Solution under Fiscal Policies

In this section, we implement the solution of the planner's problem under sustainability constraints by designing optimal taxation and subsidy policies in a market economy with profit-maximizing firms and utility-maximizing households.

4.1 Market Structure and Fiscal Policies

As in Pindyck and Wang (2013), consider a dynamically complete securities market in which the following securities can be traded at each point in time: (i) a risk-free asset with short rate r_t, (ii) the aggregate equity market, which is represented by a price-dividend pair $(\mathbf{Q}_t, \mathbf{D}_t)$, (iii) and insurance claims for disaster with every possible recovery fraction Z. We leave out the analysis of the disaster insurances as they are in zero net supply and there is no trade in equilibrium. The equilibrium risk-free rate and the stochastic discount factor do not depend on the insurance claims.

Government taxation and subsidy policies. The government taxes a representative firm's investment I_t at a rate τ_t^I and a representative household's consumption C_t at a rate τ_t^C. If a tax rate is negative, the government offers a subsidy. In equilibrium, the tax rates τ_t^C and τ_t^I are functions of belief π_t, and we write $\tau_t^C = \tau^C(\pi_t)$ and $\tau_t^I = \tau^I(\pi_t)$.

The government utilizes a package of taxes and subsidies to decentralize the planner's solution subject to the sustainability constraint. Without loss of economic efficiency, we focus on government policies that balance the government's budget every period so that

$$\tau_t^C \mathbf{C}_t + \tau_t^I \mathbf{I}_t = 0\,. \tag{26}$$

As $\mathbf{C}_t$ and $\mathbf{I}_t$ are positive, the government must finance a subsidy to one economic activity with a tax on the other activity. Importantly, subsidy and tax policies induce the private sectors to choose economic outcomes that meet the sustainability constraint.

Competitive equilibrium. We define the recursive competitive equilibrium with sustainability tax and subsidy policies as follows:

(a) Taking the stochastic discount factor (SDF) $\{\mathbb{M}_t\}$ as given, the representative firm chooses investment $\{I_t\}$ to maximize its market value. The firm's payout is

$$D_t = AK_t - (1+\tau_t^I)I_t\,. \tag{27}$$

(b) Taking the SDF $\{\mathbb{M}_t\}$ as given, the representative household chooses consumption $\{C_t\}$ and allocation to the stock market (in units of the consumption good), $\{\Gamma_t\}$, to maximize lifetime utility given in (4)-(5), without taking the sustainability constraint into account.

(c) The SDF $\{\mathbb{M}_t\}$ is consistent with the firms' and households' optimal decisions and all markets clear in equilibrium.

4.2 Firm's Optimization

The SDF is implied by the household's preferences and equilibrium consumption. In Appendix B.3, we show that the SDF is given by

$$\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} = \left[-r_{t-} - \lambda\left(\pi_{t-}\right)\mathbb{E}(\eta_{t-}-1)\right]dt - \gamma\sigma d\mathcal{B}_t + \left(\eta_{t-}-1\right)d\mathcal{J}_t\,, \tag{28}$$

where the equilibrium risk-free rate is a function of belief π: $r_{t-} = r(\pi_{t-})$. The $\gamma\sigma$ term in (28) is the equilibrium market price of diffusion shocks. Should a disaster arrive at time t, the SDF would change discretely from $\mathbb{M}_{t-}$ to $\mathbb{M}_t^{\mathcal{J}}$. Let $\eta_{t-} = \eta(\pi_{t-};Z) = \frac{\mathbb{M}_t^{\mathcal{J}}}{\mathbb{M}_{t-}}$. Taking the equilibrium SDF $\{\mathbb{M}_t\}$ and the tax-rate process $\{\tau_t^I\}$ as given, the representative firm chooses investment $\{I_s; s\geq t\}$ to maximize the value of its dividend process $\{D_s; s\geq t\}$ given in (27):

$$\max_{\{I_s;\,s\geq t\}}\ \mathbb{E}_t\left[\int_t^\infty \frac{\mathbb{M}_s}{\mathbb{M}_t}D_s ds\right]. \tag{29}$$

Let Q_t denote the firm's market value, the value function associated with the optimization problem (29). Exploiting the model's homogeneity property, we conjecture and later verify that the equilibrium firm value at time t, $Q_t = Q(K_t,\pi_t)$, satisfies

$$Q(K_t,\pi_t) = q(\pi_t)K_t\,, \tag{30}$$

where $q(\pi_t)$ is Tobin's average q as a function of belief π_t. The HJB equation for $q(\pi)$ is:

$$
\begin{aligned}
r(\pi)q(\pi) \; = \; & \max_i \; A - (1+\tau^I(\pi))i + \phi(i)q(\pi) + \mu_\pi(\pi)q'(\pi) - \gamma\sigma^2 q(\pi) \\
& + \lambda(\pi)\mathbb{E}\Big[\eta(\pi;Z)\big(q\left(\pi^{\mathcal{J}}\right)Z - q(\pi)\big)\Big] \,.
\end{aligned} \tag{31}
$$

We provide a derivation in Appendix B.1. The FOC for investment associated with (31) is

$$
1+\tau^I(\pi) = \phi'(i(\pi))q(\pi)\,. \tag{32}
$$

The left side of (32) is the marginal cost of investing one unit (accounting for the investment tax). The right side of (32) is the marginal benefit of investing, the product of Tobin's $q(\pi)$ multiplied by the marginal increase of capital stock $\phi'(i(\pi))$.

4.3 Household's Optimization

The representative household dynamically chooses consumption C and allocation to the risky asset Γ to maximize (4). Let $J(W,\pi)$ denote the household's value function, where W is its wealth. We conjecture and later verify that $J(W,\pi)$ takes the form of

$$
J(W,\pi) = \frac{1}{1-\gamma}\left(u(\pi)W\right)^{1-\gamma}\,, \tag{33}
$$

where $u(\pi)$ is to be determined.

Let $\{\mathbf{Q}_t\}$ denote the equilibrium ex-dividend aggregate stock market value and $\{\mathbf{D}_t\}$ denote the aggregate dividends, respectively. Then the cum-dividend return is given by

$$
\frac{d\mathbf{Q}_t + \mathbf{D}_{t-}dt}{\mathbf{Q}_{t-}} = \mu_{\mathbf{Q}}(\pi_{t-})dt + \sigma d\mathcal{B}_t + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)d\mathcal{J}_t\,, \tag{34}
$$

where $\mu_{\mathbf{Q}}(\pi)$ denotes the expected stock market return, excluding jump effects. The aggregate stock market value $\mathbf{Q}_t$ is proportional to the aggregate capital stock $\mathbf{K}_t$, i.e., $\mathbf{Q}_t = \mathbf{q}(\pi_t)\mathbf{K}_t$, where $\mathbf{q}(\pi_t)$ is the Tobin's q in equilibrium. The post-jump aggregate stock market value is $\mathbf{Q}_t^{\mathcal{J}} = \mathbf{q}(\pi_t^{\mathcal{J}})\mathbf{K}_t^{\mathcal{J}}$. In Appendix B.1, we show that

$$
\mu_{\mathbf{Q}}(\pi) = \frac{A - \left(1+\tau^I(\pi)\right)\mathbf{i}(\pi)}{\mathbf{q}(\pi)} + \phi(\mathbf{i}(\pi)) + \mu_\pi(\pi)\frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\,. \tag{35}
$$

The first term on the right side of (35) is the dividend yield, the second term $\phi(\mathbf{i}(\pi))$ captures expected capital gains from capital accumulation, and the last term reflects how changes in beliefs affect valuation.

Given the cum-dividend return dynamics (34), the household's wealth evolves as

$$\begin{aligned} dW_t \;=\; & \left[r(\pi_{t-})W_{t-} + (\mu_{\mathbf{Q}}(\pi_{t-}) - r(\pi_{t-}))\Gamma_{t-} - \left(1+\tau^C(\pi_{t-})\right) C_{t-}\right] dt + \sigma\Gamma_{t-} d\mathcal{B}_t \\ & + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)\Gamma_{t-} d\mathcal{J}_t \,, \end{aligned} \tag{36}$$

where the first two terms in (36) are similar to those in a standard consumption–portfolio choice problem (Merton, 1969), and the last term captures the effect of disaster shocks.

The household's value function $J(W,\pi)$ satisfies the following HJB equation:

$$\begin{aligned} 0 \;=\; & \max_{C,\Gamma} f(C, J(W,\pi)) + \left[r(\pi)W + (\mu_{\mathbf{Q}}(\pi) - r(\pi))\,\Gamma - \left(1+\tau^C(\pi)\right) C\right] J_W(W,\pi) \\ & + \mu_\pi(\pi) J_\pi(W,\pi) + \frac{1}{2}\sigma^2\Gamma^2 J_{WW}(W,\pi) + \lambda(\pi)\mathbb{E}\left[J\left(W^{\mathcal{J}}, \pi^{\mathcal{J}}\right) - J(W,\pi)\right] \,, \end{aligned} \tag{37}$$

where the post-jump belief $\pi^{\mathcal{J}}$ is given by (18) and the post-jump wealth $W^{\mathcal{J}}$ is given by

$$W^{\mathcal{J}} = W + \left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} - 1\right)\Gamma \,. \tag{38}$$

The FOC for consumption is

$$f_C(C,J) = (1+\tau^C(\pi)) J_W(W,\pi) \,. \tag{39}$$

The left side of (39) is the marginal utility of consumption and the right side is the marginal cost of consumption, which is $(1+\tau^C(\pi))$ units of forgone savings multiplied by marginal utility of wealth J_W. Using the homogeneity property (33) to simplify (39), we obtain:

$$C(W,\pi) = \left(\frac{\rho}{1+\tau^C(\pi)}\right)^{\psi} u(\pi)^{1-\psi} W \,. \tag{40}$$

The optimal consumption rule given in (40) is linear in wealth with a belief-dependent marginal propensity to consume. In Appendix B.2, we show that the FOC for asset allocation Γ implies:

$$\Gamma = \frac{\mu_{\mathbf{Q}}(\pi) - r(\pi)}{\gamma\sigma^2} W - \frac{\lambda(\pi)}{\gamma\sigma^2}\mathbb{E}\left[\left(1 - \frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\right)\left(\frac{u\left(\pi^{\mathcal{J}}\right)}{u(\pi)}\right)^{1-\gamma}\left(\frac{W^{\mathcal{J}}}{W}\right)^{-\gamma}\right] W \,. \tag{41}$$

4.4 Market Equilibrium

In equilibrium, the household invests all wealth in the stock market, $W = \Gamma = \mathbf{Q}$, and the goods market clears[12], $(1+\tau^C)\mathbf{C} = \mathbf{D} = A\mathbf{K} - (1+\tau^I)\mathbf{I}$. We show in Appendix B.3 that, in equilibrium, the ratio of the post-jump to the pre-jump SDF, $\eta_{t-} = \mathbb{M}_t^{\mathcal{J}}/\mathbb{M}_{t-}$, is givn by

$$\eta_{t-} = \eta\left(\pi_{t-}; Z\right) = \left(\frac{u\left(\pi_t^{\mathcal{J}}\right)}{u\left(\pi_{t-}\right)}\right)^{1-\gamma}\left(\frac{\mathbf{q}\left(\pi_t^{\mathcal{J}}\right)}{\mathbf{q}\left(\pi_{t-}\right)}Z\right)^{-\gamma}. \tag{42}$$

We can further simplify the FOCs (40) and (41) to obatin

$$\mathbf{c}(\pi) = \left(\frac{\rho}{1+\tau^C(\pi)}\right)^{\psi} u(\pi)^{1-\psi}\mathbf{q}(\pi)\,, \tag{43}$$

and

$$\mu_{\mathbf{Q}}(\pi) = r(\pi) + \gamma\sigma^2 - \lambda(\pi)\mathbb{E}\left[\eta(\pi;Z)\left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} - 1\right)\right]. \tag{44}$$

The household's HJB equation (37) further reduces to

$$\begin{aligned} 0 \;=&\; \frac{\psi^{-1}\rho^{\psi}\left[(1+\tau^C(\pi))u\left(\pi\right)\right]^{1-\psi} - \rho}{1-\psi^{-1}} + \mu_{\mathbf{Q}}\left(\pi\right) + \mu_{\pi}\left(\pi\right)\frac{u'\left(\pi\right)}{u\left(\pi\right)} - \frac{\gamma\sigma^2}{2} \\ &+ \frac{\lambda\left(\pi\right)}{1-\gamma}\left[\mathbb{E}\left(\eta(\pi;Z)\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\right) - 1\right]. \end{aligned} \tag{45}$$

In sum, taking the tax rates τ^C and τ^I as given, the market solution is characterized by: (i) the ODE (31) for $q(\pi)$ and the FOC (32) for firms; and (ii) the ODE (45) for $u(\pi)$ and the FOCs (43)–(44) for households.

