

Debt, Taxes, and Liquidity*

Patrick Bolton[†] Hui Chen[‡] Neng Wang[§]

May 26, 2014

Abstract

We analyze a model of optimal capital structure and liquidity choice based on a dynamic tradeoff theory for financially constrained firms. In addition to the classical tradeoff between the expected tax advantages of debt and bankruptcy costs, we introduce a cost of external financing for the firm, which generates a precautionary demand for liquidity and an optimal liquidity management policy for the firm. An important new cost of debt financing in this context is an *endogenous debt servicing cost*: debt payments drain the firm's valuable liquidity reserves and thus impose higher expected external financing costs on the firm. The precautionary demand for liquidity also means that realized earnings are separated in time from payouts to shareholders, implying that the classical Miller-formula for the net tax benefits of debt no longer holds. Our model offers a novel perspective for the "*debt conservatism puzzle*" by showing that financially constrained firms choose to limit debt usages in order to preserve their liquidity. In some cases, they may not even exhaust their risk-free debt capacity.

*We thank Phil Dybvig, Wei Jiang, Hong Liu, Gustavo Manso, Jonathan Parker, Steve Ross, and seminar participants at MIT Sloan, Stanford, UC Berkeley Haas, Washington University, University of Wisconsin-Madison, TCFE 2013, and WFA 2013 for helpful comments.

[†]Columbia University, NBER and CEPR. Email: pb2208@columbia.edu. Tel. 212-854-9245.

[‡]MIT Sloan School of Management and NBER. Email: huichen@mit.edu. Tel. 617-324-3896.

[§]Columbia Business School and NBER. Email: neng.wang@columbia.edu. Tel. 212-854-3869.

1 Introduction

We develop a dynamic tradeoff theory for financially constrained firms by integrating classical tax versus bankruptcy cost considerations into a dynamic framework in which firms face external financing costs. As in [Bolton, Chen, and Wang \(2011, 2013\)](#), these financing costs generate a precautionary demand for holding liquid assets and retaining earnings.¹ Financially constrained firms incur an additional *endogenous* debt servicing cost arising from the cash drain associated with interest payments. Given that firms face this *endogenous debt servicing cost*, our model predicts lower optimal debt levels than those obtained for unconstrained firms with no precautionary cash buffers. We thus provide a novel perspective on the “*debt conservatism puzzle*” documented in the empirical capital structure literature (see [Graham \(2000, 2003\)](#)).

The precautionary savings motive for financially constrained firms introduces another novel dimension to the standard tradeoff theory: *personal tax capitalization* and the changes this capitalization brings to the net tax benefit of debt when the firm chooses to retain its *net earnings* (after interest and corporate tax payments) rather than pay them out to shareholders. As [Harris and Kemsley \(1999\)](#), [Collins and Kemsley \(2000\)](#), and [Frank, Singh, and Wang \(2010\)](#) have pointed out, when firms choose to build up *corporate savings*, personal taxes on future expected payouts must be capitalized, and this tax capitalization changes both the market value of equity and the net tax benefit calculation for debt. In our model, the standard *Miller formula* for the net tax benefit of debt only holds when the firm is at the *endogenous* payout boundary. When the firm is away from this payout boundary, and therefore strictly prefers to retain earnings, the net tax benefits of debt are lower than the ones implied by the Miller formula. As we show, the tax benefits can even become substantially negative when the firm is at risk of running out of cash. Importantly, this is not just a conceptual observation, it is also quantitatively important as the firm is almost always in the liquidity-hoarding region.

Our dynamic model of financially constrained firms also have new predictions on

¹Corporate cash holdings of U.S. publicly traded non-financial corporations have been steadily increasing over the past twenty years and represent a substantial fraction of corporate assets, as [Bates, Kahle, and Stulz \(2009\)](#) have shown.

the effects of changes in depreciation tax allowances on capital structure and liquidity management. Unlike the standard result in the static tradeoff theory by [DeAngelo and Masulis \(1980\)](#) that depreciation tax shields are a substitute for debt tax shields, and therefore that an increase in depreciation tax allowances lowers leverage, for a financially constrained firm depreciation tax allowances are a *complement* to debt tax shields. That is, an increase in depreciation tax allowances induces the financially constrained firm to issue more debt and hold more cash. The reason is that the firm's debt servicing costs are reduced when the firm can retain a higher fraction of its EBITDA and therefore the firm responds by taking on more debt and holding more cash.

Another important change introduced by external financing costs and precautionary savings is that the conventional assumption that *cash is negative debt* is no longer valid, as [Acharya, Almeida, and Campello \(2007\)](#) have emphasized.² In our model, drawing down debt by depleting the firm's cash stock involves an opportunity cost for the financially constrained firm, which is not accounted for when cash is treated as negative debt. As a result, standard net debt calculations tend to underestimate the value of cash. The flaw in treating cash as negative debt becomes apparent in situations where the firm chooses not even to exhaust its risk-free debt capacity given the endogenous debt servicing costs involved and the scarcity of internal funds. In addition, we show that *net debt* is a poor measure of credit risk, as the same value for net debt can be associated with two distinct levels of credit risk (a high credit risk with low debt value and low cash, and a low credit risk with high debt value and high cash).

The tradeoff theory of capital structure is often pitted against the pecking order theory, with numerous empirical studies seeking to test them either in isolation or in a horse race (see [Fama and French \(2012\)](#) for a recent example). The empirical status of the tradeoff theory has been and remains a hotly debated question. Some scholars, most notably [Myers \(1984\)](#), have claimed that they “know of no study clearly demonstrating that a firm's tax

²[Acharya, Almeida, and Campello \(2007\)](#) observe that issuing debt and hoarding the proceeds in cash is not equivalent to preserving debt capacity for the future. In their model, risky debt is disproportionately a claim on high cash-flow states, while cash savings are equally available in all future cash-flow states. Therefore, preserving debt capacity or saving cash has different implications for future investment by a financially constrained firm.

status has predictable, material effects on its debt policy.” In a later review of the capital structure literature Myers (2001) further added “A few such studies have since appeared . . . and none gives conclusive support for the tradeoff theory.”

However, more recently a number of empirical studies that build on the predictions of structural models in the vein of Fischer, Heinkel, and Zechner (1989), Leland (1994), Goldstein, Ju, and Leland (2001) and Ross (2005) – but augmented with various transaction costs incurred when the firm changes its capital structure – have found empirical support for the dynamic tradeoff theory (see e.g., Hennessy and Whited (2005), Leary and Roberts (2005), Strebulaev (2007), and Lemmon, Roberts, and Zender (2008)). But it is important to observe that in reality corporate financial decisions are not only shaped by tax-induced tradeoffs, but also by external-financing-cost and liquidity considerations. We therefore need to better understand how capital structure and other corporate financial decisions are jointly determined, and how the firm is valued, when it responds to tax incentives while simultaneously managing its cash reserves in order to relax its financial constraints. This is what we attempt to model in this paper, by formulating a tractable dynamic model of a financially constrained firm that seeks to make tax-efficient corporate financial decisions.

As Decamps, Mariotti, Rochet, and Villeneuve (2011) (DMRV) and Bolton, Chen, and Wang (2011) (BCW) show, when a financially constrained firm has low cash holdings, its *marginal value of cash* can be significantly higher than one. In this context, the firm incurs a significant flow shadow cost for every dollar it pays out to creditors. This cost has to be set against the tax shield benefits of debt. As a result, a financially constrained firm could optimally choose a debt level that trades off tax shield benefits against the *endogenous debt servicing costs* such that the firm would never default on this debt. In such a situation, it would not pay the firm to take on a little bankruptcy risk in order to increase its tax shield benefits because the increase in endogenous debt servicing costs would outweigh the incremental tax shield benefits.

The financial constraint can make the firm’s equity value *concave* in the cash holdings even when debt is risky. This is in contrast to the standard risk-shifting intuition as in Jensen and Meckling (1976) and Leland (1998). Nonetheless, a conflict between

equityholders and debtholders still arises in our model because they have different exposures to the risk of liquidation and hence different degrees of effective risk aversion. For this reason, we show that covenants can have important effects on corporate financial policies.

By retaining its earnings, the firm is making a choice on behalf of its shareholders to defer the payment of their personal income tax liabilities on this income. It may actually be tax-efficient sometimes to let shareholders accumulate savings inside the firm, as [Miller and Scholes \(1978\)](#) have observed. Thus, the tax code influences not just leverage policies but also corporate savings. This in turn has important implications for standard corporate valuation methods such as the *adjusted present value method* (APV, see [Myers \(1974\)](#)), which are built on the assumption that the firm does not face any financial constraints. The APV method is commonly used to value highly levered transactions, as for example in the case of leveraged buyouts (LBOs). A standard assumption when valuing such transactions is that the firm pays down its debt as fast as possible (that is, it does not engage in any precautionary savings). Moreover, the shadow cost of draining the firm of cash in this way is assumed to be zero. As a result, highly levered transactions tend to be overvalued and the risks for shareholders that the firm may be forced to incur costly external financing to raise new funds are not adequately accounted for by this method.

We model a firm in continuous time with a single productive asset generating a cumulative stochastic cash flow, which follows an arithmetic Brownian motion process with drift (or mean profitability) μ and volatility σ . The asset costs K to set up and the entrepreneur who founds the firm must raise funds to both cover this set-up cost and endow the firm with an initial cash buffer. These funds may be raised by issuing either equity or term debt to outside investors. The firm may also obtain a line of credit (LOC) commitment from a bank. Once an LOC is set up, the firm can accumulate cash through retained earnings. As in BCW, the firm only makes payouts to its shareholders when it attains a sufficiently large cash buffer. And in the event that the firm exhausts all its available sources of internal cash and LOC, it can either raise new costly external funds or it is liquidated. Corporate earnings are subject to a corporate income tax and investors are subject to a personal income taxes on interest income, dividends, and capital gains.

There are two main cases to consider. The first is when the firm is liquidated when it exhausts all its sources of liquidity (cash and credit line) and the second is when the firm raises new external funds when it runs out of cash. In the former case term debt issued by the firm is risky, while in the latter it is default-free. Most of our analysis focuses on the case where term debt involves credit risk. We solve for the optimal capital structure of the firm, which involves both a determination of the liability structure (how much debt to issue) and the asset structure (how much cash to hold). This also involves solving for the value of equity and term debt as a function of the firm's cash holdings, determining the optimal line of credit commitment, and characterizing the firm's optimal payout policy. We then analyze how firm leverage varies in response to changes in tax policy, or in the underlying risk-return characteristics of the firm's productive asset.

The financially constrained firm has two main margins of adjustment in response to a change in its environment. It can either adjust its debt or its cash policy. In contrast, an unconstrained firm only adjusts its debt policy when its environment changes. We show that this key difference produces fundamentally different predictions on debt policy, so much so that existing tradeoff theories of capital structure for unconstrained firms offer no reliable predictions for the debt policy of constrained firms. Consider for example the effects of a cut in the corporate income tax rate from 35% to 25%. This significantly reduces the net tax advantage of debt and should result in a reduction in debt financing under the standard tradeoff theory. But this is not how a financially constrained firm responds. The main effect for such a firm is that the after-tax return on corporate savings is increased, so that it responds by increasing its cash holdings. The increase in cash holdings is so significant that the servicing costs of debt decline and compensate for the reduced tax advantage of debt. On net, the firm barely changes its debt policy in response to the reduction in corporate tax rates.

Consider next the effects of an increase in profitability of the productive asset (an increase in the drift rate μ). Under the standard tradeoff theory the firm ought to respond by increasing its debt and interest payments so as to shield the higher profits from corporate taxation. In contrast, the financially constrained firm leaves its debt policy unchanged but

modifies its cash policy by paying out more to its shareholders, as it is able to replenish its cash stock faster as a result of the higher profitability. Once again, the cash policy adjustment induces an indirect increase in the firm's debt servicing costs so that the firm chooses not to change its debt policy. Interestingly, the adjustment we find is in line with the empirical evidence and provides a simple explanation for why financially constrained firms do not adjust their leverage to changes in profitability.

The effects of an increase in volatility of cash flows σ are also surprising. While financially constrained firms substantially increase their cash buffers in response to an increase in σ , they also choose to *increase debt!* Indeed, as a result of their increased cash savings, the debt servicing costs decline so much that it is worth increasing leverage in response to an increase in volatility. This is the opposite to the predicted effect under the standard tradeoff theory, whereby the firm ought to respond by reducing leverage to lower expected bankruptcy costs.

The importance of endogenous debt servicing costs is most apparent in the case where the firm raises new financing whenever it runs out of cash. In this situation, the firm's debt is risk free. In contrast, under the classical tradeoff theory a financially unconstrained firm always issues risky debt. The reason why the financially constrained firm limits its indebtedness is that it seeks to avoid running out of cash too often and paying an external financing cost, and it wants to avoid creating a debt overhang situation, which could induce equityholders to inefficiently liquidate the firm ex post.

As relevant as it is to analyze an integrated framework combining both tax and precautionary-savings considerations, there are, surprisingly, only a few attempts in the literature at addressing this problem. [Hennessy and Whited \(2005, 2007\)](#) consider a dynamic tradeoff model for a firm facing equity flotation costs in which the firm can issue short-term debt.³ Unlike in our analysis, they do not fully characterize the firm's cash-management policy nor do they solve for the value of debt and equity as a function

³[Gamba and Triantis \(2008\)](#) extend [Hennessy and Whited \(2005\)](#) by introducing debt issuance costs and hence obtain the simultaneous existence of debt and cash. [Riddick and Whited \(2009\)](#) develop a corporate savings model and show that corporate savings and cash flow can be negatively related after controlling for q , because firms may use cash reserves to invest when receiving a positive productivity shock.

of the firm's stock of cash. Also, we allow for term debt while [Hennessy and Whited \(2005, 2007\)](#) assume one-period debt. More recently, [DeAngelo, DeAngelo, and Whited \(2011\)](#) have developed and estimated a dynamic capital structure model with taxes and external financing costs of debt and show that while firms have a target leverage ratio, they may temporarily deviate from it in order to economize on debt servicing costs.

An important strength of our analysis is that it allows for a quantitative valuation of debt and equity as well as a characterization of corporate financial policy that can be closely linked to methodologies applied in reality, such as the adjusted present value method. Importantly, our model highlights that the classical structural credit-risk valuation models in the literature are missing an important explanatory variable: the firm's cash holdings, which affect both equity and debt value. Starting with [Merton \(1974\)](#) and [Leland \(1994\)](#), the standard structural credit risk models mainly focus on how shocks to asset fundamentals or cash flows affect the risk of default, but do not explicitly consider liquidity management.

The remainder of the paper proceeds as follows. Section 2 sets up the model. Section 3 presents the Miller Benchmark. Section 4 characterizes the solution for a financially constrained firm. Section 5 continues with the main quantitative analysis. Section 6 discusses key comparative statics results. Section 7 introduces debt covenants. Section 8 introduces depreciation shocks as in [Holmstrom and Tirole \(1997\)](#) and depreciation tax allowances. Section 9 considers the solution when the firm raises new external funds when it runs out of cash. Section 10 concludes.

2 Model

A financially constrained risk-neutral entrepreneur has initial liquid wealth W_{0-} and a valuable investment project which requires an up-front setup cost $K > 0$ at time 0.

Investment project. Let Y denote the project's (undiscounted) cumulative cash flows (profits). For simplicity, we assume that operating profits are independently and identically

distributed (*i.i.d.*) over time and that cumulative operating profits Y follow an arithmetic Brownian motion process,

$$dY_t = \mu dt + \sigma dZ_t, \quad t \geq 0, \quad (1)$$

where Z is a standard Brownian motion. Over a time interval Δt , the firm's profit is normally distributed with mean $\mu\Delta t$ and volatility $\sigma\sqrt{\Delta t} > 0$. This earnings process is widely used in the corporate finance literature.⁴ Note that the earnings process (1) can potentially accumulate large losses over a finite time period. The project can be liquidated at any time (denoted by T) with a liquidation value $L < \mu/r$. That is, liquidation is inefficient. To avoid or defer inefficient liquidation, the firm needs funds to cover operating losses and to meet various payments. Should it run out of liquidity, the firm either liquidates or raises new funds in order to continue operations. Therefore, liquidity can be highly valuable under some circumstances as it allows the firm to continue its profitable but risky operations.

Tax structure. As in Miller (1977), DeAngelo and Masulis (1980), and the subsequent corporate taxation literature, we suppose that earnings after interest (and depreciation allowances) are taxed at the corporate income tax rate $\tau_c > 0$. At the personal level, income from interest payments is taxed at rate $\tau_i > 0$, and income from equity is taxed at rate $\tau_e > 0$. For simplicity, we ignore depreciation tax allowances for now and will incorporate in Section 8. At the personal level, given that capital gains may be deferred, we generally expect that $\tau_e < \tau_i$ even when interest, dividend and capital gains income is taxed at the same marginal personal income tax rate.

External financing: equity, debt, and credit line. Firms often face significant external financing costs due to asymmetric information and managerial incentive problems. We do not explicitly model informational asymmetries nor incentive problems. Rather, to be able to work with a model that can be calibrated, we directly model the costs arising

⁴See, for example, DeMarzo and Sannikov (2006) and Decamps, Mariotti, Rochet, and Villeneuve (2011), who use the same continuous-time process (1) in their analyses). Bolton and Scharfstein (1990), Hart and Moore (1994, 1998), and DeMarzo and Fishman (2007) model cash flow processes using the discrete-time counterpart of (1).

from informational and incentive frictions in reduced form. To begin with, we assume that the firm can only raise external funds once at time 0 by issuing equity, term debt and/or credit line, and that it cannot access capital markets afterwards. In later sections, we allow the firm to repeatedly access capital markets.

As in [Leland \(1994\)](#) and [Goldstein, Ju, and Leland \(2001\)](#) we model debt as a potentially risky *perpetuity* issued at par P with regular coupon payment b . Should the firm be liquidated, the debtholders have seniority over other claimants for the residual value from the liquidated assets. In addition to the risky perpetual debt, the firm may also issue external equity. We assume that there is a fixed cost Φ for the firm to initiate external financing (either debt or equity or both). As in BCW, equity issuance involves a marginal cost γ_E and similarly, debt issuance involves a marginal cost γ_D .

We next turn to the firm's liquidity policies. The firm can save by holding cash and also by borrowing via the credit line. At time 0, the firm chooses the size of its credit line C , which is the maximal credit commitment that the firm obtains from the bank. This credit commitment is fully collateralized by the firm's physical capital. For simplicity, we assume that the credit line is risk-free for the lender. Under the terms of the credit line the firm has to pay a fixed commitment fee $\nu(C)C$ per unit of time on the (unused) amount of the credit line. We specify $\nu(C) = \eta C$ where $\eta > 0$ is the credit line commitment fee parameter. Intuitively, once it draws down an amount $|W_t| < C$ it must pay the commitment fee on the residual, $\nu(C)(C + W_t)$. The economic logic behind this cost function is that the bank providing the LOC has to either incur more monitoring costs or higher capital requirement costs when it grants a larger LOC. The firm can tap the credit line at any time for any amount up the limit C after securing the credit line C at time 0. For the amount of credit that the firm uses, the interest spread over the risk-free rate r is δ . This spread δ is interpreted as an intermediation cost in our setting as credit is risk-free. Note that the credit line only incurs a flow commitment fee and no up-front fixed cost. [Sufi \(2009\)](#) documents that the typical firm on average pays about 25 basis points per annum on C , which implies that $\eta = 2.8\%$. For the tapped risk-free credit, the typical firm pays roughly 25 basis points per year, so that $\delta = 0.25\%$.

Liquidity management: cash and credit line. Liquidity hoarding is at the core of our analysis. Let W_t denote the firm's liquidity holdings at time t . When $W_t > 0$, the firm is in the cash region. When $W_t < 0$, the firm is in the credit region. As will become clear, it is suboptimal for the firm to draw down the credit line if the firm's cash holding is positive. Indeed, the firm can always defer using the costlier credit line option as long as it has unused cash on its balance sheet.

Cash region: $W \geq 0$. We denote by U_t the firm's cumulative (non-decreasing) after-tax payout to shareholders up to time t , and by dU_t the incremental after-tax payout over time interval dt . When the firm does not pay out, $dU_t = 0$, which often happens in the model, as we will show. Distributing cash to shareholders may take the form of a special dividend or a share repurchase.⁵ The firm's cash holding W_t accumulates as follows in the region where the firm has a positive cash reserve:

$$dW_t = (1 - \tau_c) [dY_t + (r - \lambda)W_t dt - \nu(C)C dt - b dt] - dU_t, \quad (2)$$

where λ is a *cash-carry cost*, which reflects the idea that cash held by the firm is not always optimally deployed. That is, the before-tax return that the firm earns on its cash inventory is equal to the risk-free rate r minus a *carry cost* λ that captures in a simple way the agency costs that may be associated with *free cash* in the firm.⁶ The firm's cash accumulation before corporate taxes is thus given by operating earnings dY_t plus earnings from investments $(r - \lambda)W_t dt$ minus the credit line commitment fee $\nu(C)C dt$ minus the interest payment on term debt $b dt$. The firm pays a corporate tax rate τ_c on these earnings net of interest payments and retains after-tax earnings minus the payout dU_t .

⁵A commitment to regular dividend payments is suboptimal in our model. For simplicity we assume that the firm faces no fixed or variable payout costs. These costs can, however, be added at the cost of a slightly more involved analysis.

⁶This assumption is standard in models with cash. For example, see [Kim, Mauer, and Sherman \(1998\)](#) and [Riddick and Whited \(2009\)](#). Abstracting from any tax considerations, the firm would never pay out cash when $\lambda = 0$, since keeping cash inside the firm then incurs no opportunity costs, while still providing the benefit of a relaxed financing constraint. If the firm is better at identifying investment opportunities than investors, we would have $\lambda < 0$. In that case, raising funds to earn excess returns is potentially a positive NPV project. We do not explore cases in which $\lambda < 0$.

Note that an important simplifying assumption implicit in this cash accumulation equation is that profits and losses are treated symmetrically from a corporate tax perspective. In practice losses can be carried forward or backward only for a limited number of years, which introduces complex non-linearities in the after-tax earnings process. As [Graham \(1996\)](#) has shown, in the presence of such non-linearities one must forecast future taxable income in order to estimate current-period effective tax rates. To avoid this complication we follow the literature in assuming that after-tax earnings are linear in the tax rate (see e.g., [Leland \(1994\)](#) and [Goldstein, Ju, and Leland \(2001\)](#)).

Credit region: $W \leq 0$. In the credit region, credit W_t evolves similarly as W_t does in the cash region, except for one change, which results from the fact that in this region the firm is partially drawing down its credit line:

$$dW_t = (1 - \tau_c) [dY_t + (r + \delta)W_t dt - \nu(C)(W_t + C)dt - bdt] - dU_t, \quad (3)$$

where δ denotes the interest rate spread over the risk-free rate, and the commitment fee is charged on the unused LOC commitment $W_t + C$. If the firm exhausts its maximal credit capacity, so that $W_t = -C$, it has to either close down and liquidate its assets or raise external funds to continue operations. In the baseline analysis of our model, we assume that the firm will be liquidated if it runs out of all available sources of liquidity including both cash and credit line. In an extension, we give the firm the option to raise new funds through external financing. But in the baseline case, after raising funds via external financing and establishing the credit facility at time 0, the firm can only continue to operate as long as $W_t > -C$.

Optimality. We solve the firm's optimization problem in two steps. Proceeding by backward induction, we consider first the firm's *ex post* optimization problem after the initial capital structure (external equity, debt, and credit line) has been chosen. Then, we determine the *ex ante* optimal capital structure.

The firm's ex post optimization problem. The firm chooses its payout policy U

and liquidation timing T to maximize the ex post value of equity subject to the liquidity accumulation equations (2) and (3):⁷

$$\max_{U, T} \mathbb{E} \left[\int_0^T e^{-r(1-\tau_i)t} dU_t + e^{-r(1-\tau_i)T} \max\{L_T + W_T - P - G_T, 0\} \right]. \quad (4)$$

Note that P denotes the proceeds from the debt issue. The first term in (4) is the present discounted value of payouts to equityholders until stochastic liquidation, and the second term is the expected liquidation payoff to equityholders. Here, G_T is the tax bill for equityholders at liquidation. It is possible that equityholders realize a capital gain upon liquidation. In this event liquidation triggers capital gains taxes for them. Capital gains taxes at liquidation are given by:

$$G_T = \tau_e \max\{W_T + L_T - P - (W_0 + K), 0\}. \quad (5)$$

Note that the basis for calculating the capital gain is $W_0 + K$, the sum of liquid and illiquid initial asset values. Let $E(W_0)$ denote the value function (4).

The ex ante optimization problem. What should the firm's initial cash holding W_0 be? And in what form should W_0 be raised? The firm's financing decision at time 0 is to jointly choose the initial cash holding W_0 , the line of credit with limit C , and the optimal capital structure (debt and equity). Specifically, the entrepreneur chooses any combination of (i) a perpetual debt issue with coupon b , (ii) a credit line with limit C , and (iii) an equity issue of a fraction a of total shares outstanding.

There is a positive fixed cost $\Phi > 0$ in tapping external financial markets, so that securities issuance is lumpy as in BCW. We also assume that there is a positive variable cost in raising debt ($\gamma_D \geq 0$) or equity ($\gamma_E \geq 0$). Let F denote the proceeds from the equity issue. We focus on the economically interesting case where some amount of external financing is optimal. After paying the set-up cost $K > 0$, and the total issuance costs

⁷Note that this objective function does not take into account the benefits of cash holdings to debtholders. We later explore the implications of constraints on equityholders' payout policies that might be imposed by debt covenants.

$(\Phi + \gamma_D P + \gamma_E F)$ the firm ends up with an initial cash stock of:

$$W_0 = W_{0-} - K - \Phi + (1 - \gamma_D)P + (1 - \gamma_E)F, \quad (6)$$

where W_{0-} is the entrepreneur's initial cash endowment before financing at time 0.

The entrepreneur's *ex ante* optimization problem can then be written as follows:

$$\max_{a, b, C} (1 - a)E(W_0; b, C), \quad (7)$$

where $E(W)$ is the solution of (4), and where the following competitive pricing conditions for debt and equity must hold:

$$P = D(W_0), \quad (8)$$

and

$$F = aE(W_0). \quad (9)$$

In addition, the value of debt $D(W_0)$ must satisfy the following equation:

$$D(W_0) = \mathbb{E} \left[\int_0^T e^{-r(1-\tau_i)s} (1 - \tau_i)b ds + e^{-r(1-\tau_i)T} \min\{L_T - C, P\} \right]. \quad (10)$$

Note that implicit in the debt pricing equation (10) is the assumption that in the event of liquidation the 'revolver debt' due under the credit line is *senior* to the 'term debt' P . We use θ_D and θ_E to denote the Lagrange multipliers for (8) and (9), respectively.

There are then two scenarios, one where the term debt is risk-free and the other where it is risky. When term debt is risk-free, debtholders collect P , the principal value of debt. In this case the price of debt is simply the value of perpetuity:

$$P = \frac{b}{r}. \quad (11)$$

When debt is risky, creditors demand an additional *credit spread* to compensate for the default risk they are exposed to under the term debt.

Before formulating debt value $D(W_0)$ and equity value $E(W_0)$ as solutions to differ-

ential equations and proceeding to characterize the solutions to the ex post and ex ante optimization problems we begin by describing the classical *Miller irrelevance solution* in our model for the special case where the firm faces no financing constraints.