Proposition 2 summarizes the asset-pricing implications (see Appendix B.3 for a proof).

Proposition 2. *Tobin's q for the aggregate economy is $\mathbf{q}(\pi) = (1+\tau^I(\pi))/\phi'(\mathbf{i}(\pi))$. The equilibrium risk-free rate, $r(\pi)$, is given by*

$$\begin{aligned} r(\pi) \;=&\; \rho + \psi^{-1}\phi(\mathbf{i}(\pi)) - \frac{\gamma\left(\psi^{-1}+1\right)}{2}\sigma^2 \\ &- \left[\left(1-\psi^{-1}\right)\left(\frac{u'(\pi)}{u(\pi)} + \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right) - \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right]\mu_{\pi}(\pi) \\ &- \lambda(\pi)\frac{\psi^{-1}-\gamma}{1-\gamma}\left[1 - \mathbb{E}\left(\eta(\pi;Z)\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\right)\right] - \lambda(\pi)\left[\mathbb{E}(\eta(\pi;Z)) - 1\right]. \end{aligned} \tag{46}$$

[12] Goods-market clearing implies $A\mathbf{K} - \mathbf{I} - \mathbf{C} = \tau^C\mathbf{C} + \tau^I\mathbf{I}$. Hence, in equilibrium, the resource constraint (15) holds if and only if the balanced-budget condition (26) holds.

and the equity premium, $\mathrm{rp}(\pi)$*, is given by*

$$\mathrm{rp}(\pi) = \gamma\sigma^2 - \lambda(\pi)\mathbb{E}\left[(\eta(\pi;Z)-1)\left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}-1\right)\right]. \tag{47}$$

4.5 Sustainability Taxation and Subsidy Design

In this subsection, we show how the government chooses its tax and subsidy policies to attain the planner's solution subject to the sustainability constraint (7). Mathematically, the government chooses $\tau^C(\pi)$ and $\tau^I(\pi)$ so that the household's value function $J(W,\pi)$, the optimal scaled consumption $\mathbf{c}(\pi)$ given by (43), and the optimal scaled investment $\mathbf{i}(\pi)$ implied by (32) in the market economy coincide with the corresponding objects in the planner's solution:

$$J(W,\pi) = V^*(\mathbf{K},\pi), \quad \mathbf{c}(\pi) = \mathbf{c}^*(\pi), \quad \text{and} \quad \mathbf{i}(\pi) = \mathbf{i}^*(\pi). \tag{48}$$

Recall that the starred objects are the planner's solutions under the sustainability constraint, with $V^*(\mathbf{K},\pi)$, $\mathbf{c}^*(\pi)$, and $\mathbf{i}^*(\pi)$ given by (21) and (23).

In Appendix B.4, we show that

$$(1-\alpha^*(\pi))\left(1+\tau^C(\pi)\right) = 1+\tau^I(\pi). \tag{49}$$

Intuitively, to fully capture the effect of the sustainability constraint (7), the planner chooses the tax and subsidy policies to equate the after-tax (societal) cost for consumption, the left side of (49), with the after-tax cost of investment, the right side of (49). To provide further intuition for (49), recall that in the market economy with taxation and subsidies, the FOCs for investment and consumption are respectively given by: $1+\tau^I(\pi) = \phi'(\mathbf{i}(\pi))\mathbf{q}(\pi)$ and $f_{\mathbf{C}}(\mathbf{C},J) = (1+\tau^C(\pi))J_W$. Using these condition, we obtain the following condition:

$$\frac{1+\tau^I(\pi)}{1+\tau^C(\pi)} = 1-\alpha^*(\pi) = \frac{\Phi_{\mathbf{I}}(\mathbf{I}^*,\mathbf{K})V^*_{\mathbf{K}}(\mathbf{K},\pi)}{f_{\mathbf{C}}(\mathbf{C}^*,V^*(\mathbf{K},\pi))}. \tag{50}$$

The first equality in (50) is the private sector's tradeoff between investment and consumption and the second equality in (50) follows from the planner's first-order condition (20).

The scaled version of the government's period-by-period balanced-budget condition (26) can be written as

$$\tau^C(\pi)\mathbf{c}^*(\pi) + \tau^I(\pi)\mathbf{i}^*(\pi) = 0. \tag{51}$$

Solving (49) and (51), we express the tax (subsidy) rates as functions of the sustainability constraint's multiplier $\alpha^*(\pi)$ and the planner's choices of $\mathbf{c}^*(\pi)$ and $\mathbf{i}^*(\pi)$:

$$\tau^C(\pi) = \frac{\alpha^*(\pi)\mathbf{i}^*(\pi)}{A - \alpha^*(\pi)\mathbf{i}^*(\pi)} \geq 0 \quad \text{and} \quad \tau^I(\pi) = -\frac{\alpha^*(\pi)\mathbf{c}^*(\pi)}{A - \alpha^*(\pi)\mathbf{i}^*(\pi)} \leq 0\,. \tag{52}$$

When the sustainability constraint (7) does not bind, $\alpha^*(\pi) = 0$, there is no need to tax investment nor consumption: $\tau^C(\pi) = \tau^I(\pi) = 0$. When the constraint (7) binds and the planner's desired level of investment is positive: $\mathbf{i}^*(\pi) > 0$, the planner discourages consumption by taxing it at the rate $\tau^C(\pi)$ to subsidize investment for the future at the rate $\tau^I(\pi)$.

Using (48) and the equilibrium result $W = \mathbf{q}(\pi)\mathbf{K}$, we can also show that the welfare measure $b^*(\pi)$ in the planner's economy under the sustainability constraint satisfies:

$$b^*(\pi) = u(\pi)\mathbf{q}(\pi)\,, \tag{53}$$

where $\mathbf{q}(\pi)$ and $u(\pi)$ are given in the market economy under the fiscal policies given in (52).

We can verify that the implied differential equation for $\mathbf{q}(\pi)u(\pi)$ in the market economy with fiscal policies given in (52) is the same as (24) for $b^*(\pi)$ in the planner's problem with the sustainability constraint. We summarize the main results in the following proposition and provide a proof in Appendix B.4.

Proposition 3. *Under the tax/subsidy policies given in* (52)*, the competitive market solution recovers the planner's solution subject to the sustainability constraint.*

5 Lowering the Social Discount Rate via Fiscal Policies

A common alternative to the sustainability approach is to promote intergenerational equity by lowering the social discount rate used to evaluate welfare across time. Following the tradition of Stern (2006), the government may wish to act as if society discounts the future at a lower "ethical" rate $\hat{\rho} < \rho$, even though private agents continue to discount at the higher market rate ρ. This section derives the fiscal instruments required to decentralize such a policy and clarifies how its implications differ from those of the sustainability-constraint approach in Section 4.

A counterfactual economy. Consider a counterfactual competitive-market economy in which all agents discount the future at the rate of $\hat{\rho}$. All other ingredients of the economy, e.g., technology, and diaster shocks, are the same as in Section 2. The equilibrium solution of this counterfactual economy is the same as the planner's solution for this economy without the sustainability constraint. Specifically, for both the planner's solution and the market solution in this counterfactual economy, the scaled consumption $\mathbf{c}^u(\pi;\hat{\rho})$ and investment $\mathbf{i}^u(\pi;\hat{\rho})$ are characterized by (A.6)–(A.7) in Appendix A.3 with a discount rate of $\hat{\rho}$. Let $b^u(\pi;\hat{\rho})$ denote the resulting welfare measure for the household in this counterfactual economy.

Setting taxes in the actual economy. Now we return to the actual economy where agents discount at the rate of ρ. To implement the allocation $\mathbf{c}^u(\pi;\hat{\rho})$ and $\mathbf{i}^u(\pi;\hat{\rho})$ obtained above, the government imposes state-dependent taxes on consumption and investment, $\hat{\tau}^C$ and $\hat{\tau}^I$, subject to its period-by-period balanced-budget constraint (26). Taking $\hat{\tau}^C$ and $\hat{\tau}^I$ as given, firms and the household (with a discount rate of ρ) optimally choose their investment and consumption policies to $\mathbf{c}^u(\pi;\hat{\rho})$ and $\mathbf{i}^u(\pi;\hat{\rho})$, respectively.

In Appendix C, we show that the tax rates for consumption and investment that implement the planner's desired allocations, $\mathbf{c}^u(\pi;\hat{\rho})$ and $\mathbf{i}^u(\pi;\hat{\rho})$, are

$$\hat{\tau}^C(\pi) = \frac{A}{A-(1-\varepsilon(\pi))\mathbf{i}^u(\pi;\hat{\rho})} - 1 \quad \text{and} \quad \hat{\tau}^I(\pi) = -\frac{\mathbf{c}^u(\pi;\hat{\rho})}{\mathbf{i}^u(\pi;\hat{\rho})}\hat{\tau}^C(\pi)\,, \tag{54}$$

where

$$\varepsilon(\pi) = \frac{\hat{\rho}}{\rho}\left(\frac{\hat{b}(\pi;\rho)}{b^u(\pi;\hat{\rho})}\right)^{1-\psi^{-1}}. \tag{55}$$

The $\hat{b}(\pi;\rho)$ term in (55) that solves (C.5)–(C.6) in Appendix C is the (certainty equivalent) welfare measure for the representative agent in the actual market economy under the government policy given in (54). Importantly, $\hat{b}(\pi;\rho)$ is different from the welfare measure for the agent with a discount rate of $\hat{\rho}$ in the counterfactual economy described above: $b^u(\pi;\hat{\rho})$. Intuitively, $\varepsilon(\pi)$ captures the implications of the government's using a lower ethical discount rate $\hat{\rho} < \rho$ on its tax and subsidy policies. If the government aims to reduce consumption by making investment relatively cheaper, then $\varepsilon(\pi) < 1$, which implies a positive consumption tax rate. As a special case, $\varepsilon(\pi) = 1$ when $\hat{\rho} = \rho$, there is neither tax nor subsidy: $\hat{\tau}^C(\pi) = \hat{\tau}^I(\pi) = 0$.

Note that the tax and subsidy policy given in (54) balances the government's budget period by period:

$$\hat{\tau}^C(\pi)\mathbf{c}^u(\pi;\hat{\rho}) + \hat{\tau}^I(\pi)\mathbf{i}^u(\pi;\hat{\rho}) = 0\,. \tag{56}$$

Because a lower discount rate raises the value of savings, thus increasing the shadow value of capital, the fiscal wedge must tilt incentives toward more saving. Formally, matching the planner's optimality conditions with the private agents' conditions in the actual market economy implies that the following fiscal-wedge condition must hold (see Appendix C):

$$\frac{1+\hat{\tau}^I(\pi)}{1+\hat{\tau}^C(\pi)} = \varepsilon(\pi)\,. \tag{57}$$

This condition requires that the relative price of investment goods to consumption goods equals $\varepsilon(\pi)$.

In our quantitative analysis below, we show that these taxes are largely constant across states of the world, in that they do not vary much even as the likelihood of disasters vary significantly. Consequently, this policy achieves intergenerational equity only by imposing an almost uniformly high consumption tax, largely independent of whether the economy is currently in a high- or low-risk climate state. One potential weakness of this "lowering the discount rate" policy proposal is that it is not obvious which discount rate society shall use. Given that policies are highly sensitive to the choice of the discount rate $\hat{\rho}$, a moderate disagreement about which discount rate to use, e.g., 2% versus 2.5% or 1.5%, can lead to very different consumption tax and investment subsidies. This is because a calculation of welfare often boils down to a variation of perpetual (stochastic) growth calculation where the discount rate appears in the denominator. This is why policy recommendations vary substantially with $\hat{\rho}$, as we show in subsection 6.4.

6 Quantitative Analysis

This section quantitatively evaluates how the sustainability constraint alters optimal fiscal policy, asset prices, and intergenerational welfare. We first calibrate the model to data from tourism-dependent economies *before* the adoption of sustainability taxes. We then use the estimated parameters to compare intergenerational welfare outcomes with and without the sustainability constraint that ensures expected welfare is non-decreasing across generations. We further compare policy implications of imposing the sustainability constraint versus those of using an ethical discount rate as proposed in Stern (2006).

6.1 Distributional and Functional Form Specifications

Following Barro (2006) and Pindyck and Wang (2013), the recovery fraction Z upon a disaster arrival is assumed to follow a power-law distribution over $(0, 1)$:

$$\Xi(Z) = Z^{\beta}\,, \tag{58}$$

where $\beta > 0$ governs the expected capital loss. Conditional on a disaster, the expected fractional loss is $\mathbb{E}(1 - Z) = 1/(\beta + 1)$. Larger β implies smaller expected destruction. We adopt the standard quadratic adjustment-cost specification:

$$\phi(i) = i - \frac{1}{2}\theta i^2 - \delta\,, \tag{59}$$

where θ captures capital adjustment costs and δ is the depreciation rate.

6.2 Calibration and Parameter Choices

Table 1 reports the key parameters. The following parameters are taken directly from the literature—productivity A, the time rate of preference ρ, the elasticity of intertemporal substitution ψ, and the disaster damage distribution parameter β. As shown by Bansal and Yaron (2004), an elasticity of intertemporal substitution $\psi > 1$ is required to match observed risk premia and risk-free rates. Production parameters are in line with those in Eberly, Rebelo, and Vincent (2012) and Hong, Wang, and Yang (2023).

The disaster arrival process parameters (λ_G, λ_B, and π_0), for typical country exposed to heatwaves, are taken from Hong, Ng, and Xu (2025).[13] The large gap between λ^G and λ^B reflects the uncertainty documented in climate-science projections of extreme-event frequency (National Academies of Sciences, Engineering, and Medicine, 2016). The disaster-damage

[13] In the good state, a disaster arrives once every 7.2 years ($1/\lambda_G$), while in the bad state, a disaster arrives once every 1.35 years ($1/\lambda_B$). The expected arrival rate is $\pi_0\lambda_B + (1 - \pi_0)\lambda_G$ or 0.20 — a disaster once every five years, which matches the actual frequency of heatwaves in our sample.

Table 1: Summary of Key Parameters

Parameter	Symbol	Value
Productivity parameter	A	0.3
Rate of time preference	ρ	0.05
Elasticity of intertemporal substitution	ψ	2
Prior of being in State B	π_0	0.11
Disaster jump arrival rate in State G	λ^G	0.138
Disaster jump arrival rate in State B	λ^B	0.727
Power law exponent	β	99
Capital diffusion volatility	σ	0.12
Adjustment cost parameter	θ	4.7
Depreciation rate of capital	δ	0.06
Coefficient of relative risk aversion	γ	4.53

parameter β reflects the empirical magnitude of temperature-related output losses reported in Dell, Jones, and Olken (2014) and Bilal and Känzig (2024).

The remaining parameters—$(\theta, \sigma, \delta, \gamma)$—are calibrated via the simulated method of moments to match four empirical moments: the investment–capital ratio, equity risk premium, real risk-free rate, real GDP per capita growth rate (Table 2).[14] See Appendix O.2 for more details for the calibration.

Table 2: Empirical Moments and Model-Implied Moments

Moments	Data	Model
Real risk-free rate (%)	1.59	1.76
Equity risk premium (%)	6.21	6.56
Investment-capital ratio I/K	0.10	0.14
Real GDP per capita growth rate (%)	3.21	3.35

[14]The moments are from a set of tourism-dependent countries, which include Aruba, Antigua and Barbuda, Bahamas, Spain, Fiji, France, Grenada, Greece, Croatia, Iceland, Italy, Jamaic, St. Kitts and Nevis, St. Lucia, Maldives, Mexico, Malta, Seychelles, Thailan, Virgin islands and Vanuatu. Countries missing an observation for a particular variable are dropped from the calculation.

6.3 Quantitative Findings

Planner's solution with and without the sustainability criterion. Panel A of Figure 1 plots the welfare measure $b(\pi)$ with and without the sustainability criterion. Under the sustainability constraint (the solid blue line), welfare declines more steeply with the perceived probability of the bad climate state (π) than under the standard benchmark which features no sustainability constraint (the dashed red line).

Panel B of Figure 1 plots the Lagrange multiplier $\alpha^*(\pi)$ for the sustainability criterion. For $\pi > 0.15$, the constraint binds ($\alpha^*(\pi) > 0$), and moreover, $\alpha^*(\pi)$ increases as the disaster belief worsens. At $\pi = 1$, the certainty-equivalent welfare under the sustainability criterion, $b(\pi)$, is about 0.005 lower than under the standard benchmark without the sustainability constraint, reflecting the short-run sacrifice needed to preserve long-run interrogational equity.

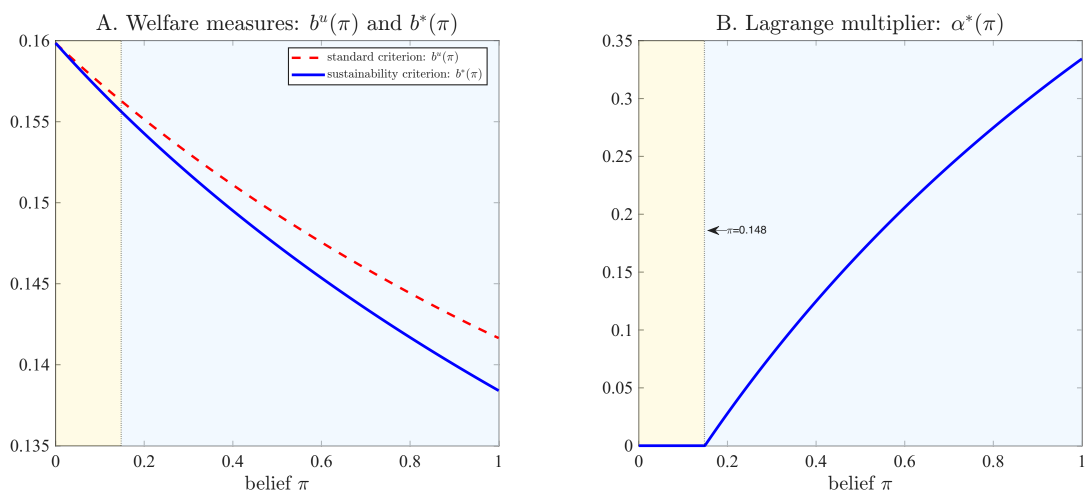

Figure 1: Certainty-equivalent welfare measures, $b^u(\pi)$ under the standard benchmark and $b^*(\pi)$ under the sustainability criterion (panel A), and the Lagrangian multiplier for the sustainability criterion, $\alpha^*(\pi)$ (panel B).

Consumption and investment. Figure 2 compares the planner's scaled consumption and investment policies with and without the sustainability criterion. For low or moderate π, the sustainability constraint is slack and investment $\mathbf{i}^*(\pi)$ declines with the disaster belief as in Hong, Wang, and Yang (2023). When $\pi > 0.15$, the sustainability constraint binds and investment $\mathbf{i}^*(\pi)$ rises with π: the planner increases the investment-capital ratio to offset future welfare losses from more frequent disasters. Consumption $\mathbf{c}(\pi) = A - \mathbf{i}(\pi)$ mirrors this pattern—rising initially and then falling once sustainability binds.

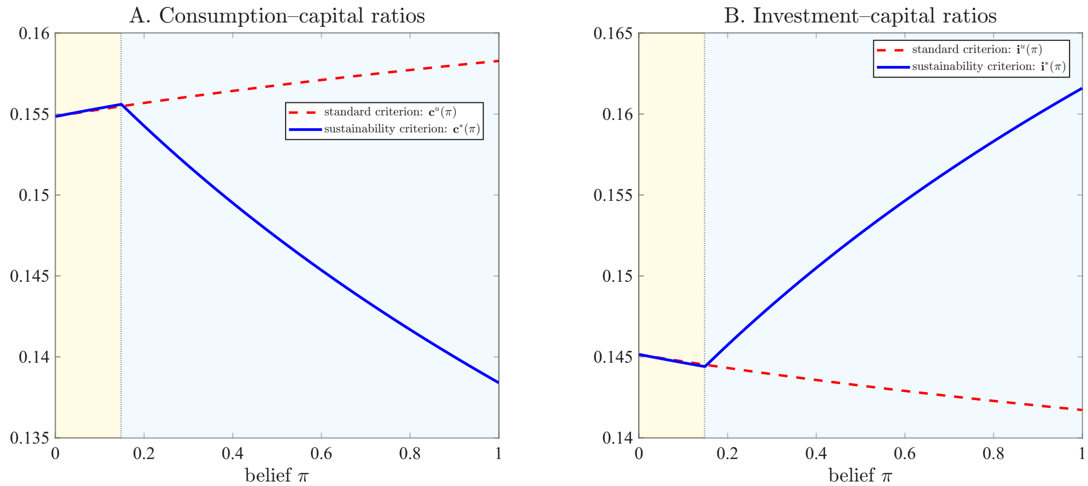

Figure 2: Optimal consumption and investment policies under the sustainability criterion and under the standard welfare benchmark (without the sustainability criterion).

Optimal consumption taxes and investment subsidies. Figure 3 plots the decentralized fiscal instruments that implement the planner's allocation under the sustainability criterion. The optimal consumption tax $\tau^C(\pi)$ is zero when sustainability is slack and increases sharply once the sustainability constraint binds. The associated investment subsidy $-\tau^I(\pi)$ (negative of the tax rate) increases as disaster belief π worsens. These results illustrate how state-contingent fiscal policy—rather than a fixed tax rule—achieves intergenerational equity.

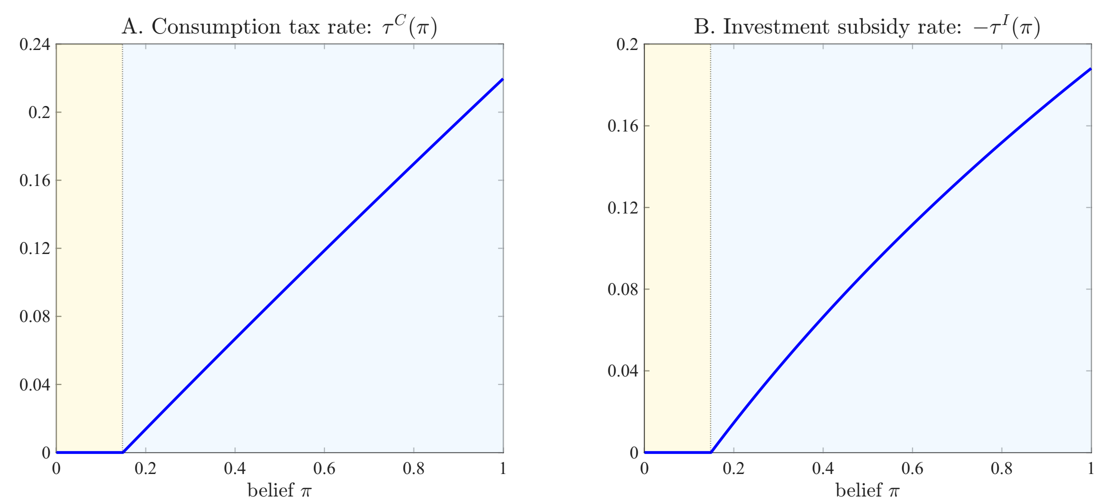

Figure 3: Optimal belief-dependent consumption tax rate $\tau^C(\pi)$ and belief-dependent investment subsidy rate $-\tau^I(\pi)$ under the sustainability criterion.