3 The Miller Benchmark

Under the Miller benchmark, the firm faces neither external financing costs ($\Phi = \gamma_P = \gamma_F = \eta = \delta = 0$) nor any cash carry cost ($\lambda = 0$). Without loss of generality we shall assume that in this idealized world the firm never relies on a credit line and simply issues new equity if it is in need of cash to service the term debt. Given that shocks are *i.i.d.* the firm then never defaults. Miller (1977) argues that the *effective tax benefit of debt*, which takes into account both corporate and personal taxes, is

$$\tau^* = \frac{(1 - \tau_i) - (1 - \tau_c)(1 - \tau_e)}{(1 - \tau_i)} = 1 - \frac{(1 - \tau_c)(1 - \tau_e)}{(1 - \tau_i)}. \quad (12)$$

For a firm issuing a perpetual interest-only debt with coupon payment b , its *ex post* equity value is then:

$$E^* = \mathbb{E} \left[\int_0^\infty e^{-r(1-\tau_i)t} (1 - \tau_c)(1 - \tau_e) (dY_t - bdt) \right] = \frac{1}{r} (1 - \tau^*) (\mu - b) . \quad (13)$$

For a perpetual debt with no liquidation ($T = \infty$), *ex post* debt value is simply $D^* = b/r$ as both the after-tax coupon and the after-tax interest rate are proportional to before-tax coupon b and before-tax interest rate r with the same coefficient $(1 - \tau_i)$.

The firm's total value, denoted by V^* , is given by the sum of its debt and equity value:

$$V^* = E^* + D^* = \frac{\mu}{r} (1 - \tau^*) + \frac{b}{r} \tau^* , \quad (14)$$

where the first term is the value of the *unlevered firm* and the second term is the *present value of tax shields*. First, as long as $\tau^* > 0$, (14) implies that the optimal leverage for a financially unconstrained firm is the maximally allowed coupon b . Given that the firm

cannot borrow more than its value (or debt capacity), it may pledge at most 100% of its cash flow by setting $b^* = \mu$. In this case, firm value satisfies the familiar formula $V^* = \mu/r$. As we will show, for a financially constrained firm, even with $\tau^* > 0$, liquidity considerations will lead the firm to choose moderate leverage.

4 Analysis

We now characterize the solutions to the ex post and ex ante problems for the firm. We show that the firm will find itself in one of the following three possible regions: (i) the liquidation region, (ii) the interior (internal financing) region, and (iii) the payout region. As we will show below, the firm is in the payout region when its cash stock W exceeds an endogenous upper barrier \bar{W} , and liquidates itself once it exhausts its LOC (when $W = -C$). Finally, in the interior region, $-C \leq W \leq \bar{W}$, the firm services interest payments for its term debt and accumulates liquidity.

4.1 Optimal Payout Policy and the Value of Debt and Equity

There are two sub-regions in the interior region, the cash-hoarding region and the credit region. In the interior credit region, $-C \leq W \leq 0$, the firm's *after-tax* credit evolution equation is given by

$$dW_t = (1 - \tau_c) (\mu + (r + \delta)W - \nu(C)(W_t + C) - b) dt + (1 - \tau_c) \sigma dZ_t. \quad (15)$$

In the interior cash-hoarding region, $0 < W \leq \bar{W}$, the firm's *after-tax* cash accumulation is given by

$$dW_t = (1 - \tau_c) (\mu + (r - \lambda)W - \nu(C)C - b) dt + (1 - \tau_c) \sigma dZ_t. \quad (16)$$

Note that the corporate tax rate τ_c lowers both the drift and the volatility of the liq-

uidity accumulation processes (15-16).⁸ In this *cash-hoarding region*, the firm effectively accumulates savings for its shareholders inside the firm. Shareholders' interest income on their corporate savings is then taxed at the corporate income tax rate τ_c rather than the personal interest income tax rate τ_i if earnings were disbursed and accumulated as personal savings. An obvious question for the firm with respect to corporate versus personal savings is: which is more tax efficient? If $(r - \lambda)(1 - \tau_c) > r(1 - \tau_i)$ it is always more efficient to save inside the firm and the firm will never pay out any cash to its shareholders. Thus, a necessary and sufficient condition for the firm to *eventually* payout its cash is:

$$(r - \lambda)(1 - \tau_c) < r(1 - \tau_i). \quad (17)$$

By holding on to its cash and investing it at a return of $(r - \lambda)$ the firm earns

$$[1 + (r - \lambda)(1 - \tau_c)](1 - \tau_e)$$

per unit of savings. If instead the firm pays out a dollar to its shareholders, they only collect $(1 - \tau_e)$ and earn an after-tax rate of return $r(1 - \tau_i)$. Therefore, when $[1 + r(1 - \tau_i)](1 - \tau_e) \geq [1 + (r - \lambda)(1 - \tau_c)](1 - \tau_e)$, which simplifies to (17), the firm will eventually disburse cash to its shareholders. It may not immediately pay out its earnings so as to reduce the risk that it may run out of cash. Thus, the payout boundary is optimally chosen by equityholders to trade off the after-tax efficiency of personal savings versus the expected costs of premature liquidation when the firm runs out of cash.

Equity value $E(W)$. Let $E(W)$ denote the after-tax value of equity. In the interior cash hoarding region $0 \leq W \leq \bar{W}$, equity value $E(W)$ satisfies the following ODE:

$$(1 - \tau_i)rE(W) = (1 - \tau_c)(\mu + (r - \lambda)W - \nu(C)C - b)E'(W) + \frac{1}{2}\sigma^2(1 - \tau_c)^2E''(W). \quad (18)$$

⁸The tax implications on the volatility of after-tax labor income have first been explored by [Kimball and Mankiw \(1989\)](#) in a precautionary savings model for households.

Note that we discount the after-tax cash flow using the after-tax discount rates $(1 - \tau_i)r$, as the alternative of investing in the firm's equity is to invest in the risk-free asset earning an after-tax rate of return $(1 - \tau_i)r$.

Next, we turn to the interior credit region, $-C \leq W \leq 0$. Using a similar argument as the one for the cash hoarding region, $E(W)$ satisfies the following ODE:

$$(1 - \tau_i)rE(W) = (1 - \tau_c)(\mu + (r + \delta)W - \nu(C)(C + W) - b)E'(W) + \frac{1}{2}\sigma^2(1 - \tau_c)^2E''(W). \quad (19)$$

Note that the firm pays the spread δ over the risk-free rate r on the amount $|W|$ that it draws down from its LOC.⁹

Next, we turn to various boundary conditions. At the *endogenous* payout boundary \bar{W} , equityholders must be indifferent between retaining cash inside the firm and distributing it to shareholders, so that:

$$E'(\bar{W}) = 1 - \tau_e. \quad (20)$$

In addition, since equityholders optimally choose the payout boundary \bar{W} the following *super-contact condition* must also be satisfied:

$$E''(\bar{W}) = 0. \quad (21)$$

Substituting (20) and (21) into the ODE (18), we then obtain the following valuation equation at the payout boundary \bar{W} :

$$E(\bar{W}) = \frac{(1 - \tau^*)(\mu + (r - \lambda)\bar{W} - \nu(C)C - b)}{r}, \quad (22)$$

where τ^* is the *Miller tax rate* given by (12). The expression (22) for the value of equity $E(\bar{W})$ at the payout boundary \bar{W} can be interpreted as a “steady-state” perpetuity valuation equation by slightly modifying the Miller formula (13) with the added term $(r - \lambda)\bar{W} - \nu(C)C$ for the interest income on the maximal corporate cash holdings \bar{W} and

⁹Using a standard argument, one can show that $E(W)$ is continuously differentiable at $W = 0$. See Karatzas and Shreve (1991).

the running cost of the whole unused LOC C . Because \bar{W} is a *reflecting boundary*, the value attained at this point should match this steady-state level as though we remained at \bar{W} forever. If the value is below this level, it is optimal to defer the payout and allow cash holdings W to increase until (21) is satisfied. At that point the benefit of further deferring payout is balanced by the cost due to the lower rate of return on corporate cash as implied by condition (17).

At the payout boundary \bar{W} , each unit of cash is valued at $(1 - \tau^*)(r - \lambda)/r < 1$ by equity investors for two reasons: (i) the effective *Miller tax rate* $\tau^* > 0$ and (ii) the cash carry cost λ . That is, cash is disadvantaged without a precautionary value of cash-holdings, and hence the firm pays out for $W \geq \bar{W}$.

At the left boundary $W = -C$, equity value is given by

$$E(-C) = \max \{0, L - C - P - G\}, \quad (23)$$

where G denotes the capital gains taxes given in (5) at the moment of exit. There are two scenarios to consider. First, term debt is fully repaid at liquidation and debt is risk-free. If debt is risky, the seniority of debt over equity implies that equity is worthless, so that: $E(-C) = 0$. Recall that credit line is fully repaid.

Debt value $D(W)$. Let $D(W)$ denote the after-tax value of debt. Taking the firm's payout policy \bar{W} as given, investors price debt accordingly. In the cash hoarding region, $D(W)$ satisfies the following ODE:

$$(1 - \tau_i)rD(W) = (1 - \tau_i)b + (1 - \tau_c)(\mu + (r - \lambda)W - \nu(C)C - b)D'(W) + \frac{1}{2}\sigma^2(1 - \tau_c)^2D''(W), \quad (24)$$

And, in the credit region, $-C < W < 0$, the ODE for debt pricing $D(W)$ is

$$(1 - \tau_i)rD(W) = (1 - \tau_i)b + (1 - \tau_c)(\mu + (r + \delta)W - \nu(C + W) - b)D'(W) + \frac{1}{2}\sigma^2(1 - \tau_c)^2D''(W). \quad (25)$$

The boundary conditions are:

$$D(-C) = \min \{L - C, P\}, \text{ and} \quad (26)$$

$$D'(\bar{W}) = 0. \quad (27)$$

Condition (26) follows from the absolute priority rule which states that debt payments have to be serviced in full before equityholders collect any liquidation proceeds. Condition (27) follows from the fact that the expected life of the firm does not change as W approaches \bar{W} (since \bar{W} is a reflective barrier),

$$\lim_{\varepsilon \rightarrow 0} \frac{D(K, \bar{W}) - D(K, \bar{W} - \varepsilon)}{\varepsilon} = 0.$$

Firm value $V(W)$ and Enterprise value $Q(W)$. Since debtholders and equityholders are the firm's two claimants and credit line use is default-free and is fully priced in the equity value $E(W)$, we define the firm's total value $V(W)$ as

$$V(W) = E(W) + D(W). \quad (28)$$

Following the standard practice in both academic and industry literatures, we define enterprise value as firm value $V(W)$ netting out cash:

$$Q(W) = V(W) - W = E(W) + D(W) - W. \quad (29)$$

Note that $Q(W)$ is purely an accounting definition and may not be very informative about the economic value of the productive asset under financial constraints. Indeed, under MM, the (net) marginal value of liquidity is zero, $Q'(W) = 0$. Here, we know that's not the case indicating the value of liquidity for a financially constrained firm.

Having characterized the market values of debt and equity as a function of the firm's stock of cash W , we now turn to the firm's ex ante optimization problem, which involves the choice of an optimal 'start-up' cash reserve W_0 , an optimal credit line commitment

with limit C , and an optimal debt and equity structure.

4.2 Optimal Capital Structure

At time 0, the entrepreneur chooses the fraction of outside equity a , the coupon on the perpetual risky debt b , and the credit line limit C (with implied W_0) to solve the following problem:

$$\max_{a,b,C} (1-a)E(W_0; b, C), \quad (30)$$

where

$$W_0 = W_{0-} + F + P - (\gamma_E F + \gamma_D P + \Phi) - K, \quad (31)$$

$$F = aE(W_0; b, C), \text{ and} \quad (32)$$

$$P = D(W_0; b, C). \quad (33)$$

Without loss of generality we set $W_{0-} = 0$. The optimal amount of cash W_0 the firm starts out with (after the time-0 financing arrangement) is then given by the solution to the following equation, which defines a fixed point for W_0 :

$$(1 - \gamma_E) aE(W_0) + (1 - \gamma_D) D(W_0) = W_0 + K + \Phi. \quad (34)$$

The entrepreneur is juggling with the following issues in determining the firm's start-up capital structure. The first and most obvious consideration is that by raising funds through a term debt issue with coupon b , the entrepreneur is able to both obtain a tax shield benefit and to hold on to a larger fraction of equity ownership. That is in essence the benefit of (term) debt financing. One cost of debt financing is that the perpetual interest payments b must be serviced out of liquidity W and may drain the firm's stock of cash or use up the credit line. To reduce the risk that the firm may run out of cash, the entrepreneur can start the firm with a larger cash cushion W_0 , and she can take out an LOC commitment with a larger limit C . The benefit of building a large cash buffer is obviously that the firm can collect a larger debt tax shield and reduce the risk of premature liquidation. The

cost is, first that the firm will pay a larger issuance cost at time 0, and second that the firm will invest its cash inside the firm at a suboptimal after-tax rate $(1 - \tau_c)(r - \lambda)$. To reduce the second cost the firm may choose to start with a lower cash buffer W_0 but a larger LOC commitment C . The tradeoff the firm faces here is that while it economizes on issuance costs and on the opportunity cost of inefficiently saving cash inside the firm, it has to incur a commitment cost $\nu(C)C$ on its LOC. In addition, by committing to a larger LOC C , the firm will pay a spread δ when tapping the credit line. Finally, as the credit line is senior to term debt, the firm increases credit risk on its term debt b and the likelihood of inefficient liquidation.

Depending on underlying parameter values, the firm's time-0 optimal capital structure can admit three possible solutions: (i) no term debt (equity issuance only, with possibly an LOC); (ii) term debt issuance only (with, again, a possible LOC); and (iii) a combination of equity and term debt issuance (with a possible LOC).

Solution procedure. We now briefly sketch out our approach to solving numerically for the optimal capital structure at date 0. We focus our discussion on the most complex solution where the firm issues both debt and equity. The objective function in this case is given by (30). We begin by fixing a pair of (b, C) and solving for $E(W)$ and $D(W)$ from the ODEs for E and D . We then proceed to solve for the range of a , as specified by (a_{\min}, a_{\max}) , for which there is a solution W_0 to the budget constraint (31). Next, we solve for W_0 from the fixed point problem (31) for a given triplet (b, C, a) . There is either one or two fixed points, each representing an equilibrium. The intuition for the case of multiple equilibria is that outside investors can give the firm high or low valuation depending on the initial cash holding W_0 being high or low, which in turn result in the actual W_0 being high or low. Finally, we find (b^*, C^*, a^*) that maximizes $(1 - a)E(W_0; b, C)$.

4.3 Net Tax Benefit of Debt for a Financially Constrained Firm

Miller (1977) provides a simple formula of the net tax benefit of debt for an unconstrained firm, which nets out the tax benefit of debt at firm level against the tax disadvantage of

debt at individual level. How does the net tax benefit of debt for the financially constrained firm compare to that for an unconstrained firm? To address this question, we consider how an extra dollar in income generated inside the firm may be used. A marginal Δ increment in income can be used in one of the following ways: (i) paid out to service debt, (ii) paid out to equityholders as a dividend, or (iii) retained inside the firm as part of the liquidity reserve. The after-tax interest income to debt holders is $(1 - \tau_i)\Delta$, while the after-tax dividend income to equity holders is $(1 - \tau_c)(1 - \tau_e)\Delta$. If the amount Δ is retained, the firm's cash reserve will increase by $(1 - \tau_c)\Delta$, resulting in an after-tax capital gain of $E(W_t + (1 - \tau_c)\Delta) - E(W_t)$, or approximately $E'(W_t)(1 - \tau_c)\Delta$ for small Δ .

In the absence of external financing costs there is no need to retain cash. The net tax benefit of debt is then based on the comparison between choices (i) and (ii), which yields the effective Miller tax rate in (12). In the presence of external financing costs, the firm prefers to retain cash instead of paying it out whenever W_t is away from the endogenous payout boundary, $W_t < \bar{W}$. The net marginal tax benefit of debt then becomes

$$\tau^*(W_t) = \frac{(1 - \tau_i)\Delta - (1 - \tau_c)E'(W_t)\Delta}{(1 - \tau_i)\Delta} = 1 - \frac{(1 - \tau_c)E'(W_t)}{(1 - \tau_i)}. \quad (35)$$

In other words, for a financially constrained firm the payout choice (2), and hence the Miller formula for the net tax benefit of debt, is only relevant when a firm is indifferent between paying out and retaining cash inside the firm, i.e., when $W = \bar{W}$. Note that since $E'(\bar{W}) = 1 - \tau_e$, the right-hand side of (35) reduces to Miller's effective tax rate in (12) at the payout boundary \bar{W} .

5 Quantitative Results

Parameter values and calibration. We choose the model parameters as follows. First, we set the corporate income tax rate at $\tau_c = 35\%$ as in Leland (1994), the personal equity income tax rate at $\tau_e = 12\%$, as well as personal interest income tax rate at $\tau_i = 30\%$, as in Hennessy and Whited (2007) and Goldstein, Ju, and Leland (2001). The tax rate τ_e on equity income is lower than the tax rate on interest income τ_i to reflect the fact that

Table 1: **Parameters.** This table reports the parameter values for the baseline model. All the parameter values are annualized when applicable.

Risk-free rate	r	6%	Fixed financing cost	Φ	1%
Risk-neutral mean ROA	μ	12%	Prop. debt financing cost	γ_D	1%
Volatility of ROA	σ	10%	Prop. equity financing cost	γ_E	6%
Initial investment	K	1	Cash-carrying cost	λ	0.5%
Liquidation value	L	0.9	Credit line spread	δ	0.25%
Tax rate on corporate income	τ_c	35%	LOC commitment fee parameter	η	2.8%
Tax rate on equity income	τ_e	12%			
Tax rate on interest income	τ_i	30%			

capital gains are typically taxed at a lower rate than interest, as well as the fact that the taxation of capital gains can be deferred until capital gains are realized. Based on our assumed tax rates, the Miller effective tax rate as defined in (12) is $\tau^* = 18.3\%$.

In most dynamic structural models following [Leland \(1994\)](#), the Miller tax rate τ^* is sufficient to capture the combined effects of the three tax rates (corporate, personal equity, and personal interest incomes) on leverage choices. However, in a dynamic model with financing frictions and cash accumulation (as our model), the Miller tax rate τ^* is no longer sufficient to capture the effects of corporate and personal equity/debt tax rates because the time when the firm earns its profit is generally separate from the time when it pays out its earnings. The reason for this separation is that it is often optimal for a financially constrained firm to hoard cash rather than immediately pay out its earnings. Hence, most of the time, the conventional double-taxation Miller calculation for tax shields is not applicable for a financially constrained firm.

Second, we set the annual risk-free rate to $r = 6\%$. We set the annual risk-neutral expected return on capital to $\mu = 12\%$ based on the estimates of [Acharya, Almeida, and Campello \(2013\)](#), and the volatility of the annual return on capital to $\sigma = 10\%$ based on [Sufi \(2009\)](#). For the interest spread on the LOC, we choose $\delta = 0.25\%$ to capture the costs for banks to monitor the firm (there is no default risk for the LOC). For the LOC commitment fee, we calibrate $\eta = 0.028$ to match the average (unused) LOC-to-asset ratio

Table 2: **Optimal capital structure.** This table reports the optimal capital structure results from the baseline case.

credit line	coupon rate	outside equity	payout boundary	initial cash	project value	debt value	interest coverage	market leverage
C	b	a	\bar{W}	W_0	J_0	D_0	μ/b	L_0
0.100	0.080	0	0.326	0.303	0.763	1.326	1.497	0.635

($C = 0.10$) in Sufi (2009). The implied average proportional commitment fee is 28 basis points, close to the median commitment fee of 25 basis points as documented by Sufi (2009).

Third, we set external financing costs as follows: we take the fixed cost to be $\Phi = 1\%$ of the setup cost K as in BCW. The firm incurs this cost when it raises external funds, whether in the form of debt or equity or both. We further take the marginal debt issuance cost $\gamma_D = 1\%$ and the marginal equity issuance cost $\gamma_E = 6\%$ based on the empirical findings of Altinkilic and Hansen (2000).

Fourth, we set the cash-carrying cost to $\lambda = 0.5\%$, which is somewhat lower than in BCW. The reason is that here λ only reflects the cash-carry costs that are due to agency or governance factors, while the parameter λ in BCW also includes the tax disadvantage of holding cash, which we model explicitly here. Finally, the liquidation value is set at $L = 0.9$ as in Hennessy and Whited (2007). Table 1 summarizes all the parameter values.

The Ex-Post Value of Equity $E(W)$. Based on our baseline parameter values, the optimal LOC commitment is $C = 0.10$, the optimal coupon is $b = 0.08$, and there is no outside equity stake, $a = 0$. Moreover, the optimal start-up cash buffer is $W_0 = 0.303$ and the optimal payout boundary is $\bar{W} = 0.326$. The entrepreneur obtains an initial equity value of $U_0 = 0.763$ under the optimal capital structure. These results are summarized in Table 2.

Figure 1 plots the value of equity $E(W)$ in Panel A and the marginal equity value of liquidity $E'(W)$ in Panel B. When W reaches the endogenous lower boundary $\underline{W} = -C =$

-0.10 , the firm has run out of its maximal liquidity supply and is liquidated. At that point equity is worthless, because the liquidation value of the assets is lower than the face value of debt. When W hits $\bar{W} = 0.326$, it is optimal for the firm to pay out any cash in excess of \bar{W} . Indeed, at that point the marginal value of liquidity inside the firm for equityholders is equal to the after-tax value of a dollar of payout: $E'(\bar{W}) = 1 - \tau_e = 0.88$, as can be seen in Panel B. In the interior region, equity value $E(W)$ increases with W with a slope $E'(W) > 1 - \tau_e$, reflecting the value of a higher cash buffer as insurance against the risk of early liquidation. As Panel B shows, when the firm is close to running out of cash, the marginal value of one dollar of cash to equity holders exceeds six dollars.

Remarkably, Figure 1 reveals that $E(W)$ is concave in W even though the firm is levered with *risky debt*. The standard intuition in corporate finance suggests that the value of equity for a firm with risky debt is equivalent to the value of a call option with strike price equal to the face value of the firm's debt. It follows that the value of equity is convex in the value of the firm's underlying assets. However, in our baseline model, default is entirely driven by (diffusive) liquidity shocks. When a firm is forced to liquidate, it generates discrete losses for both debt holders and equity holders. In particular, it makes the equity holders dynamically risk averse and engage in precautionary corporate savings.¹⁰ Consistent with our results, Rauh (2009) finds that firms with more poorly funded pension plans tend to hold more safe assets in their portfolios suggesting that they are reluctant to engage in risk-shifting by holding risky financial assets. In our calibrated model, the firm starts with a relatively large cash buffer $W_0 = 0.303$, sets the payout boundary at $\bar{W} = 0.326$, and secures a LOC commitment of $C = 0.10$, all of which contribute to reducing liquidity risk. In Section 8, we show that large liquidity shocks can generate convexity in the equity value.

The Ex-Post Value of Debt. In Figure 2, we examine the property of debt in our model. A first observation that emerges from Figure 2 is that the market value of debt $D(W)$ is increasing and concave in W (Panel A), while the credit spread is decreasing in W

¹⁰Technically, the concavity of the equity value is easiest to see in the case without uncertainty, where the equity value will be linearly increasing in cash holding when $W > -C$ but drops discretely at $W = -C$.

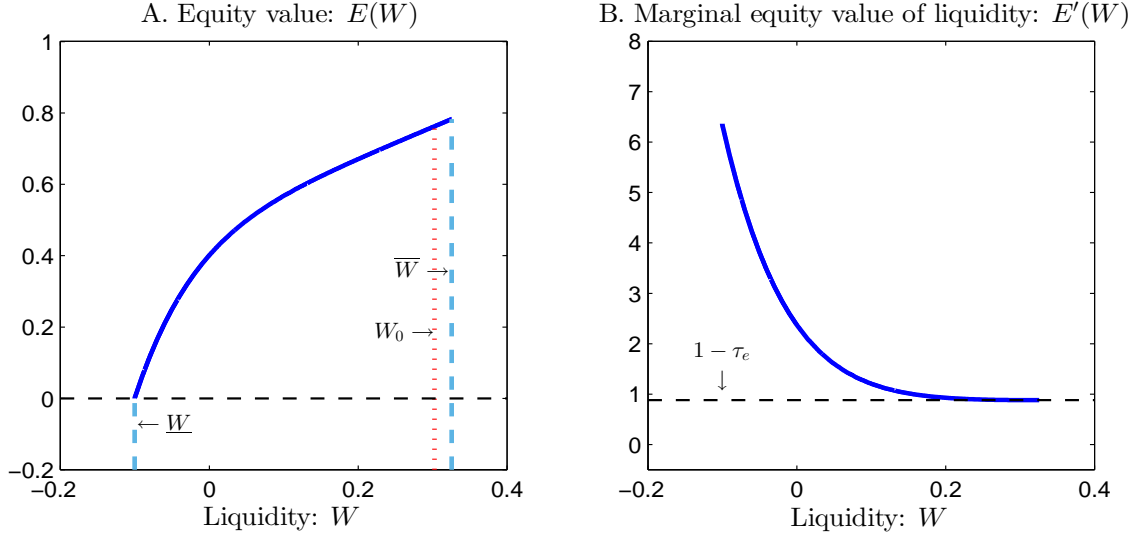


Figure 1: **Equity value $E(W)$ and the marginal equity value of liquidity $E'(W)$.** This figure plots $E(W)$ and $E'(W)$ for the baseline case.

(Panel D). This is intuitive, given that the firm is less likely to default when it has a higher cash buffer. This is also in line with the evidence provided in [Acharya, Davydenko, and Strebulaev \(2012\)](#).¹¹ Second, the marginal debt value of liquidity $D'(W)$ is highly sensitive to W (Panel B). As W increases towards the endogenous payout boundary $\bar{W} = 0.326$, $D'(W)$ approaches zero, indicating that debt becomes insensitive to changes in W .

A third striking observation is that net debt, $D(W) - W$, is non-monotonic in W (Panel C), which suggests that net debt (based on market value of debt) is a poor measure of a firm's credit risk. Analysts commonly use net debt as a measure of a firm's credit risk based on the logic that the firm could at any time use its cash hoard to retire some or all of its outstanding debt. As our results show, information may be lost by netting debt with cash. For example, a netted value of 1.1 could reflect either a high credit risk (if the firm is drawing down on its LOC) or a low credit risk if the firm holds a comfortable cash buffer W in excess of 0.2.

¹¹[Acharya, Davydenko, and Strebulaev \(2012\)](#) first run an OLS regression of yield spreads on cash-to-total assets and other variables. They obtain a positive coefficient, suggesting that surprisingly higher cash holdings are associated with higher spreads. However, when they run an instrumental variable regression (using the ratio of intangible-to-total assets as an instrument) they find that the coefficient on the cash-to-total assets variable is negative.