Asset-pricing implications. Figure 4 presents the model's asset-pricing implications. Panel A shows that Tobin's q initially declines with disaster belief π, consistent with the standard long-run-risk model's prediction, but reverses once the sustainability constraint binds for $\pi > 0.15$. Investment subsidies lift asset values despite worsening beliefs, producing higher Tobin's q in the high-risk state. This mechanism rationalizes the observed increases in equity valuations and Tobin's q following the adoption of sustainability taxes in countries such as Greece and Spain. Panels B and C show that the risk-free rate and equity premium vary with belief π in subtler and more nuanced ways. The risk-free rate and equity premium are asset *return* measures that involve ratio calculations. As investors have to forecast whether the sustainability constraint will bind or not upon a sudden arrival of a disaster shock, these ratios can be discontinuous as the Lagrange multipliers can jump from zero to a positive value upon the arrival of a disaster.

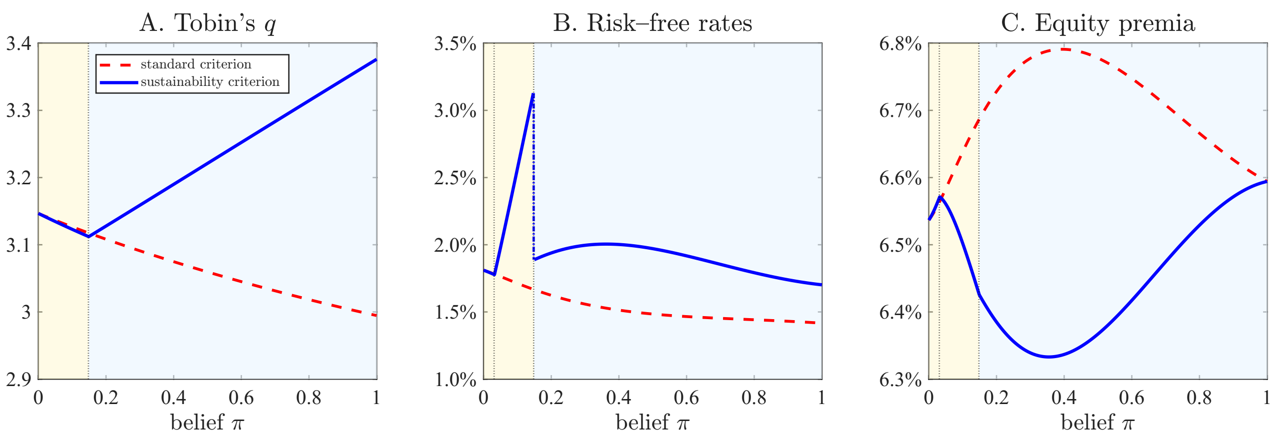

Figure 4: Implications of the sustainability criterion on Tobin's q, the risk-free rate $r(\pi)$, and equity premium $\mathrm{rp}(\pi)$.

Next we compare the effects of imposing the sustainability constraint with the effects of lowering the social discount rate $\hat{\rho} < \rho$ proposed by Stern (2006) and analyzed in Section 5 .

6.4 Stern's Proposal of using a Low Ethical Discount-Rate

Figure 5 plots the optimal consumption tax rates and investment subsidies when the government adopts a social rate of time preference $\hat{\rho}$ that is lower than the representative household's time rate of preference ρ. We consider three levels of $\hat{\rho}$ for our analysis: 0.04, 0.03, and 0.02, all of which are lower than the representative household's time rate of preference, which is $\rho = 0.05$ used in our calibration.

Setting $\hat{\rho} = 0.02$ requires a 70% consumption tax to induce private agents to behave as if they discounted at that rate, while $\hat{\rho} = 0.04$ still demands over a 10% consumption tax —a level far exceeding that required under the sustainability-constraint policy, which reacts endogenously to disaster risk and typically calls for a lower tax which is only a few percent of consumption.

Importantly we note that these tax rates based on Stern's proposal of adopting a lower ethical discount rate approach hardly vary with disaster risk π—and if anything are actually decreasing with π. Thus, while lowering the ethical discount rate can in principle promote intergenerational fairness, it does so in a risk-insensitive and fiscally much more expensive way for the current generation. This is in sharp contrast to the risk-contingent sustainability policy that we promote and analyzed earlier. The sustainability policy achieves intergenerational equity with lower consumption taxes but with rates that increase as disasters become more likely.

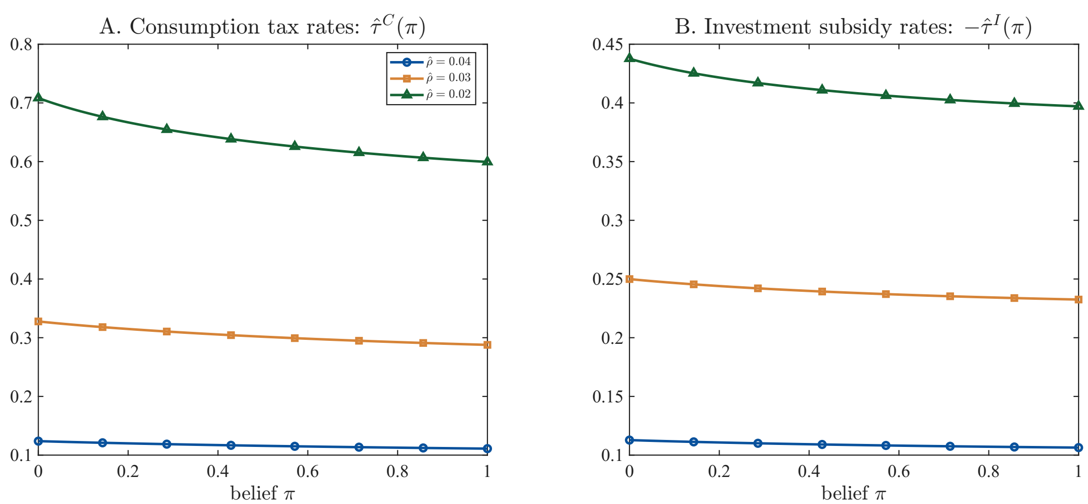

Figure 5: Optimal consumption taxes and investment subsidies under Stern's proposal of using a social discount rate $\hat{\rho}$ that is lower than the household's discount rate ρ. Optimal tax and subsidy policies are mostly flat with respect to diaster belief π in contrast to the highly belief-dependent policies under the sustainability constraint.

Figure 6 shows consumption and investment do not vary much with π but the choice of $\hat{\rho}$ has a significant "level" effect on consumption taxes and investment subsidies. Stern's proposal based on a social discount rate lower than the agent's ($\hat{\rho} < \rho$) achieves higher long-run capital accumulation but at the cost of very large and nearly state-independent fiscal distortions for the current generation. Finally, Figure 7 shows how asset prices vary with the chosen social discount rate $\hat{\rho}$. For all levels of $\hat{\rho}$, Tobin's q declines with π. This is in contrast to the prediction that Tobin's q increases with π under the sustainability criterion. This is because the sustainability criterion treats high disaster-risk states very different from low disaster-risk states.

The sustainability criterion leads to economic outcomes that are risk-sensitive compared to the Stern's proposal. The Stern's proposal based on a low social discount rate therefore performs poorly on both normative and quantitative grounds. The choice of $\hat{\rho}$ is inherently arbitrary, whereas the sustainability criterion provides a transparent welfare target with a clear mandate that the future generations are no worse off (in expectation) than the current generation. The implied taxes under Stern's proposal are unrealistically high, far exceeding observed consumption-tax levels. The required wedges in Stern's proposal are highly sensitive to the selected social discount rate but nearly insensitive to disaster risks. Hence, while lowering the ethical discount rate can conceptually capture concerns for future generations, the sustainability-criterion framework delivers the same goal with smaller fiscal distortions, stronger responsiveness to climate risk, and more plausible empirical relevance.

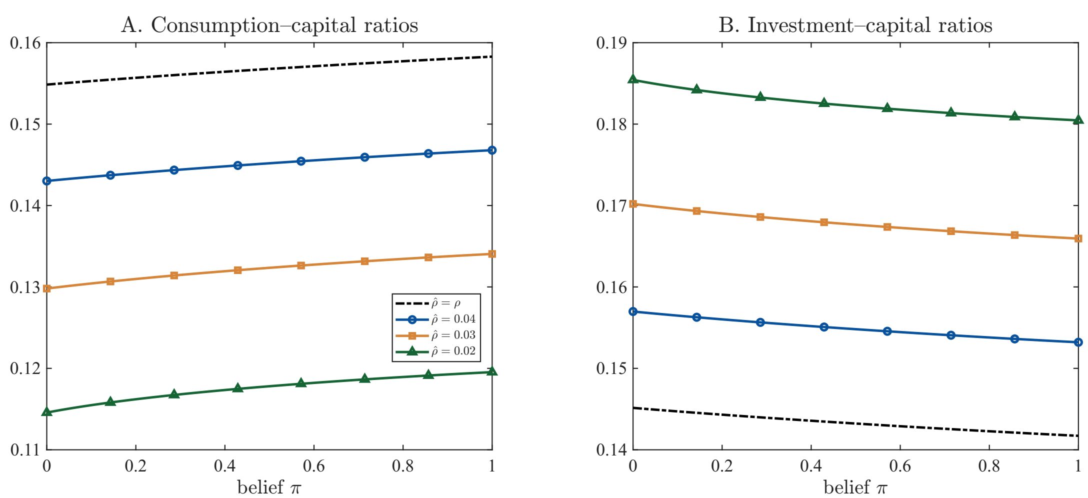

Figure 6: Optimal consumption and investment policies under Stern's proposal of using a social discount rate $\hat{\rho}$ that is lower than the household's discount rate ρ.

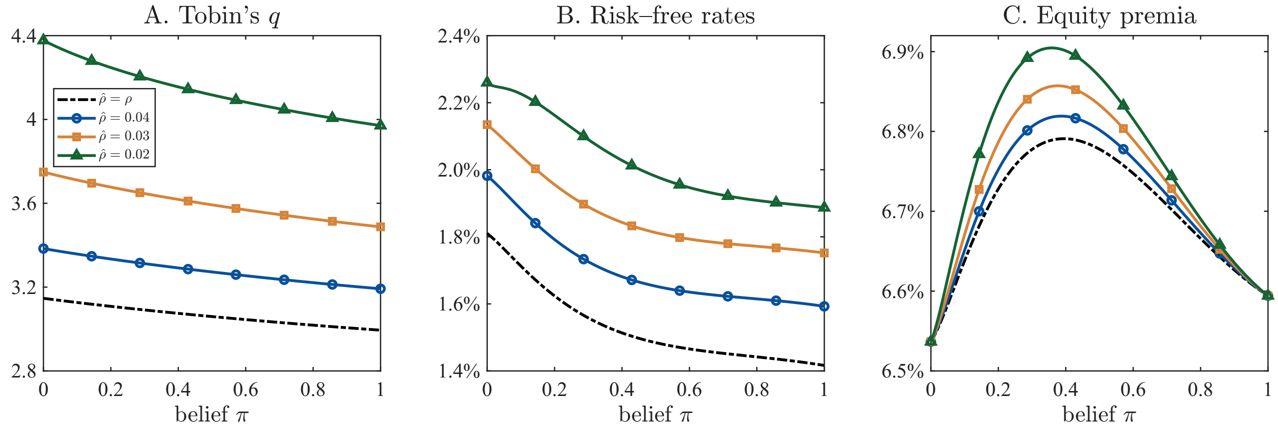

Figure 7: Implications of Stern's proposal of using a social discount rate $\hat{\rho} < \rho$ on Tobin's q, the risk-free rate $r(\pi)$, and the equity premium $\mathrm{rp}(\pi)$.

7 Conclusion

Climate disasters threaten intergenerational equity by exposing capital stocks to greater downside risk for future generations. We develop a stochastic general-equilibrium framework in which a social planner learns about disaster risk and enforces a sustainability criterion requiring that expected social welfare be non-decreasing over time. The planner's allocation can be decentralized through state-contingent fiscal instruments: a consumption tax that rises with perceived disaster risk and finances investment subsidies for climate-resilient capital. This fiscal rule formalizes how governments can operationalize the principle underlying the UN Sustainable Development Goals—shifting resources from current consumption to safeguard future generations' welfare. Quantitatively, the policy achieves intergenerational equity with modest distortions and higher asset valuations despite adverse climate news. Compared with Stern's proposal of the government adopting a social discount rate (lower than the private agents'), the sustainability-criterion policy is more risk-responsive, fiscally efficient, and more acceptable to current generations. Recent initiatives such as Greece's climate-resilience levy and the premium-flyers solidarity tax exemplify how governments act as planners enforcing sustainability on behalf of households, translating normative goals into actionable fiscal design.

Several extensions merit further research. First, our framework can be adapted to incorporate endogenous mitigation and adaptation investment, allowing the government to allocate resources between reducing disaster probabilities and cushioning their impacts. Second, explicitly modeling heterogeneous capital stocks could illuminate how sustainability taxes and subsidies change the relative prices of manufactured versus natural capital. Third, introducing international linkages—through cross-border spillovers of disaster risk, capital flows, or climate-resilience financing—would enable the analysis of coordinated fiscal policies and global sustainability transfers. Exploring these extensions would deepen our understanding of how fiscal design can balance efficiency, equity, and resilience in a warming world.

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Appendices

A Planner's Solution in Section 3

A.1 The Sustainability Constraint

Recall that, for a given consumption process C, the utility process V is defined by (4). Note that the process

$$\int_0^t f(C_s, V_s)\, ds + V_t = \mathbb{E}_t \left[\int_0^\infty f(C_s, V_s)\, ds \right] \tag{A.1}$$

is a martingale and hence has zero drift. Consequently, $f(C_{t-}, V_{t-})dt + \mathbb{E}_{t-}[dV_t] = 0$, so that $\mathbb{E}_{t-}[dV_t]/dt = -f(C_{t-}, V_{t-})$.

Substituting (5) into the sustainability constraint $f(C_{t-}, V_{t-}) \le 0$ given by (7), we obtain

$$\begin{cases} C_{t-}^{1-\psi^{-1}} - ((1-\gamma)V_{t-})^{\omega} \le 0\,, & \text{if } \psi > 1\,, \\ C_{t-}^{1-\psi^{-1}} - ((1-\gamma)V_{t-})^{\omega} \ge 0\,, & \text{if } \psi < 1\,, \end{cases}$$

which simplifies to $C_{t-} \le ((1-\gamma)V_{t-})^{1/(1-\gamma)}$. This establishes (8). When the EIS ψ tends to one, the aggregator becomes $f(C, V) = \rho(1-\gamma)V \left[\ln C - \frac{1}{1-\gamma} \ln((1-\gamma)V) \right]$, and the sustainability constraint also simplifies to $U(C_{t-}) \le V_{t-}$.

A.2 Derivation of the HJB Equation (16)

Let $V_t = V(\mathbf{K}_t, \pi_t)$ denote the *optimal* value function associated with (4)–(5). By Itô's lemma,

$$\begin{aligned} dV_t \ &= \Big[\Phi(\mathbf{I}_{t-}, \mathbf{K}_{t-}) V_{\mathbf{K}}(\mathbf{K}_{t-}, \pi_{t-}) + \mu_\pi(\pi_{t-}) V_\pi(\mathbf{K}_{t-}, \pi_{t-}) + \frac{1}{2}\sigma^2 \mathbf{K}_{t-}^2 V_{\mathbf{KK}}(\mathbf{K}_{t-}, \pi_{t-}) \Big] dt \\ &\quad + \sigma \mathbf{K}_{t-} V_{\mathbf{K}}(\mathbf{K}_{t-}, \pi_{t-}) d\mathcal{B}_t + \left[V\left(\mathbf{K}_t^{\mathcal{J}}, \pi_t^{\mathcal{J}}\right) - V(\mathbf{K}_{t-}, \pi_{t-}) \right] d\mathcal{J}_t\,, \end{aligned} \tag{A.2}$$

where $\mathbf{K}^{\mathcal{J}}$ and $\pi^{\mathcal{J}}$ are defined by (17) and (18), respectively. For the optimal value function, the drift of (A.1) is zero at the maximum:

$$0 = \max_{\mathbf{C}_{t-}, \mathbf{I}_{t-}} f(\mathbf{C}_{t-}, V_{t-})dt + \mathbb{E}_{t-}[dV_t]\,. \tag{A.3}$$

Taking conditional expectations in (A.2), and substituting $\mathbb{E}_{t-}[dV_t]$ into (A.3), we obtain (16).

A.3 The Economy without the Sustainability Constraint

In this section, we study the planner's problem without the sustainability constraint (7). We use the superscript u to denote the solution in this *unconstrained* economy.

The planner chooses aggregate consumption $\mathbf{C}$ and investment $\mathbf{I}$ to maximize the representative household's utility in (4)–(5), subject to the representative firm's production/capital-accumulation technology and the aggregate resource constraint: $A\mathbf{K} = \mathbf{C}^u + \mathbf{I}^u$.

Let $V^u(\mathbf{K},\pi)$ denote the value function, which satisfies (16). Because of the resource constraint, it suffices to take $\mathbf{C}^u$ as the only control variable. The FOC for consumption is

$$f_{\mathbf{C}}(\mathbf{C}^u, V^u(\mathbf{K},\pi)) = \Phi_{\mathbf{I}}(\mathbf{I}^u,\mathbf{K})V^u_{\mathbf{K}}(\mathbf{K},\pi)\,. \tag{A.4}$$

Exploiting the homogeneity of the model, we conjecture that the value function takes the form

$$V^u(\mathbf{K},\pi) = \frac{1}{1-\gamma}\left(b^u(\pi)\mathbf{K}\right)^{1-\gamma}\,, \tag{A.5}$$

where $b^u(\pi)$ is to be determined as part of the solution. Substituting (A.5) into (16) yields the following ODE:

$$\begin{aligned} 0 \;=\; & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{\mathbf{c}^u(\pi)}{b^u(\pi)}\right)^{1-\psi^{-1}} - 1\right] + \phi\left(\mathbf{i}^u(\pi)\right) + \mu_\pi(\pi)\frac{\left(b^u\right)'(\pi)}{b^u(\pi)} - \frac{\gamma\sigma^2}{2} \\ & + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b^u\left(\pi^{\mathcal{J}}\right)}{b^u(\pi)}\right)^{1-\gamma}\mathbb{E}\left[Z^{1-\gamma}\right] - 1\right]\,, \end{aligned} \tag{A.6}$$

where $\mathbf{c}^u = \mathbf{C}^u/\mathbf{K}$ and $\mathbf{i}^u = A - \mathbf{c}^u$ are scaled consumption and investment, respectively. Finally, simplifying the FOC (A.4) yields

$$\mathbf{c}^u(\pi) = \left(\frac{\rho}{\phi'\left(\mathbf{i}^u(\pi)\right)}\right)^{\psi} b^u(\pi)^{1-\psi}\,. \tag{A.7}$$

B Market Equilibrium Solution in Section 4

B.1 Firm's Optimization

Applying Itô's Lemma, we obtain the following dynamics for $Q_t = Q(K_t,\pi_t)$:

$$\begin{aligned} dQ_t = & \left(\Phi(I_{t-},K_{t-})Q_K + \frac{1}{2}\sigma^2 K_{t-}^2 Q_{KK} + \mu_\pi(\pi_{t-})Q_\pi\right)dt + \sigma K_{t-}Q_K d\mathcal{B}_t \\ & + \left(Q\left(K_t^{\mathcal{J}},\pi_t^{\mathcal{J}}\right) - Q(K_{t-},\pi_{t-})\right)d\mathcal{J}_t\,. \end{aligned} \tag{B.1}$$

From (29), under the optimal investment policy, $Q(K_t,\pi_t) = \mathbb{E}_t\left[\int_t^\infty \frac{\mathbb{M}_s}{\mathbb{M}_t}D_s ds\right]$. Hence, $\mathbb{M}_t Q_t + \int_0^t \mathbb{M}_s D_s ds = \mathbb{E}_t\left[\int_0^\infty \mathbb{M}_s D_s ds\right]$ forms a martingale at the optimum and therefore has zero drift (see Back (2017), Chapter 14.4). Applying Itô's lemma and using (28) and (B.1), we obtain the following HJB equation:

$$\begin{aligned} 0 \;=\; & \max_I\, D(I,K) + \Phi(I,K)Q_K(K,\pi) + \frac{1}{2}\sigma^2K^2Q_{KK}(K,\pi) + \mu_\pi(\pi)Q_\pi(K,\pi) \\ & + \left[-r(\pi) - \lambda(\pi)\left(\mathbb{E}[\eta(\pi;Z)] - 1\right)\right]Q(K,\pi) - \gamma\sigma^2 K Q_K(K,\pi) \\ & + \lambda(\pi)\mathbb{E}\left[\eta(\pi;Z)Q(K^{\mathcal{J}},\pi^{\mathcal{J}}) - Q(K,\pi)\right]\,, \end{aligned} \tag{B.2}$$

where $D(I,K) = AK - (1+\tau^I(\pi))I$. The FOC for investment implied by (B.2) is

$$1 + \tau^I(\pi) = \Phi_I(I,K)Q_K(K,\pi)\,. \tag{B.3}$$

Using the homogeneity property $Q(K,\pi) = q(\pi)K$ to simplify (B.2) and (B.3) yields (31) and (32), respectively.

Now we turn to the aggregate level and derive the dynamics of the cum-dividend return. Since $\mathbf{Q}(\mathbf{K},\pi) = \mathbf{q}(\pi)\mathbf{K}$, we have $\mathbf{Q}_{\mathbf{K}}(\mathbf{K},\pi) = \mathbf{q}(\pi)$, $\mathbf{Q}_{\mathbf{KK}}(\mathbf{K},\pi) = 0$, and $\mathbf{Q}_\pi(\mathbf{K},\pi) = \mathbf{q}'(\pi)\mathbf{K}$. The dynamics of $\mathbf{Q}_t = \mathbf{Q}(\mathbf{K}_t,\pi_t)$ implied by (B.1) therefore simplify to

$$\frac{d\mathbf{Q}_t}{\mathbf{Q}_{t-}} = \left(\phi(\mathbf{i}(\pi_{t-})) + \mu_\pi(\pi_{t-})\frac{\mathbf{q}'(\pi_{t-})}{\mathbf{q}(\pi_{t-})}\right)dt + \sigma d\mathcal{B}_t + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)d\mathcal{J}_t\,. \tag{B.4}$$

Combining $\mathbf{D}_{t-}dt/\mathbf{Q}_{t-} = [A - (1+\tau^I(\pi_{t-}))\mathbf{i}(\pi_{t-})]/\mathbf{q}(\pi_{t-})$ with (B.4) yields (34)–(35).