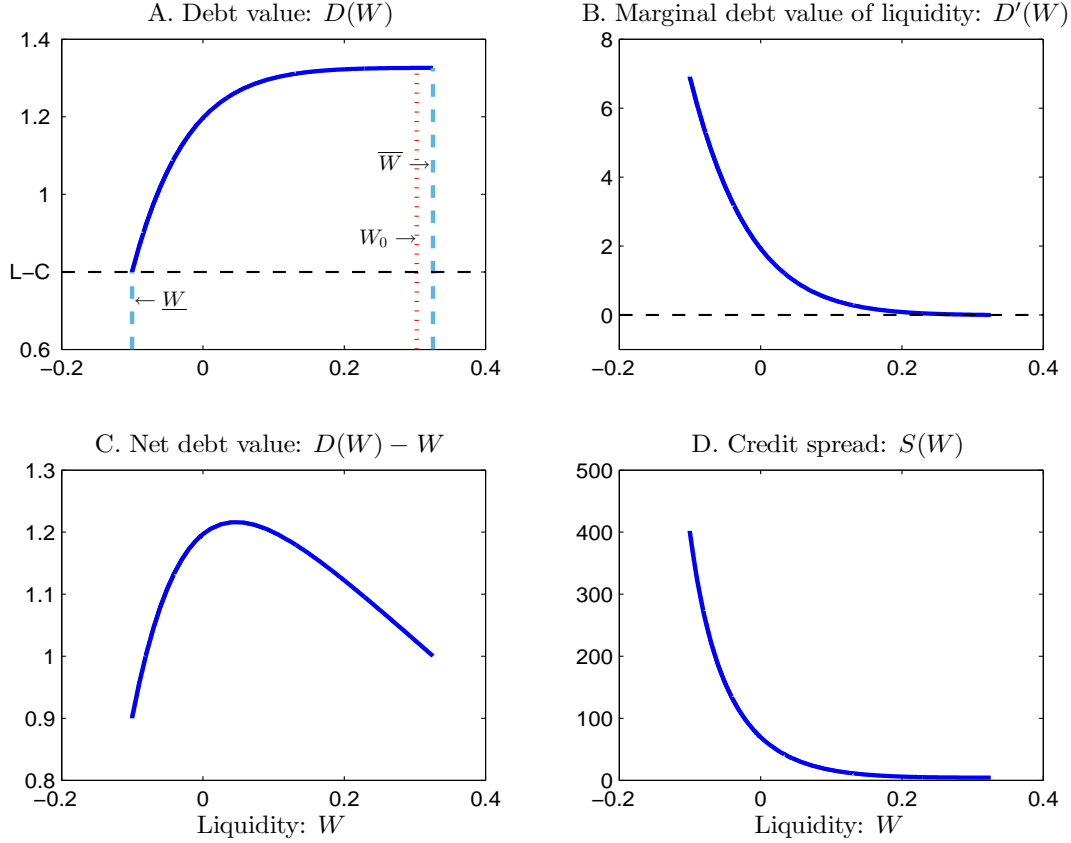


Figure 2: **Debt value $D(W)$, the marginal debt value of liquidity $D'(W)$, net debt value $D(W) - W$, and credit spread $S(W) = b/D(W) - r$.**

Finally, Panel D plots the credit spread, defined as the difference between the yield on long-term debt and the risk-free rate, $S(W) = b/D(W) - r$. The spread drops to merely 5 basis points as W approaches the endogenous payout boundary, and when the firm exhausts its credit line limit the credit spread increases beyond 400 basis points. Note that even at the payout boundary, the firm's debt is not risk free as there is still a small probability that the firm ends up in liquidation in the future.

The Ex-Post Enterprise Value. Figure 3 plots the enterprise value $Q(W) = V(W) - W$ in Panel A, and the marginal enterprise value of liquidity $Q'(W)$ in Panel B. Given that both equity value and debt value are increasing and concave in W , we expect that enterprise value $Q(W)$ is more concave than equity value $E(W)$. This means that an

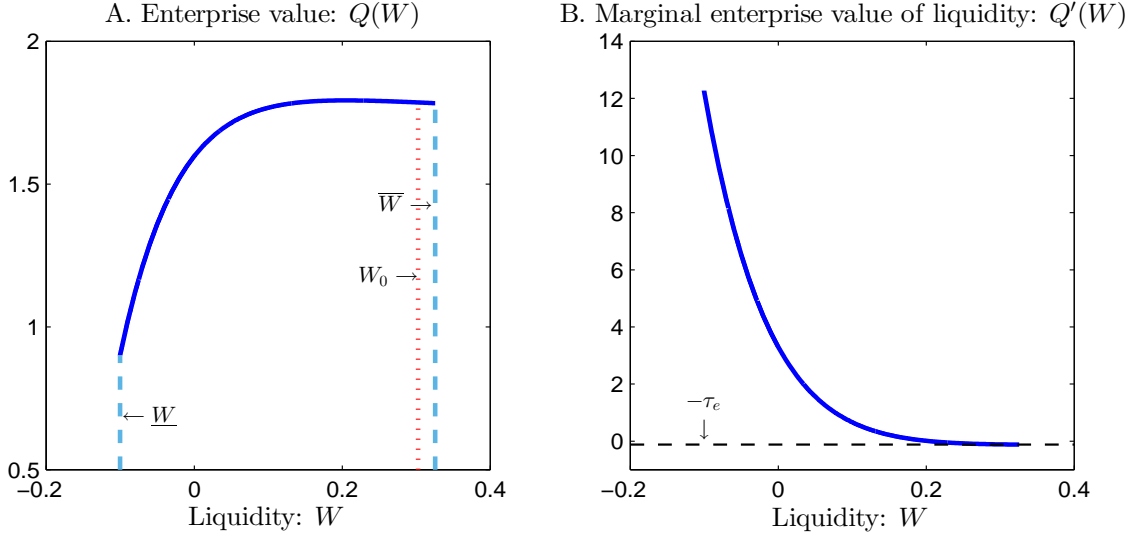


Figure 3: **Enterprise value $Q(W) = V$ and the marginal enterprise value $Q'(W)$.** This figure plots the enterprise value $Q(W) = V(W) - W$ and $Q'(W)$. Note that $Q'(W)$ can be negative near the payout boundary \bar{W} .

investor holding a portfolio of debt and equity in this firm would be more averse to cash-flow risk than an equity holder. Thus, although equity holders are dynamically risk-averse, they are not risk-averse enough to be in a position to optimally control risk from the point of view of total firm value. This is why it is optimal in general to include debt covenants into the term debt contract that limit equity holders' ability to control risk or pay out dividends ex post, which we study in Section 7.

Note that the marginal enterprise value of liquidity $Q'(W)$ can become negative for values of W that exceed 0.205. This is simply due to the fact that paying out excess cash triggers personal equity income tax at 12%. Therefore, at the payout boundary $\bar{W} = 0.326$, $Q'(0.326) = -0.12 < 0$. Because $Q'(W)$ can be negative, we should be somewhat cautious with the economic interpretation of enterprise value $Q(W)$ in environments with (personal equity) taxes.

Leverage. We define market leverage, $L(W)$, as the ratio between the market value of debt $D(W)$ and the firm’s market value $V(W)$,

$$L(W) = \frac{D(W)}{V(W)}. \quad (36)$$

Another common definition of leverage replaces debt value with net debt $D(W) - W$ and accordingly replaces firm value $V(W)$ by enterprise value $Q(W)$. We refer to this leverage ratio as “net leverage” and denote it by $L^N(W)$,¹²

$$L^N(W) = \frac{D(W) - W}{Q(W)}. \quad (37)$$

Figure 4 plots both leverage $L(W)$ and net leverage $L^N(W)$ as a function of W in the interior region. Both measures of leverage are decreasing in W . Under our baseline parameters, the value of equity is zero when the firm is liquidated, $E(-C) = 0$, so that leverage takes its maximum value of 100%. At the payout boundary \bar{W} , market leverage reaches its minimum value of 62.9%. This may appear to be a high level of leverage. However, note that for an unconstrained firm, the *Miller solution* described above prescribes optimal market leverage of 100%. An important reason why we obtain high leverage ratios even for a financially constrained firm is that underlying cash flows in our model are *i.i.d.* This assumption implies that the issuance of long-term debt does not generate any solvency risk. It only affects the liquidity risk of the firm.

The Tax Advantage of Debt for a Financially Constrained Firm. Equation (35) shows that the net tax benefit of debt depends crucially on the financing constraint. The net tax benefit is reduced when the firm’s precautionary savings motive is high, i.e., when the marginal value of liquidity is high. As Figure 5 shows, the size of this effect on the net tax benefit of debt can be quite large when the firm’s cash holdings are low. The net tax benefit turns negative for $W < 0.13$, and can be as low as -4.91 when the firm is close to

¹²Acharya, Almeida, and Campello (2007) argue that cash should not be treated as negative debt. Cash can help financially constrained firms hedge future investment against income shortfalls, so that constrained firms would value cash more. Our model provides a precise measure of this distortion.

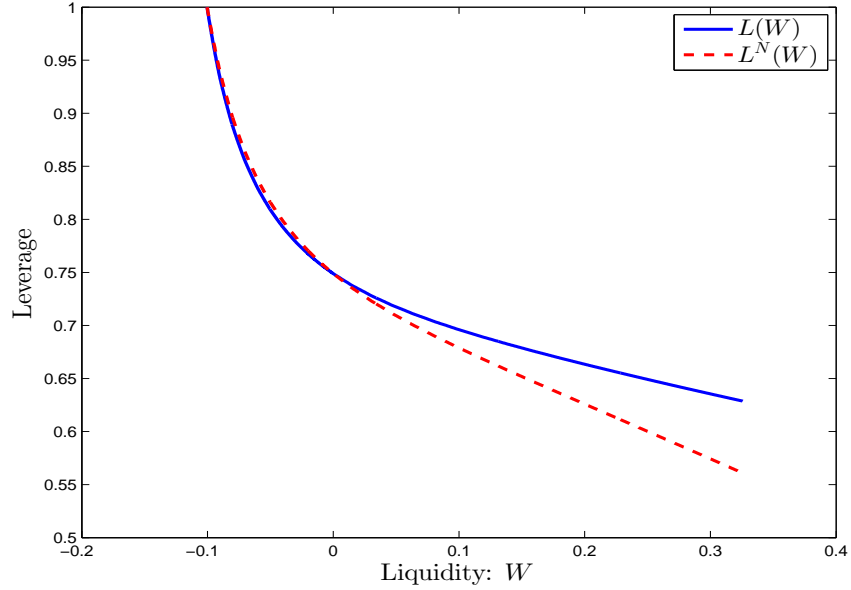


Figure 4: **Leverage $L(W)$ and net leverage $L^N(W)$.** This figure plots market leverage $L(W)$ and net leverage $L^N(W)$ against liquidity W .

running out of cash, compared to 0.183 in the Miller formula.

6 Comparative Statics

The solution illustrated above shows that a financially constrained firm will only exploit the tax advantage of debt to a very limited extent. For example, when it is close to the endogenous payout boundary \bar{W} , our financially constrained firm's market leverage $L(W)$ is about 40% lower than the optimal market leverage for an unconstrained firm. In this section, we further examine the impact of financing constraints on the capital structure using comparative statics analysis.

Unlike for the classical dynamic tradeoff theory where the firm's only margin of adjustment is debt versus equity financing, a financially constrained firm has an additional margin along which it can respond to, say, a change in tax policy: it can change its cash policy. The tradeoff between the tax benefits of debt and the costs of financial distress implies that debt financing increases with the effective tax benefit of debt τ^* ; moreover,

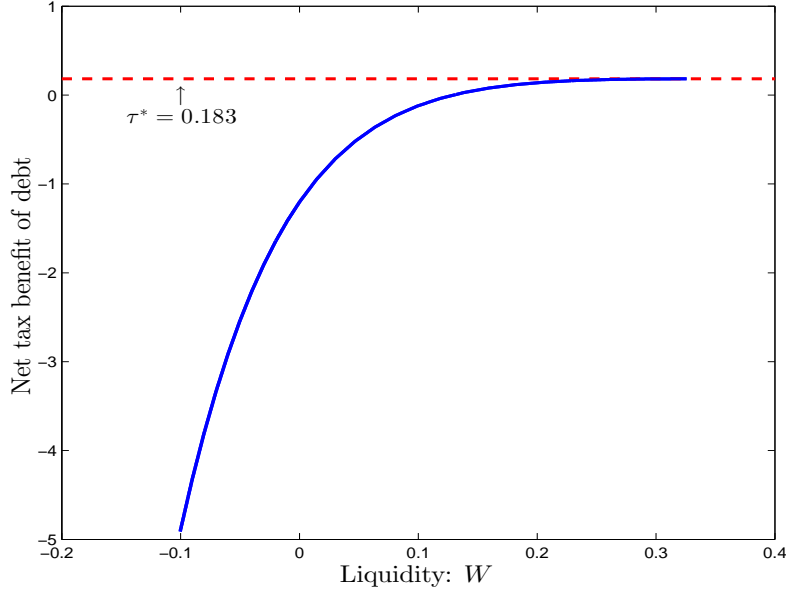


Figure 5: **Measuring tax benefits.** This figure plots the net tax benefits of debt for the financially constrained firm (conditional on its cash holding W). The dash-line denotes the Miller tax rate, which is the effective tax benefit for an unconstrained firm.

debt usage increases with the firm’s profitability but decreases with higher cash-flow risk. However, the presence of external financing constraints alter these predictions both quantitatively and qualitatively, because the constrained firm will now be concerned with the impact that changes in financial policies have on liquidity. We examine this question by exploring how the optimal capital structure of a financially constrained firm varies in response to changes in tax rates and the underlying cash-flow characteristics.

6.1 Corporate Financial Policy and Taxation

In Table 3, we report the financially constrained firm’s optimal financial policy, market value, and leverage under three different tax policy scenarios. In the first scenario, we lower the corporate tax rate from $\tau_c = 35\%$ to 25% , keeping other parameters the same. The effects of this change on corporate financial policy are reported in the second row of Table 3 and plotted in Figure 6. Cutting the corporate tax rate substantially reduces the Miller tax rate τ^* from 18.3% to 5.7% . Based on the intuition of the classical dynamic tradeoff

Table 3: **Comparative statics: tax rates.** This table reports the results from comparative statics on different tax rates.