B.2 Household's Optimization

Recall the belief dynamics (11) and the wealth dynamics (36). Following the same steps as in Appendix A.2, we can show that the representative household's value function $J(W,\pi)$ satisfies the HJB equation (37):

$$0 = \max_{C,\Gamma}\ f(C, J(W,\pi)) + \mathcal{D}^{(C,\Gamma)}J(W,\pi)\,, \tag{B.5}$$

where

$$\begin{aligned}\mathcal{D}^{(C,\Gamma)}J(W,\pi) \ &= \ \left[r(\pi)W + (\mu_{\mathbf{Q}}(\pi) - r(\pi))\,\Gamma - \left(1+\tau^C(\pi)\right)C\right]J_W(W,\pi)\\ &\quad + \mu_\pi(\pi)J_\pi(W,\pi) + \frac{1}{2}\sigma^2\Gamma^2 J_{WW}(W,\pi) + \lambda(\pi)\mathbb{E}\left[J\left(W^{\mathcal{J}},\pi^{\mathcal{J}}\right) - J(W,\pi)\right]\,.\end{aligned}$$

The FOC for consumption is $f_C(C,J) = (1+\tau^C(\pi))J_W(W,\pi)$ as in (39). The FOC for asset allocation is

$$\begin{aligned}\sigma^2\Gamma J_{WW}(W,\pi) \ &= \ -\left(\mu_{\mathbf{Q}}(\pi) - r(\pi)\right)J_W(W,\pi)\\ &\quad + \lambda(\pi)\mathbb{E}\left[\left(1 - \frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\right)J_W\left(W^{\mathcal{J}},\pi^{\mathcal{J}}\right)\right]\,.\end{aligned} \tag{B.6}$$

Substituting (33) into (B.6) yields (41).

B.3 Market Equilibrium

Equilibrium SDF. Following Duffie and Epstein (1992a), we show that, under mild regularity conditions, the equilibrium SDF in the decentralized market is

$$\mathbb{M}_t = \exp\left[\int_0^t f_J\left(\mathbf{C}_s, J_s\right)ds\right]\frac{f_{\mathbf{C}}\left(\mathbf{C}_t, J_t\right)}{1+\tau_t^C}\,, \tag{B.7}$$

with dynamics given by (28).

To simplify notation, let $\mathcal{D}$ denote the operator $\mathcal{D}^{(\mathbf{C},\Gamma)}$ evaluated at the optimal policies $\mathbf{C}$ and Γ in (B.5). The HJB equation (B.5) implies that

$$f_J(\mathbf{C}, J)J_W(W,\pi) = -\frac{\partial}{\partial W}\mathcal{D}J(W,\pi) = -\mathcal{D}J_W(W,\pi) - r(\pi)J_W(W,\pi)\,. \tag{B.8}$$

Define $\Lambda_t = \exp\left[\int_0^t f_J\left(\mathbf{C}_s, J_s\right)ds\right]$, so that $d\Lambda_t = \Lambda_t f_J(\mathbf{C}_t, J_t)dt$. By the consumption FOC (39), $\mathbb{M}_t$ in (B.7) can be written as $\mathbb{M}_t = \Lambda_t J_W(W_t, \pi_t)$. Itô's lemma implies

$$d\mathbb{M}_t = \Lambda_{t-}\,dJ_W(W_t,\pi_t) + J_W(W_{t-},\pi_{t-})\,d\Lambda_t\,, \tag{B.9}$$

and

$$\begin{aligned} dJ_W(W_t,\pi_t) \;&= \Big\{\mathcal{D}J_W(W_{t-},\pi_{t-}) - \lambda(\pi_{t-})\mathbb{E}_{t-}\left[J_W\left(W_t^{\mathcal{J}},\pi_t^{\mathcal{J}}\right) - J_W(W_{t-},\pi_{t-})\right]\Big\}dt \\ &\quad + \sigma\Gamma_{t-}J_{WW}(W_{t-},\pi_{t-})d\mathcal{B}_t + \left[J_W\left(W_t^{\mathcal{J}},\pi_t^{\mathcal{J}}\right) - J_W(W_{t-},\pi_{t-})\right]d\mathcal{J}_t\,. \end{aligned} \tag{B.10}$$

Using (B.8), we can rewrite (B.10) as

$$\begin{aligned} \frac{dJ_W(W_t,\pi_t)}{J_W(W_{t-},\pi_{t-})} \;&= -\left[f_J(\mathbf{C}_{t-},J_{t-}) + r_{t-} + \lambda(\pi_{t-})\mathbb{E}_{t-}\left(\eta_{t-}-1\right)\right]dt \\ &\quad + \sigma\Gamma_{t-}\frac{J_{WW}(W_{t-},\pi_{t-})}{J_W(W_{t-},\pi_{t-})}d\mathcal{B}_t + (\eta_{t-}-1)d\mathcal{J}_t\,, \end{aligned}$$

where

$$\eta_{t-} = \frac{J_W\left(W_t^{\mathcal{J}},\pi_t^{\mathcal{J}}\right)}{J_W(W_{t-},\pi_{t-})}\,. \tag{B.11}$$

Therefore, (B.9) can be rewritten as

$$\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} = \mu_{\mathbb{M},t-}dt + \sigma_{\mathbb{M},t-}d\mathcal{B}_t + (\eta_{t-}-1)\,d\mathcal{J}_t\,, \tag{B.12}$$

where

$$\mu_{\mathbb{M},t-} = -r_{t-} - \lambda(\pi_{t-})\mathbb{E}_{t-}\left(\eta_{t-}-1\right)\,,$$

and

$$\sigma_{\mathbb{M},t-} = \sigma\Gamma_{t-}\frac{J_{WW}(W_{t-},\pi_{t-})}{J_W(W_{t-},\pi_{t-})} = -\frac{\mu_{\mathbf{Q}}(\pi_{t-}) - r_{t-}}{\sigma} - \frac{\lambda(\pi_{t-})}{\sigma}\mathbb{E}_{t-}\left[\eta_{t-}\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)\right]\,, \tag{B.13}$$

with the second equality in (B.13) following from the asset allocation FOC (B.6).

Next, consider a security with cum-dividend return given by (34). By Itô's lemma,

$$\begin{aligned} d(\mathbb{M}_t\mathbf{Q}_t) \;&= -\left(\mathbb{M}_{t-}\mathbf{D}_{t-} + \lambda(\pi_{t-})\mathbb{E}_{t-}\left[\mathbb{M}_t^{\mathcal{J}}\mathbf{Q}_t^{\mathcal{J}} - \mathbb{M}_{t-}\mathbf{Q}_{t-}\right]\right)dt \\ &\quad + \left(\sigma_{\mathbb{M},t-} + \sigma\right)\mathbb{M}_{t-}\mathbf{Q}_{t-}d\mathcal{B}_t + \left(\mathbb{M}_t^{\mathcal{J}}\mathbf{Q}_t^{\mathcal{J}} - \mathbb{M}_{t-}\mathbf{Q}_{t-}\right)d\mathcal{J}_t\,, \end{aligned} \tag{B.14}$$

where $\mathbb{M}_t^{\mathcal{J}} = \eta_{t-}\mathbb{M}_{t-}$. Integrating (B.14) over $[t, T]$ gives, for any $T > t$,

$$\begin{aligned}\mathbb{M}_t\mathbf{Q}_t &= \mathbb{M}_T\mathbf{Q}_T + \int_t^T \mathbb{M}_{s-}\mathbf{D}_{s-}ds - \int_t^T (\sigma_{\mathbb{M},s-} + \sigma)\,\mathbb{M}_{s-}\mathbf{Q}_{s-}d\mathcal{B}_s \\ &\quad - \left(\sum_{t<s_i\leq T}\left(\mathbb{M}_{s_i}^{\mathcal{J}}\mathbf{Q}_{s_i}^{\mathcal{J}} - \mathbb{M}_{s_i-}\mathbf{Q}_{s_i-}\right) - \int_t^T \lambda_{s-}\mathbb{E}_{s-}\left[\mathbb{M}_s^{\mathcal{J}}\mathbf{Q}_s^{\mathcal{J}} - \mathbb{M}_{s-}\mathbf{Q}_{s-}\right]ds\right),\end{aligned} \tag{B.15}$$

where s_i denotes the time of the ith jump. Under standard integrability conditions, taking conditional expectation $\mathbb{E}_t[\,\cdot\,]$ of both sides of (B.15) and proceeding as in Duffie, Pan, and Singleton (2000, Lemma 1) yields

$$\mathbb{M}_t\mathbf{Q}_t = \mathbb{E}_t[\mathbb{M}_T\mathbf{Q}_T] + \mathbb{E}_t\left[\int_t^T \mathbb{M}_s\mathbf{D}_s ds\right]. \tag{B.16}$$

Assume the transversality condition $\lim_{T\to\infty}\mathbb{E}_t[\mathbb{M}_T\mathbf{Q}_T] = 0$. Letting $T \to \infty$ in (B.16) yields

$$\mathbf{Q}_t = \mathbb{E}_t\left[\int_t^\infty \frac{\mathbb{M}_s}{\mathbb{M}_t}\mathbf{D}_s ds\right].$$

This confirms that $\mathbb{M}_t$ defined by (B.7) is indeed the SDF.

We now determine the coefficients in the equilibrium SDF dynamics (B.12). In equilibrium, the household invests all wealth in the stock market, so that $\Gamma = W = \mathbf{Q}$. Substituting (33) into (B.11) yields the expression for $\eta_{t-} = \eta(\pi_{t-}, Z)$ given in (42). Next, simplifying the FOCs (40) and (41) delivers (43) and (44). Substituting (44) into (B.13) implies $\sigma_{\mathbb{M},t-} = -\gamma\sigma$. Therefore, by (B.12), the equilibrium SDF follows the dynamics in (28).

Asset-pricing implications. We next pin down the equilibrium interest rate r and the equity risk premium rp stated in Proposition 2.

Using the equilibrium conditions together with (33) and (43), we can simplify the household's HJB equation (37) as follows:

$$\begin{aligned}0 &= \frac{1}{1-\psi^{-1}}\left(\left(1+\tau^C(\pi)\right)\frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} - \rho\right) + \left(\mu_{\mathbf{Q}}(\pi) - \left(1+\tau^C(\pi)\right)\frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)}\right) \\ &\quad + \mu_\pi(\pi)\frac{u'(\pi)}{u(\pi)} - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\pi)}{1-\gamma}\left[\mathbb{E}\left(\eta(\pi;Z)\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\right) - 1\right].\end{aligned} \tag{B.17}$$

Substituting (43) into (B.17) yields (45).

The goods-market clearing condition implies $\left(1+\tau^C(\pi)\right)\mathbf{c}(\pi) = A - \left(1+\tau^I(\pi)\right)\mathbf{i}(\pi)$. Hence (35) can be written as

$$\mu_{\mathbf{Q}}(\pi) = \left(1+\tau^C(\pi)\right)\frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} + \phi(\mathbf{i}(\pi)) + \mu_\pi(\pi)\frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}. \tag{B.18}$$

The first term on the right-hand side of (B.18) is the tax-inclusive consumption–wealth ratio (equivalently, the dividend yield) in equilibrium, while the last two terms capture the percentage drift in prices given in (B.4). Plugging (B.18) into (B.17) gives

$$\begin{aligned}\left(1+\tau^C(\pi)\right)\frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} &= \rho-\left(1-\psi^{-1}\right)\left[\phi(\mathbf{i}(\pi))+\mu_\pi(\pi)\left(\frac{u'(\pi)}{u(\pi)}+\frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right)-\frac{\gamma\sigma^2}{2}\right] \\ &\quad -\lambda(\pi)\frac{1-\psi^{-1}}{1-\gamma}\left[\mathbb{E}\left(\eta(\pi;Z)\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\right)-1\right].\end{aligned} \tag{B.19}$$

Equation (44) comes from the household's portfolio optimality condition, whereas (B.18) follows from the cum-dividend return dynamics. In equilibrium, the two coincide. Equating them and using (B.19) delivers the equilibrium risk-free rate $r(\pi)$ in (46).

To compute the risk premium, note from (34) that the expected return on equity, $r^e_{t-}=r^e(\pi_{t-})$, satisfies

$$r^e_{t-}dt=\mathbb{E}_{t-}\left[\frac{d\mathbf{Q}_t+\mathbf{D}_{t-}dt}{\mathbf{Q}_{t-}}\right]=\mu_{\mathbf{Q}}(\pi_{t-})dt+\lambda(\pi_{t-})\mathbb{E}_{t-}\left[\frac{\mathbf{Q}^{\mathcal{J}}_t}{\mathbf{Q}_{t-}}-1\right]dt\,.$$

Using (44), the equity risk premium $\mathrm{rp}(\pi)=r^e(\pi)-r(\pi)$ is then given by (47).

B.4 Sustainability Taxation and Subsidy Design

We now prove Proposition 3, which states that under the fiscal policies (52), the market solution replicates the planner's solution subject to the sustainability constraint. In particular: (i) the household's value function in the market economy coincides with the planner's value function, and (ii) the equilibrium allocation coincides with the planner's allocation.

To elaborate, for (i), to verify $J(W_t,\pi_t)=V(\mathbf{K}_t,\pi_t)$, note that the equilibrium relation $W_t=\mathbf{q}(\pi_t)\mathbf{K}_t$ implies it suffices to show that $u(\pi)\mathbf{q}(\pi)=b^*(\pi)$. To this end, we show that the ODE for $u(\pi)\mathbf{q}(\pi)$ coincides with the ODE (24) for $b^*(\pi)$, and that the boundary conditions at $\pi=0$ and $\pi=1$ are the same in the two economies. For (ii), the resource constraint (15) implies that once the consumption rules coincide, the investment rules coincide as well. We now provide the proof.

We start from the market economy. Using (43) and the FOC for aggregate investment, $1+\tau^I(\pi)=\phi'(\mathbf{i}(\pi))\mathbf{q}(\pi)$ implied by (32), we obtain

$$\mathbf{c}(\pi)=\left(\frac{\rho}{\phi'(\mathbf{i}(\pi))}\frac{1+\tau^I(\pi)}{1+\tau^C(\pi)}\right)^{\psi}\left[u(\pi)\mathbf{q}(\pi)\right]^{1-\psi}\,. \tag{B.20}$$

Substituting (B.18) into (B.17) and using (43) together with $\eta(\pi;Z)$ in (42) yields

$$\begin{aligned}0 &= \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{\mathbf{c}(\pi)}{u(\pi)\mathbf{q}(\pi)}\right)^{1-\psi^{-1}}-1\right]+\phi(\mathbf{i}(\pi))+\mu_\pi(\pi)\frac{(u(\pi)\mathbf{q}(\pi))'}{u(\pi)\mathbf{q}(\pi)}-\frac{\gamma\sigma^2}{2} \\ &\quad+\frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{u\left(\pi^{\mathcal{J}}\right)\mathbf{q}(\pi^{\mathcal{J}})}{u(\pi)\mathbf{q}(\pi)}\right)^{1-\gamma}\mathbb{E}\left[Z^{1-\gamma}\right]-1\right].\end{aligned} \tag{B.21}$$

Applying the same arguments to the boundaries $\pi=0$ and $\pi=1$ shows that $u(0)\mathbf{q}(0)=b^*(0)$

and $u(1)\mathbf{q}(1) = b^*(1)$.

Next, comparing (B.20) with $\mathbf{c}^*(\pi)$ in (23), and comparing (B.21) with the ODE (24) for $b^*(\pi)$, we conclude that $\mathbf{c}(\pi) = \mathbf{c}^*(\pi)$ and $u(\pi)\mathbf{q}(\pi) = b^*(\pi)$ hold simultaneously provided that

$$\frac{1+\tau^I(\pi)}{1+\tau^C(\pi)} = 1 - \alpha^*(\pi)\,,$$

which establishes (49).

C Ethical Discount Rate Policy in Section 5

Let $b^u(\pi;\hat{\rho})$, $\mathbf{c}^u(\pi;\hat{\rho})$, and $\mathbf{i}^u(\pi;\hat{\rho})$ be the solution to (A.6)–(A.7) with ρ replaced by $\hat{\rho}$, so that

$$\mathbf{c}^u(\pi;\hat{\rho}) = \left(\frac{\hat{\rho}}{\phi'\left(\mathbf{i}^u(\pi;\hat{\rho})\right)}\right)^{\psi} b^u(\pi;\hat{\rho})^{1-\psi}\,. \tag{C.1}$$

Imposing $\mathbf{c}(\pi) = \mathbf{c}^u(\pi;\hat{\rho})$ and $\mathbf{i}(\pi) = \mathbf{i}^u(\pi;\hat{\rho})$, and comparing (B.20) with (C.1), we obtain that the target allocation is replicated if and only if the tax rates $\hat{\tau}^C(\pi)$ and $\hat{\tau}^I(\pi)$ satisfy

$$\frac{1+\hat{\tau}^I(\pi)}{1+\hat{\tau}^C(\pi)} = \frac{\hat{\rho}}{\rho}\left(\frac{u(\pi)\mathbf{q}(\pi)}{b^u(\pi;\hat{\rho})}\right)^{1-\psi^{-1}}\,. \tag{C.2}$$

Comparing the ODE (B.21) for $u(\pi)\mathbf{q}(\pi)$ with the ODE (A.6) (under discount rate $\hat{\rho}$) for $b^u(\pi;\hat{\rho})$ shows that $u(\pi)\mathbf{q}(\pi) \neq b^u(\pi;\hat{\rho})$. Consider instead the recursion

$$\widehat{V}_t = \mathbb{E}_t\left[\int_t^{\infty} f\left(\widehat{\mathbf{C}}_s, \widehat{V}_s;\rho\right) ds\right]\,,$$

where allocations are targeted at $\widehat{\mathbf{C}} = \mathbf{c}^u(\pi;\hat{\rho})\mathbf{K}$ and $\widehat{\mathbf{I}} = \mathbf{i}^u(\pi;\hat{\rho})\mathbf{K}$, while the discount rate is ρ. The value function is homogeneous in $\mathbf{K}$:

$$\widehat{V}(\mathbf{K},\pi;\rho) = \frac{(\hat{b}(\pi;\rho)\mathbf{K})^{1-\gamma}}{1-\gamma}\,. \tag{C.3}$$

We show below that $u(\pi)\mathbf{q}(\pi) = \hat{b}(\pi;\rho)$, and hence $J(W,\pi) = \widehat{V}(\mathbf{K},\pi;\rho)$.

Using (A.1), we can show that $\widehat{V}(\mathbf{K},\pi;\rho)$ solves the PDE:

$$\begin{aligned} 0 \;=\; & f(\widehat{\mathbf{C}}, \widehat{V}(\mathbf{K},\pi;\rho)) + \Phi(\widehat{\mathbf{I}},\mathbf{K})\widehat{V}_{\mathbf{K}}(\mathbf{K},\pi;\rho) + \mu_\pi(\pi)\widehat{V}_\pi(\mathbf{K},\pi;\rho) + \frac{1}{2}\sigma^2\mathbf{K}^2\widehat{V}_{\mathbf{KK}}(\mathbf{K},\pi;\rho) \\ & + \lambda(\pi)\mathbb{E}\left[\widehat{V}\left(\mathbf{K}^{\mathcal{J}},\pi^{\mathcal{J}};\rho\right) - \widehat{V}(\mathbf{K},\pi;\rho)\right]\,. \end{aligned} \tag{C.4}$$

Substituting (C.3) into (C.4) yields the ODE for $\hat{b}(\pi;\rho)$:

$$\begin{aligned} 0 \ =& \ \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{\mathbf{c}^u(\pi;\hat{\rho})}{\hat{b}(\pi;\rho)}\right)^{1-\psi^{-1}}-1\right]+\phi\left(\mathbf{i}^u(\pi;\hat{\rho})\right)+\mu_\pi(\pi)\frac{\hat{b}'(\pi;\rho)}{\hat{b}(\pi;\rho)}-\frac{\gamma\sigma^2}{2} \\ &+\frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{\hat{b}\left(\pi^{\mathcal{J}};\rho\right)}{\hat{b}(\pi;\rho)}\right)^{1-\gamma}\mathbb{E}\left[Z^{1-\gamma}\right]-1\right]. \end{aligned} \tag{C.5}$$

The states $\pi=0$ and $\pi=1$ are absorbing, so the economy stays permanently in states G and B, respectively. In these cases, ODE (C.5) reduces to two algebraic equations, whose solutions $\hat{b}(0;\rho)$ and $\hat{b}(1;\rho)$ provide the boundary conditions for (C.5). Specifically, for $\pi\in\{0,1\}$,

$$\hat{b}(\pi;\rho)=\mathbf{c}^u(\pi;\hat{\rho})\left\{1-\frac{1-\psi^{-1}}{\rho}\left[\phi\left(\mathbf{i}^u(\pi;\hat{\rho})\right)-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\pi)}{1-\gamma}\big(\mathbb{E}[Z^{1-\gamma}]-1\big)\right]\right\}^{\frac{1}{\psi^{-1}-1}}. \tag{C.6}$$

Now suppose (C.2) holds, so that $\mathbf{c}(\pi)=\mathbf{c}^u(\pi;\hat{\rho})$ and $\mathbf{i}(\pi)=\mathbf{i}^u(\pi;\hat{\rho})$. Comparing ODE (B.21) for $u(\pi)\mathbf{q}(\pi)$ with ODE (C.5) for $\hat{b}(\pi;\rho)$, and noting that the same boundary conditions apply, we conclude that $u(\pi)\mathbf{q}(\pi)=\hat{b}(\pi;\rho)$.

Substituting $\hat{b}(\pi;\rho)=u(\pi)\mathbf{q}(\pi)$ into (C.2) delivers (57). Combining (57) with the balanced-budget condition (56) then yields the closed-form expressions for the tax rates in (54).

We summarize the steps for computing the tax rates $\hat{\tau}^C(\pi)$ and $\hat{\tau}^I(\pi)$. First, solve for $b^u(\pi;\hat{\rho})$, $\mathbf{c}^u(\pi;\hat{\rho})$, and $\mathbf{i}^u(\pi;\hat{\rho})$ from (A.6)–(A.7) with the discount rate set to $\hat{\rho}$. Second, solve for $\hat{b}(\pi;\rho)$ from (C.5)–(C.6). Finally, compute the tax rates using (54).

Supplementary Appendix for Online Publication Only

The following supplementary material is for Hong, Huang and Wang "Climate Disasters and Intergenerational Equity: A Fiscal Policy for Sustainable Development".

O.1 Numerical Implementation

In this section, we describe the numerical procedures used to solve the planner's problems without and with the sustainability constraint, corresponding to Appendix A.3 and Section 3, respectively.

O.1.1 The Economy without the Sustainability Constraint

The boundary conditions at $\pi = 0$ and $\pi = 1$. The states $\pi = 0$ and $\pi = 1$ are absorbing, meaning the economy remains permanently in states G and B, respectively. In these two absorbing states, the belief does not change and the equations (A.6) and (A.7) reduce to the following algebraic equations:

$$0 = \frac{\rho}{1-\psi^{-1}}\left[\frac{(A-\mathbf{i}^u)\phi'(\mathbf{i}^u)}{\rho}-1\right]+\phi(\mathbf{i}^u)-\frac{\gamma\sigma^2}{2}+\frac{\lambda}{1-\gamma}\left[\mathbb{E}\left[Z^{1-\gamma}\right]-1\right], \quad \text{(O.1)}$$

$$b^u = (A-\mathbf{i}^u)^{1/(1-\psi)}\left(\frac{\phi'(\mathbf{i}^u)}{\rho}\right)^{\psi/(1-\psi)}. \quad \text{(O.2)}$$

Given $\lambda \in \{\lambda^G, \lambda^B\}$, we first solve for $\mathbf{i}^u$ from equation (O.1) and then substitute it into (O.2) to obtain b^u. The solution with $\lambda = \lambda^G$ (i.e., $\pi = 0$) is denoted by $(b^u(0), \mathbf{i}^u(0))$, while the solution with $\lambda = \lambda^B$ (i.e., $\pi = 1$) is $(b^u(1), \mathbf{i}^u(1))$.