	Miller tax rate τ^*	credit line C	coupon rate b	payout bound \bar{W}	initial cash W_0	project value J_0	debt value D_0	interest coverage μ/b	market leverage L_0
baseline	18.3%	0.100	0.080	0.326	0.303	0.763	1.326	1.497	0.635
$\tau_c = 25\%$	5.7%	0.097	0.077	0.507	0.258	0.887	1.281	1.557	0.591
$\tau_e = 0\%$	7.1%	0.100	0.080	0.326	0.303	0.867	1.326	1.497	0.605
$\tau_i = 15\%$	32.7%	0.098	0.086	0.288	0.661	0.960	1.688	1.393	0.637

model, one would expect such a decline to significantly lower the firm’s reliance on debt financing. However, this is not the case for our financially constrained firm, which barely changes its reliance on term debt, with the coupon b changing from 0.08 to 0.077. The reason is that a financially constrained firm issues term debt not only to take advantage of the tax shield, but also to build a liquidity reserve. In fact, even when the tax shield is completely removed, the firm will still have strong incentive to issue debt.

Instead of significantly reducing its term debt, by far the main change in response to the lower corporate tax rate concerns the firm’s cash management. On the one hand, the reduction in the corporate tax rate raises the firm’s after-tax return on its savings $(r - \lambda)(1 - \tau_c)$ from 3.575% to 4.125%. On the other hand, it also raises the volatility of the after-tax cash flows $(1 - \tau_c)\sigma$ from 7.8% to 9%. Both changes will encourage the firm to increase its cash savings, which is reflected in the large upward shift in the payout boundary from $\bar{W} = 0.326$ to 0.507. Thus, should such a tax reform be introduced in the US, our model predicts that corporations would increase their already substantial cash holdings. The other changes that such a tax reform would induce is a small reduction in the reliance on LOC (from $C = 0.10$ to $C = 0.097$) as cash holding is less expensive and credit line is less valuable.¹³ Naturally, the reduction in corporate taxation will also result in higher equity value and the enterprise value, as shown in Panel A and C in Figure 6.

¹³Note that as a result of the lower amount of debt issued, the firm also decreases its initial cash hoard from $W_0 = 0.303$ to $W_0 = 0.258$.

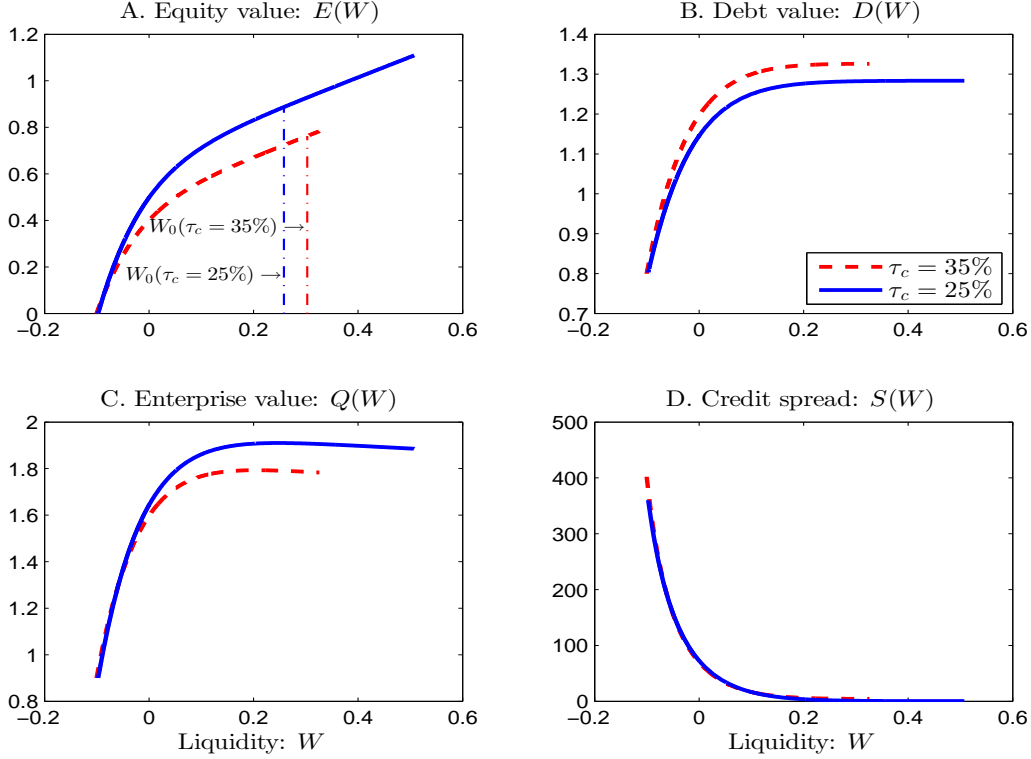


Figure 6: **Comparative statics with respect to corporate income tax rate τ_c .**

The market value of debt falls due to the smaller coupon, and the credit spread is lower for most levels of cash holding.

In the second scenario, we eliminate the personal tax rate on equity income, i.e., changing τ_e from 12% to 0%. The effects of this change are reported in the third row of Table 3 and plotted in Figure 7. This change reduces the Miller tax rate τ^* from 18.3% to 7.1%, but the firm's financial policy remains essentially unchanged. While the reduction in equity income tax τ_e does raise the equity and enterprise value, it has a negligible effect on the amount of term debt issued and on the cash policy (including the payout boundary and the initial cash holding). The reason is that these policies are predominately determined by the financial constraint. Indeed, as we have demonstrated in the net tax benefit calculation in equation (35) and in Figure 5, unlike for the Miller tax rate, changes in τ_e only directly affect the net tax benefit of debt at the payout boundary.

In the third scenario, we lower the personal tax rate on interest income, τ_i , from 30%

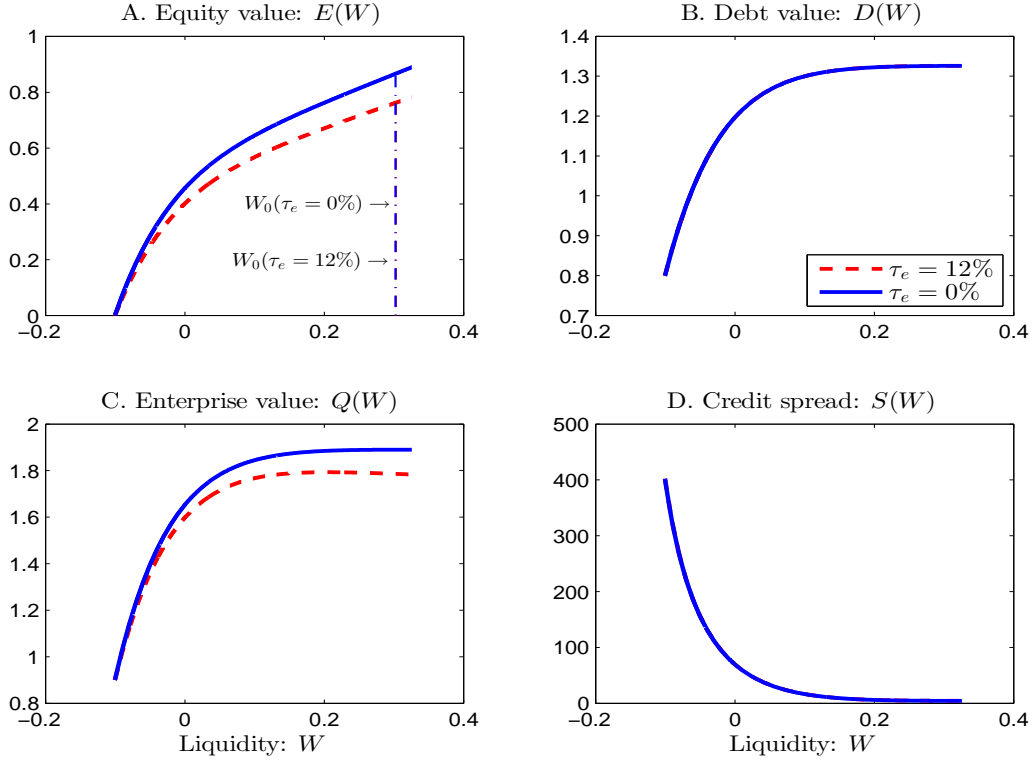


Figure 7: **Comparative statics with respect to personal equity income tax rate τ_e**

to 15%. The effects of this change are reported in the fourth row of Table 3 and plotted in Figure 8. The cut in τ_i almost doubles the Miller tax rate τ^* from 18.3% to 32.7%. As indicated by equation (35), a lower τ_i also directly increases the net tax benefit of debt, which results in a significant increase in the coupon rate from $b = 0.08$ to 0.086. The higher coupon combined with the reduction in τ_i raises the value of outstanding term debt $D(W)$, as seen in Panel A of Figure 8. In fact, the higher tax benefit of debt induces the firm to raise substantially more cash via debt issuance at time 0 than it wants for precautionary reasons, with $W_0 = 0.661$ exceeding the payout boundary $\bar{W} = 0.288$, which generates an immediate payout of $W_0 - \bar{W} = 0.373$ to the entrepreneur.

Interestingly, the other significant change in corporate financial policy is an overall reduction in the cash buffer the firm chooses to retain, with both a slight reduction in the LOC commitment C from 0.10 to 0.098 and a downward shift in the payout boundary \bar{W} from 0.326 to 0.288. The reason is that, with a higher debt burden the ex-post value

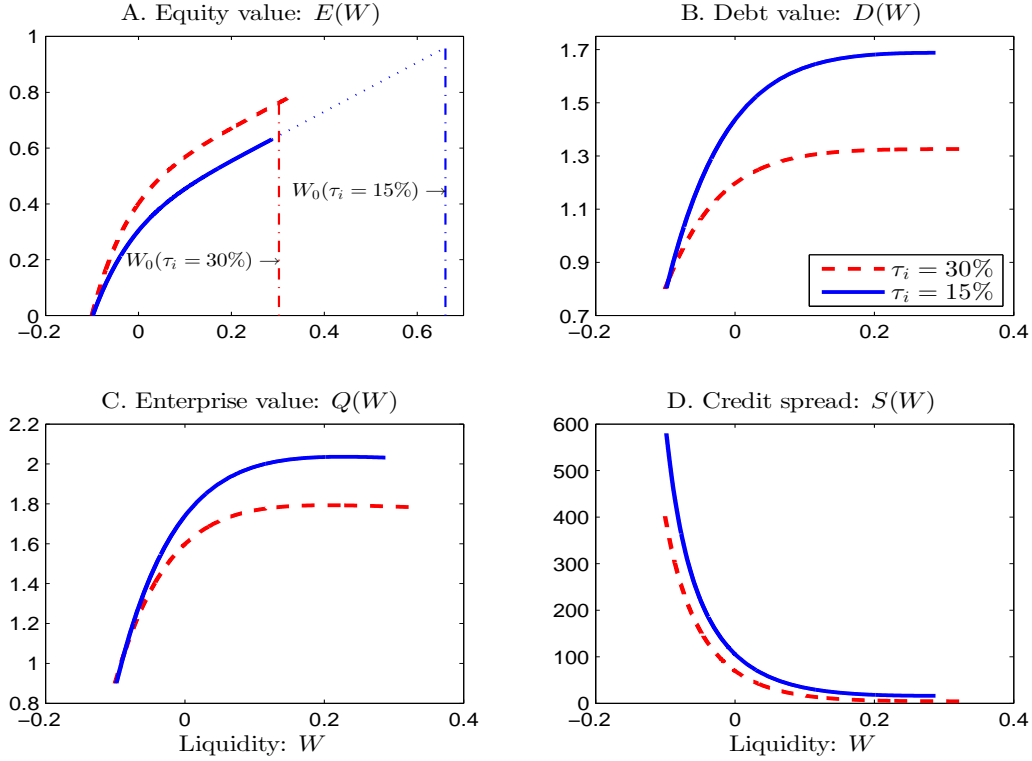


Figure 8: **Comparative statics with respect to personal interest income tax rate τ_i**

of equity $E(W)$ (plotted in Panel A of Figure 8) is now lower, so that equityholders—who determine the firm’s optimal cash policy—are now less concerned with ensuring the continuation of the firm and more interested in getting higher cash payouts. Thus, although equityholders are dynamically risk averse, from the point of view of maximizing total firm value they are in effect willing to hold less cash to be able to get a somewhat higher short-term payout. The lower interest income tax rate raises the enterprise value (see Panel C), while the credit spread is higher due to the increase in term debt and reduction in cash holding (see Panel D). Overall, the effects of the change in τ_i for financially constrained firms is close to the predictions from the classical dynamic tradeoff solution: the increase in τ^* results in higher debt financing, higher market leverage, higher credit spreads, and a higher probability of default.

Table 4: **Comparative statics: cash flow parameters and cash holding cost.** This table reports the results from comparative statics on the mean and volatility of return to capital, μ and σ , and the cash holding cost λ .

	credit line C	coupon rate b	outside equity a	payout boundary \bar{W}	initial cash W_0	project value J_0	debt value D_0	interest coverage μ/b	market leverage L_0
baseline	0.100	0.080	0	0.326	0.303	0.763	1.326	1.497	0.635
$\sigma = 20\%$	0.101	0.086	0.205	0.774	0.506	0.633	1.376	1.391	0.634
$\mu = 14\%$	0.100	0.094	0	0.313	0.538	1.049	1.564	1.481	0.599
$\lambda = 0.75\%$	0.144	0.076	0	0.266	0.233	0.752	1.255	1.579	0.625

6.2 Profitability, Earnings Volatility, and Financial Policy

In Table 4 we report how the firm’s optimal financial policy changes with: *i*) cash-flow volatility σ ; *ii*) profitability μ ; and *iii*) the cash-carrying cost λ .

The effects of an increase in σ on corporate financial policy are reported in the second row of Table 4 and plotted in Figure 9. One well known prediction from the dynamic tradeoff theory is that an increase in cash-flow risks reduces leverage. Riskier firms are expected to reduce their indebtedness because they face higher expected bankruptcy costs. In this context it is striking to observe that, as we increase the volatility from $\sigma = 10\%$ to 20% , the amount of term debt the firm issues actually increases, with the coupon b rising from 0.08 to 0.086, and the interest coverage dropping from 1.497 to 1.391.

Another significant change in the financial policy is that the firm chooses to issue outside equity in addition to term debt, with the outside equity share increasing from $a = 0$ in the baseline case to 0.205. The purpose of issuing more debt as well as outside equity is to build up a sufficient initial cash buffer W_0 , which nearly doubles from 0.303 to 0.506. Moreover, the firm also adopts a significantly more conservative payout policy, with the endogenous payout boundary \bar{W} shifting from 0.326 to 0.774. In other words, the main margin of adjustment to an increase in volatility of earnings is a substantial increase in corporate savings. Overall, an increase in volatility is bad news for the firm,

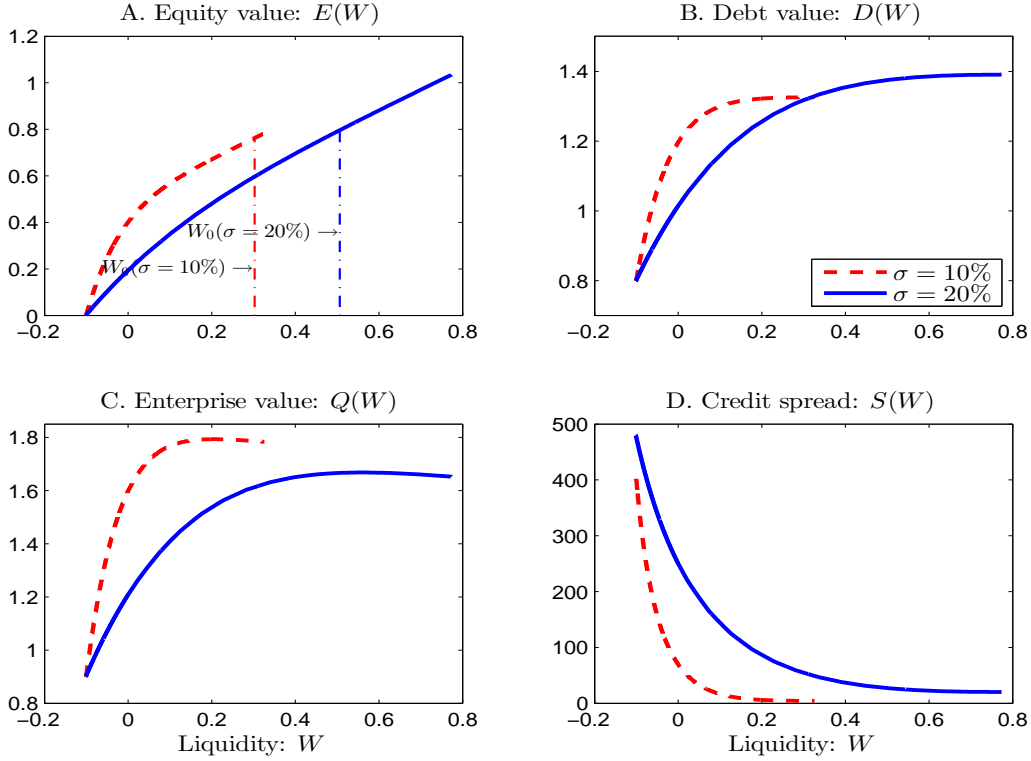


Figure 9: **Comparative statics with respect to earnings volatility σ**

as witnessed by the decline in ex-ante project value from 0.763 to 0.633. Intuitively, the firm attempts to make up for this worsening situation by holding more cash to reduce the probability of an early liquidation and by exploiting the tax-shield benefits of debt more aggressively. In sum, the main lesson emerging from this comparative statics exercise is that the observation of higher debt and leverage for riskier firms is not necessarily a violation of the tradeoff theory. It points to the importance of incorporating into the standard model a precautionary savings motive.

The effects of an increase in profitability μ on corporate financial policy are reported in the third row of Table 4 and plotted in Figure 10. Under the Miller benchmark, an increase in profitability μ will result in a proportional increase in the coupon b . Simply put, higher profits require a higher tax shield, which is obtained by committing to higher interest payments, b . Remarkably, this seemingly obvious prediction is not borne out for a financially constrained firm. As can be seen in Table 4, the firm keeps its LOC

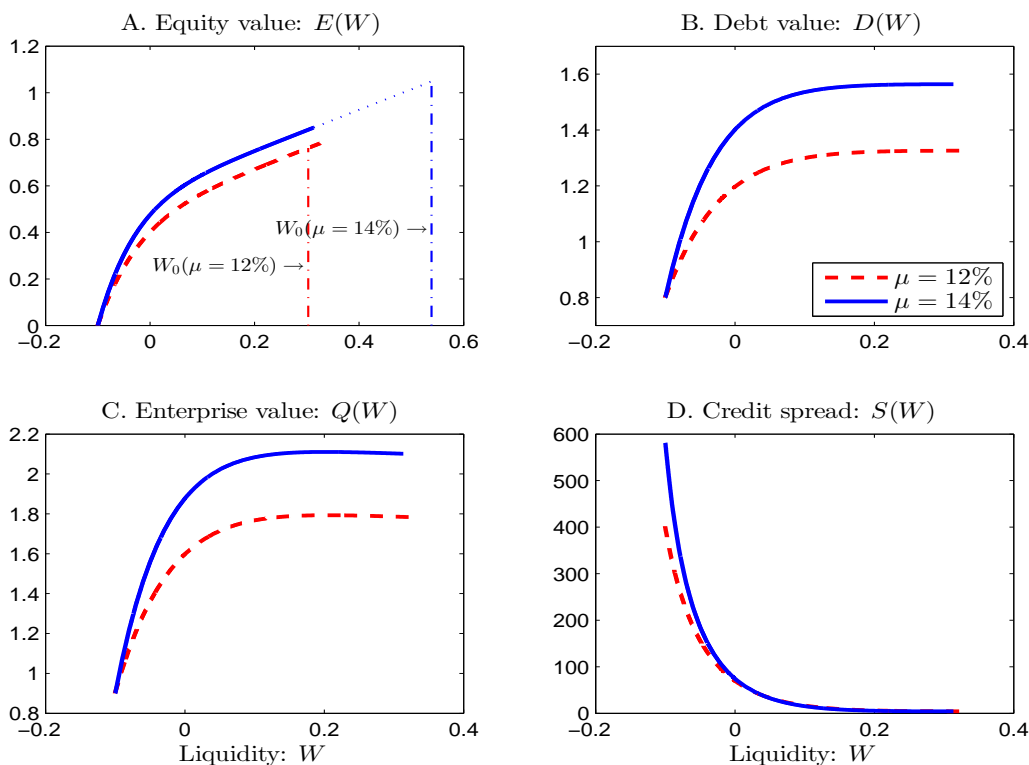


Figure 10: Comparative statics with respect to profitability μ

commitment unchanged at $C = 0.10$ and mainly adjusts its coupon from $b = 0.08$ to 0.094, and its payout boundary \bar{W} from 0.326 to 0.313. In other words, the firm raises more debt to take advantage of the tax shield, but can afford to reduce its cash holding thanks to a higher profitability μ .

Figure 10 shows that equity value, debt value, and enterprise value all increase as a result of higher profitability, and the credit spread can become higher as well when the level of cash holding is low. Initial market leverage actually is lower (dropping from 0.635 to 0.599) due to the valuation effects. It has often been pointed out that in practice market leverage appears to be unresponsive to changes in profitability, which is generally interpreted as a violation of the static tradeoff theory (see e.g. Rajan and Zingales, 1995). A common explanation given for this violation is that more profitable firms have more growth options and therefore face greater debt-overhang costs. In our model the firm does not have any growth options, yet its leverage is unresponsive to changes in profitability.

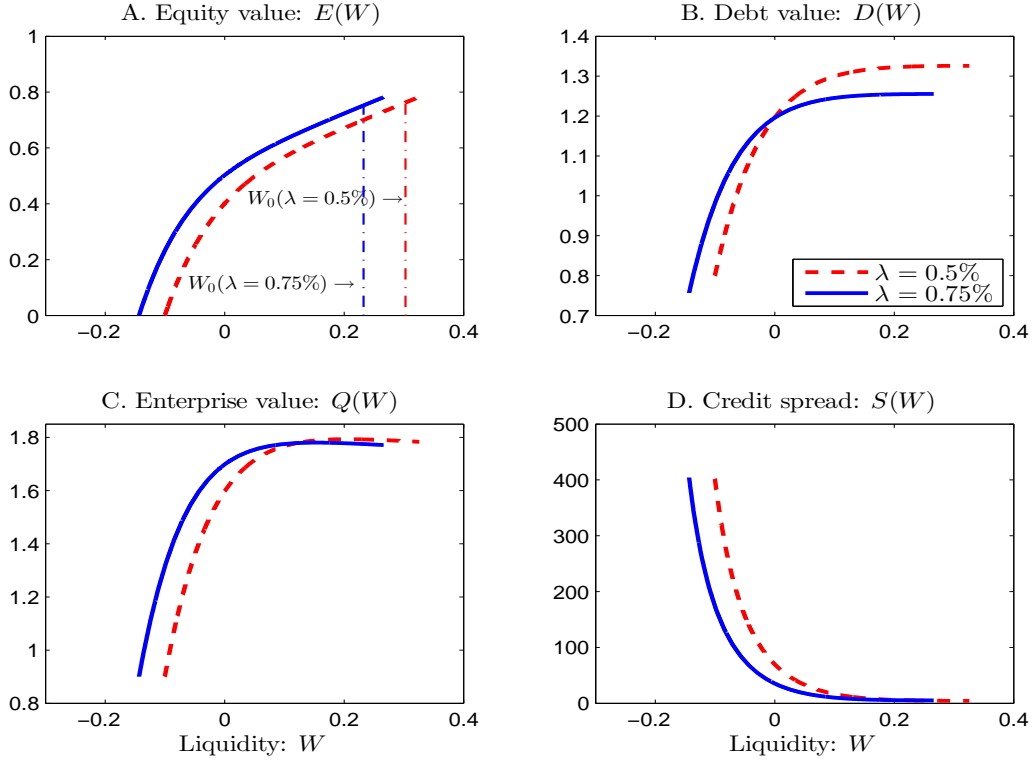


Figure 11: **Comparative statics with respect to cash-carrying cost λ**

The reason is that the firm adjusts its cash policy as well as its debt. Thus, if one takes into account the reality that financially constrained firms have a precautionary savings motive and also seek to reduce their tax burdens, their financial policies may no longer be so puzzling.

Finally, the effects of an increase in the *cash-carrying* cost λ on corporate financial policy are reported in the fourth row of Table 4 and plotted in Figure 11. The main effect of an increase in the cash-carrying cost λ from 0.5% to 0.75% is to substantially reduce the firm's cash holding. The initial stock of cash drops from $W_0 = 0.303$ to $W_0 = 0.233$, and the endogenous payout boundary \bar{W} shifts down from 0.326 to 0.266. The firm makes up for its lower cash reserves by taking out a substantially larger LOC, with the commitment C increasing from 0.10 to 0.144. The firm also reduces its term debt coupon from 0.08 to 0.076 in response to the increase in its debt servicing costs. The overall effect of this policy on the value of equity, debt, enterprise value, and credit spreads are shown in Figure 11.

6.3 Capital Structure, Cash-Flow Risk, and Setup Costs

In Figure 12, we explore more systematically how the firm's financial policy varies with the underlying cash-flow risk and the size of the required initial setup cost K , which are informative about how capital structure decisions differ in settings with and without liquidity constraints.

The three panels in the left column plot the optimal coupon b , the outside equity share a , the payout boundary and the initial cash holding as we vary cash-flow volatility σ from 4% to 40%. Under the classical tradeoff theory the firm should respond to an increase in cash-flow risk by relying less on debt financing. This is indeed how our financially constrained firm responds when σ increases from 4% to 10%: it progressively lowers the coupon b from 0.093 to 0.08. In this region, the firm chooses high coupon for the tax shield, but the precautionary demand for cash is low due to low cash-flow risks. As a result, there is excess cash that is paid out immediately after debt issuance, as captured by $W_0 - \bar{W}$ in Panel E. As cash-flow risk rises, the firm reduces the amount of debt issued, which causes the initial cash holding W_0 to fall as well, until it is equal to the payout boundary \bar{W} when the volatility rises to $\sigma = 10\%$.

When σ increases further from 10% to 16%, the firm responds by issuing more debt: the coupon *increases* from 0.08 to 0.088. The reason behind this increased reliance on debt is that the financially constrained firm at that point prefers to raise its initial cash buffer W_0 and further delay its payout to hedge against the increased cash-flow risk. Since the firm chooses not to issue any outside equity in the meantime (see Panel C), issuing more term debt is the only way to boost the firm's initial cash holding.

Finally, when σ further increases from 16% to 40%, the firm again responds by reducing the amount of debt issued. As we see in Panel C, in this region the firm starts to issue outside equity, with outside equity share a rising from 0 to 0.6. This is because when cash-flow volatility reaches sufficiently high levels ($\sigma > 0.16$), debt financing becomes more costly than equity due to the toll of debt servicing costs on corporate liquidity. As a result, the firm substitutes away from debt and into equity financing instead.