With $\phi(i) = i - \frac{1}{2}\theta i^2 - \delta$, equation (O.1) admits the solution

$$\mathbf{i}^u = \frac{1}{1+\psi^{-1}}\left[\left(A+\frac{\psi^{-1}}{\theta}\right)-\sqrt{\left(A+\frac{\psi^{-1}}{\theta}\right)^2-\frac{2}{\theta}\left(1+\psi^{-1}\right)\chi}\right],$$

where

$$\chi = A-\rho-\left(1-\psi^{-1}\right)\left(\delta+\frac{\gamma\sigma^2}{2}-\frac{\lambda}{1-\gamma}\left(\mathbb{E}[Z^{1-\gamma}]-1\right)\right), \quad \lambda \in \{\lambda^G, \lambda^B\}.$$

For the investment policies in both states G and B to be well defined, the following parameter restriction is required: $\left(A+\frac{\psi^{-1}}{\theta}\right)^2-\frac{2}{\theta}\left(1+\psi^{-1}\right)\chi > 0$. Moreover, investment is positive in both states if and only if $\chi > 0$ for $\lambda \in \{\lambda^G, \lambda^B\}$.

Numerical treatment of post-jump belief $\pi^{\mathcal{J}}$. Given a equidistant mesh $0 = \pi_0 < \pi_1 < \cdots < \pi_N = 1$ with step size $\Delta\pi = 1/N$, the post-jump belief $\pi_j^{\mathcal{J}} = \pi_j + \sigma_\pi(\pi_j)$ may not coincide with the mesh points. For each $j = 1, \cdots, N-1$, we define an integer $m(j)$ such that $\pi_{m(j)-1} < \pi_j^{\mathcal{J}} \leq \pi_{m(j)}$, i.e., $m(j) = \left\lceil \frac{\pi_j^{\mathcal{J}}}{\Delta\pi} \right\rceil$, where $\lceil \cdot \rceil$ denotes the ceiling function. To approximate $b^u\left(\pi^{\mathcal{J}}\right)$, we use piecewise linear interpolation:

$$b^u(\pi_j^{\mathcal{J}}) \approx (1 - w(j))b^u(\pi_{m(j)}) + w(j)b^u\left(\pi_{m(j)-1}\right) =: b_j^{\mathcal{J}}\,, \tag{O.3}$$

where $w(j) = m(j) - \frac{\pi_j^{\mathcal{J}}}{\Delta\pi}$, for $j = 1, 2 \cdots, N-1$.

Discretization. Let b_j and $\mathbf{i}_j$ denote $b^u(\pi_j)$ and $\mathbf{i}^u(\pi_j)$, respectively. We approximate $(b^u)'(\pi_j)$ using a backward difference scheme, and approximate $b^u\left(\pi_j^{\mathcal{J}}\right)$ by $b_j^{\mathcal{J}}$ given in (O.3). Under these discretizations, the system of nonlinear differential-algebraic equations (DAEs) (A.6)–(A.7) is approximated by:

$$\begin{aligned} 0 \;&= \frac{\rho}{1-\psi^{-1}}\left[\frac{(A-\mathbf{i}_j)\phi'(\mathbf{i}_j)}{\rho} - 1\right] + \phi(\mathbf{i}_j) + \frac{\mu_\pi(\pi_j)}{\Delta\pi}\left(1 - \frac{b_{j-1}}{b_j}\right) \\ &\quad - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\pi_j)}{1-\gamma}\left[\left(\frac{b_j^{\mathcal{J}}}{b_j}\right)^{1-\gamma}\mathbb{E}\left[Z^{1-\gamma}\right] - 1\right], \end{aligned} \tag{O.4}$$

$$0 = (A - \mathbf{i}_j)^{1/(1-\psi)}\left(\frac{\phi'(\mathbf{i}_j)}{\rho}\right)^{\psi/(1-\psi)} - b_j\,, \tag{O.5}$$

for $j = 1, 2, \cdots, N-1$.

Algorithm. We solve the system (A.6)–(A.7) using the following iterative scheme:

1. Solve equations (O.1) and (O.2) to obtain the boundary values, $(b_0^u, \mathbf{i}_0^u)$ and $(b_N^u, \mathbf{i}_N^u)$.
2. Assign initial guesses for b^u and $\mathbf{i}^u$.
3. Iteratively solve for b^u and $\mathbf{i}^u$ from (O.4)–(O.5).

O.1.2 The Economy with the Sustainability Constraint

The boundary conditions at $\pi = 0$ and $\pi = 1$. Similar to the unconstrained case, at the boundaries the ODE (24) and the associated optimality conditions reduce to the following algebraic system: for $\lambda \in \{\lambda^G, \lambda^B\}$,

$$0 \;=\; \frac{\rho}{1-\psi^{-1}}\left[\frac{(A-\mathbf{i}^*)\phi'(\mathbf{i}^*)}{(1-\alpha^*)\rho} - 1\right] + \phi(\mathbf{i}^*) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\left[\mathbb{E}\left[Z^{1-\gamma}\right] - 1\right], \tag{O.6}$$

$$b^* \;=\; (A - \mathbf{i}^*)^{1/(1-\psi)}\left(\frac{\phi'(\mathbf{i}^*)}{(1-\alpha^*)\rho}\right)^{\psi/(1-\psi)}, \tag{O.7}$$

$$0 \;=\; \alpha^* \cdot (\mathbf{c}^* - b^*)\,, \tag{O.8}$$

$$\mathbf{c}^* \;\leq\; b^*\,, \quad \alpha^* \geq 0\,. \tag{O.9}$$

Considering the following two mutually exclusive cases:

Case 1. If $\alpha^* = 0$, then (O.6) and (O.7) can be solved using the same approach as in the unconstrained case (O.1)–(O.2). After obtaining the solution b^*, $\mathbf{i}^*$, and $\mathbf{c}^* = A - \mathbf{i}^*$, we verify whether the constraint $\mathbf{c}^* \leq b^*$ holds. If it is violated, we proceed to the next case.

Case 2. If $\alpha^* \neq 0$, then the complementary slackness condition implies that the constraint is binding, i.e., $b^* = \mathbf{c}^* = A - \mathbf{i}^*$. Substituting this into the system simplifies (O.6) and (O.7) to:

$$\begin{aligned} 0 &= \phi\left(\mathbf{i}^*\right) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\left[\mathbb{E}\left[Z^{1-\gamma}\right] - 1\right], \\ \alpha^* &= 1 - \frac{1}{\rho}\phi'\left(\mathbf{i}^*\right)\left(A - \mathbf{i}^*\right). \end{aligned} \tag{O.10}$$

The above solution is admissible only if $\alpha^* > 0$.

With $\phi(i) = i - \frac{1}{2}\theta i^2 - \delta$, equation (O.10) admits a closed-form solution:

$$\mathbf{i}^* = \frac{1}{\theta} - \sqrt{\frac{1}{\theta^2} - \frac{2}{\theta}\left(\delta + \frac{\gamma\sigma^2}{2} - \frac{\lambda}{1-\gamma}\left(\mathbb{E}[Z^{1-\gamma}] - 1\right)\right)}.$$

For the solution to be well-defined, the following parameter restriction is required:

$$\frac{1}{\theta} - 2\left(\delta + \frac{\gamma\sigma^2}{2} - \frac{\lambda}{1-\gamma}\left(\mathbb{E}[Z^{1-\gamma}] - 1\right)\right) > 0.$$

Furthermore, ensuring positive investment requires that $\delta + \frac{\gamma\sigma^2}{2} - \frac{\lambda}{1-\gamma}\left(\mathbb{E}[Z^{1-\gamma}] - 1\right) > 0$.

Algorithm. To solve for $b^*(\pi)$, $\mathbf{i}^*(\pi)$, and $\mathbf{c}^*(\pi)$, we adopt the following iterative scheme.

1. Solve (O.6)–(O.9) to obtain the boundary values, $(b_0^*, \mathbf{i}_0^*, \mathbf{c}_0^*)$ and $(b_N^*, \mathbf{i}_N^*, \mathbf{c}_N^*)$.

2. Using the boundary values, initialize interior points for b^*, $\mathbf{i}^*$, and $\mathbf{c}^*$—for example, via linear interpolation. Set the iteration counter $k = 0$.

3. Check the complementary slackness condition. If the constraint is slack, i.e., $\mathbf{c}^{*(k)}(\pi) < b^{*(k)}(\pi)$, then set $\alpha^{*(k+1)}(\pi) = 0$; otherwise, update the multiplier via

$$\alpha^{*(k+1)}(\pi) = \max\left(1 - \frac{1}{\rho}\phi'(\mathbf{i}^{*(k)}(\pi))b^{*(k)}(\pi), 0\right).$$

4. Given $\alpha^{*(k+1)}(\pi)$, we update $b^{*(k+1)}(\pi)$, $\mathbf{i}^{*(k+1)}(\pi)$, and $\mathbf{c}^{*(k+1)}(\pi)$ by solving the system (23), (24), and $A = \mathbf{c}^* + \mathbf{i}^*$, subject to the constraint $\mathbf{c}^*(\pi) \leq b^*(\pi)$.

5. Repeat Steps 3–4 until convergence is achieved, i.e.,

$$\|b^{*(k+1)} - b^{*(k)}\| \leq \epsilon, \quad \|\mathbf{i}^{*(k+1)} - \mathbf{i}^{*(k)}\| \leq \epsilon, \quad \|\alpha^{*(k+1)} - \alpha^{*(k)}\| \leq \epsilon,$$

for some pre-specified tolerance $\epsilon > 0$.

O.2 Calibration

We calibrate the parameters using the model without the sustainability constraint described in Appendix A.3. The calibration employs the simulated method of moments (SMM), following McFadden (1989) and Duffie and Singleton (1993). Below, we provide a detailed description of the calibration procedure.

1. Provide a reasonable initial guess for the parameter vector to be calibrated.

2. Given the parameter vector, numerically solve the model (see Appendix O.1.1).

3. Simulate the model at time step Δt for T years.

   3.1. Simulate the belief dynamics:

   $$d\pi_t = \sigma_\pi(\pi_{t-})\left(d\mathcal{J}_t - \lambda(\pi_{t-})dt\right) .$$

   (i) Discretize $[0, T]$ on an equally spaced grid $0 = t_0 < t_1 < \cdots < t_n = T$, where $n = T/\Delta t$.
   (ii) On each interval $(t_{j-1}, t_j]$, a jump occurs with probability approximately equal to $\lambda(\pi_{t_{j-1}})\Delta t$. Draw U from the uniform distribution on $[0, 1]$. A jump occurs if $U \leq \lambda(\pi_{t_{j-1}})\Delta t$. If a jump occurs, the belief updates instantaneously from $\pi_{t_{j-1}}$ to $\pi_{t_j}^{\mathcal{J}} = \pi_{t_{j-1}} + \sigma_\pi(\pi_{t_{j-1}})$.
   (iii) If no jump occurs on $(t_{j-1}, t_j]$, then π_t evolves according to $d\pi_t = \mu_\pi(\pi_t)dt$ and admits a closed-form solution. In this case,

   $$\pi_{t_j} = \frac{\pi_{t_{j-1}} e^{-(\lambda^B - \lambda^G)\Delta t}}{1 + \pi_{t_{j-1}}\left(e^{-(\lambda^B - \lambda^G)\Delta t} - 1\right)} .$$

   3.2. Plugging the belief path into the model solution obtained in Step 2, we can compute all simulated moments.

4. Repeat Step 3 S times and average across simulations to compute the model-implied moments.

5. Update the parameter vector by minimizing a distance between the model-implied moments and the corresponding empirical moments.

6. Repeat until the model-implied moments are as close as possible to their empirical counterparts.