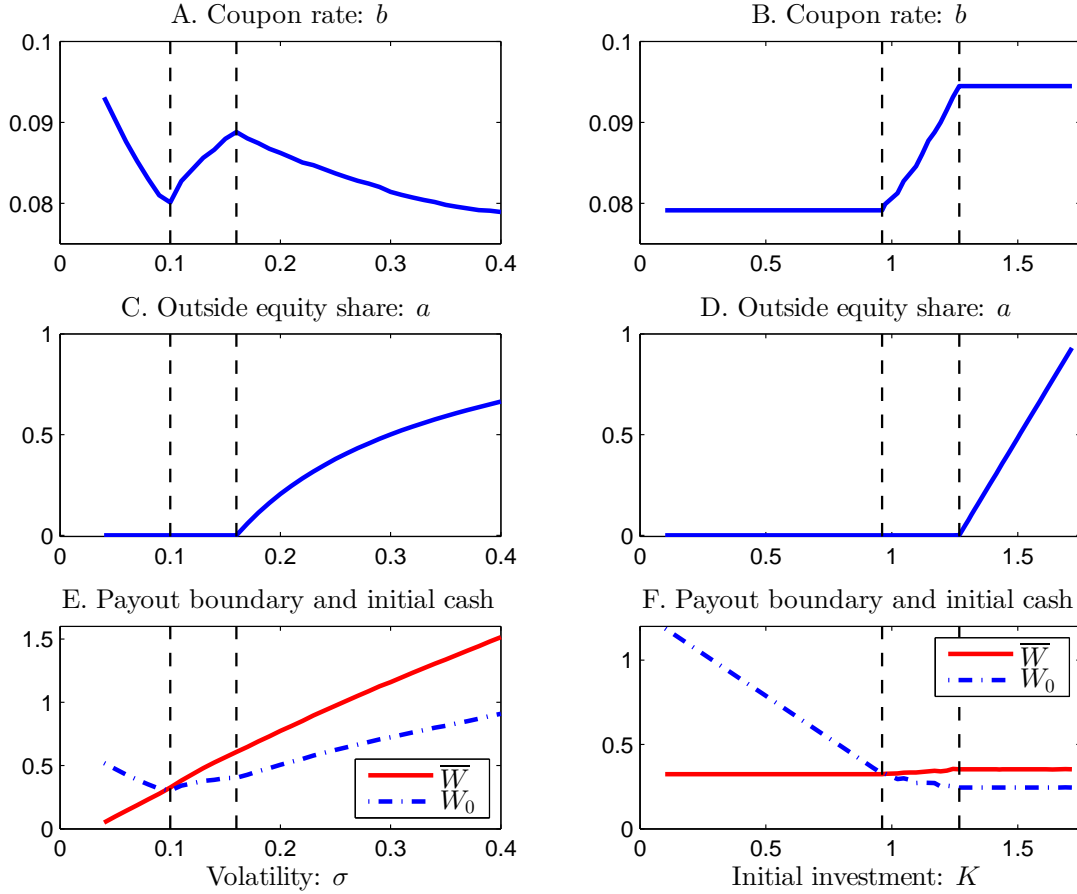


Figure 12: **Optimal long-term debt and outside equity share as functions of cashflow volatility σ (left panels) and initial setup cost K (right panels).**

In the right columns (Panels B, D, and F) of Figure 12, we vary the setup cost K from 0.1 to 1.7. When the initial amount of required funding K is low, the firm's financing decisions are purely driven by tax tradeoff considerations. Thus, for $K \leq 0.96$ the optimal coupon remains flat at $b = 0.079$. When K increases from 0.96 to 1.27, the firm responds by issuing more debt (b increases from 0.079 to 0.095) to make sure that it is able to pay for the setup costs while maintaining a sufficiently large initial cash buffer W_0 (see Panel F). And when K increases further from 1.27 to 1.7, the firm starts to finance part of the setup costs by issuing outside equity so as to avoid further increasing the debt servicing costs.

7 Covenants and Agency Conflicts

Bond covenants that protect bondholders in situations where a conflict arises between equityholders and debtholders are ubiquitous. In our model such a conflict can arise over payout policies, as the payout boundary is determined to maximize equity value and not total firm value. Although equityholders in our model are effectively risk averse, a conflict between equityholders and debtholders still arises because they have different exposures to the risk of liquidation and hence different degrees of effective risk aversion. How do the firm's financial policies change when we introduce debt covenants into the bond contract? This is the question we address in this section.

There are many different forms of bond covenants, ranging from interest service coverage ratios to restrictions on capital expenditures and new debt issuance. [Leland \(1994\)](#) considers both covenants on the firm's bankruptcy decision and its cash payouts. He models covenants that regulate "cash payouts" as a reduction in the rate of return on the firm's asset (the expected return on V is $(r - d)$ instead of r). He considers payouts d that cover both dividend payments and a fraction of the coupon payment C , and shows that any payouts $d \geq 0$ lower the value of the firm, as they increase the incidence of costly bankruptcy. The modeling of cash payouts in [Leland \(1994\)](#) is inevitably in reduced form given that neither cash-flows nor retained earnings are explicitly modeled.

In our framework covenants on cash payouts can be explicitly modeled. An *optimal covenant on cash payouts* is a payout boundary that is chosen to maximize total firm value $V(W) = E(W) + D(W)$ instead of just equity value $E(W)$. Thus, an optimal payout boundary \bar{W}^* must satisfy the optimality conditions

$$V'(\bar{W}^*) = 1 - \tau_e$$

and

$$V''(\bar{W}^*) = 0.$$

The effects of this payout covenant are reported in [Table 5](#). For comparison, the first

Table 5: **Payout covenant.** This table reports the results on the impact of payout covenant.

covenant inclusion	credit line	coupon rate	payout boundary	initial cash	project value	debt value	interest coverage	market leverage
	C	b	\bar{W}	W_0	J_0	D_0	μ/b	L_0
no	0.100	0.080	0.326	0.303	0.763	1.326	1.497	0.635
yes (fix a, b, C)	0.100	0.080	0.373	0.308	0.765	1.331	1.497	0.635
yes	0.099	0.101	0.494	0.639	0.769	1.666	1.194	0.684

row repeats the baseline model solution without covenants, while the third row reports the solution with covenants. To isolate the effect of the covenant on the payout boundary, the second row reports the results from the case where the choice of outside equity share a , term debt coupon b , and credit-line limit C are the same as in the baseline model, and only the payout boundary \bar{W} is allowed to be adjusted. Then, by comparing the three rows, we are able to break down the source of the gains from imposing a payout covenant.

Comparing the first and second rows we find that the introduction of a payout covenant shifts the payout boundary from $\bar{W} = 0.326$ to $\bar{W}^* = 0.373$, so that the firm on average holds a larger buffer of internal funds that can be used to service its term debt. As a result, debt value at issuance rises slightly from $D_0 = 1.326$ to $D_0 = 1.331$, so that the firm can start out with a slightly higher initial cash buffer ($W_0 = 0.308$ instead of $W_0 = 0.303$). Overall project value J_0 also rises slightly as a result of the introduction of the payout covenant.

These results suggest that the gains from the introduction of a payout covenant on project value or initial debt value are small in our model when we hold the other financing decisions fixed. When the firm fully re-optimizes its capital structure under payout covenant protection, the gains are larger, as the comparison between the second and third rows illustrates. The firm chooses to issue significantly more term debt under covenant protection, with the coupon increasing from $b = 0.080$ to 0.101 . The payout boundary is also significantly higher ($\bar{W} = 0.494$ instead of 0.373). This results in a higher debt capitalization at issuance ($D_0 = 1.666$ instead of 1.331), a higher initial cash buffer

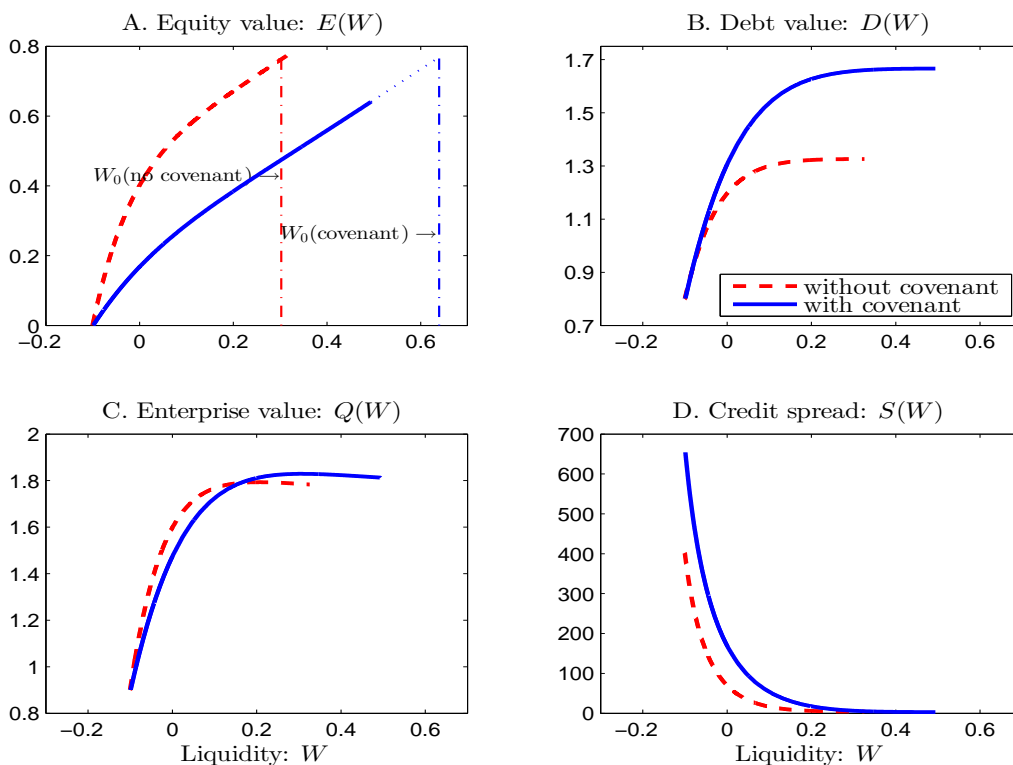


Figure 13: **Impact of debt covenant.** This figure plots the model solution when the financial policies are chosen to maximize the value of the firm instead of the value of equity.

($W_0 = 0.639$ instead of 0.308), and higher leverage ($L_0 = 0.684$ instead of 0.635).

The effects of the payout covenant can be seen in greater detail in Figure 13. Panels A and B reveal that the main effect of the covenant is a transfer of ex-post value from equity to debt. Panel C shows that the covenant increases enterprise value $Q(W)$ only for high values of W , and then not by much. Finally, Panel D illustrates a result that is surprising at first sight: the credit spread for debt protected by covenants is higher. In other words, covenant-protected debt appears to be riskier after controlling for the firm's financial slack. However, this is only due to the fact that the firm chooses to be more highly levered when its debt is protected with a covenant. Still, this result is a warning for non-structural empirical studies seeking to determine the value of a covenant based on corporate debt pricing data.

In sum, the most significant effect of the covenant is to substantially increase debt

value and optimal leverage.

8 Depreciation Shocks and Depreciation Allowances

We now extend the model in another important direction by introducing capital depreciation and depreciation-tax allowances. In reality, capital depreciates stochastically over time. For example, a machine may operate smoothly for some time interval and suddenly break down. At that point it must be repaired or replaced, which means that the firm needs to incur capital expenditure I to keep the asset productive and continue its operations. In addition, the firm will be granted depreciation-tax allowances, which should reflect the average life and replacement cost of the asset.

To capture such depreciation dynamics we now introduce into the model a depreciation shock, which is in the form of a lumpy capital expenditure I and are timed according to a Poisson arrival process with mean arrival rate ζ . For illustrative purposes, we fix I to be a constant. The firm may want to set aside depreciation allowances and increase its liquidity reserve in anticipation of such depreciation shocks. Additionally, these depreciation allowances are tax deductible. Specifically, we model depreciation-tax allowances by letting the firm subtract the expected depreciated amount of capital, ζI , from its taxable earnings in every period.

The depreciation shock is in effect a large liquidity shock to the firm, as in [Holmstrom and Tirole \(1997\)](#). Such a large liquidity shock will have qualitatively different effects on the firm's liquidity and capital structure decisions from the small diffusive shocks (the Brownian shocks in equation (1)) we have considered so far.

We continue to maintain the assumption that there is no new external financing available after $t > 0$, so that the firm has to use either cash or credit line to meet the required capital expenditure I . Following the realization of a depreciation shock the firm can continue to operate as long as it has sufficient liquidity to cover the capital expenditure I , i.e., as long as $W \geq W^* \equiv I - C$. If the firm has insufficient liquidity, it is forced to liquidate. Without loss of generality, we focus on the case where $W^* > 0$ (or $I > C$).

In addition, we restrict our attention to the case where the firm's liquidation value is sufficiently low relative to the debt issued such that the equity value will be 0 at liquidation ($L - C > P$), but it is sufficiently high relative to the credit-line limit C and the size of the depreciation shock to ensure the credit line to be risk-free ($L - C - I > 0$).

In the presence of real depreciation shocks, equity value $E(W)$ satisfies the following ODE in the region $W^* \leq W \leq \bar{W}$:

$$(1 - \tau_i) r E(W) = [(1 - \tau_c) (\mu + (r - \lambda)W - \nu(C)C - b) + \tau_c \zeta I] E'(W) \quad (38)$$

$$+ \frac{1}{2} \sigma^2 (1 - \tau_c)^2 E''(W) + \zeta (E(W - I) - E(W)).$$

The ODE (38) differs from (18) in the baseline model in two aspects: (i) the depreciation-tax allowance $\tau_c \zeta I$ in the coefficient for $E'(W)$, and (ii) the last term $\zeta (E(W - I) - E(W))$ which reflects the impact of depreciation shocks on equity value.

Next, in the region $0 \leq W < W^*$, the ODE for $E(W)$ becomes:

$$(1 - \tau_i) r E(W) = [(1 - \tau_c) (\mu + (r - \lambda)W - \nu(C)C - b) + \tau_c \zeta I] E'(W) \quad (39)$$

$$+ \frac{1}{2} \sigma^2 (1 - \tau_c)^2 E''(W) + \zeta (0 - E(W)).$$

The ODE (39) differs from (38) in that the arrival of a depreciation shock in this region will force immediate liquidation and drive the equity value to zero.

Finally, if $W < 0$, the ODE for $E(W)$ is given by:

$$(1 - \tau_i) r E(W) = [(1 - \tau_c) (\mu + (r + \delta)W - \nu(C)(W + C) - b) + \tau_c \zeta I] E'(W) \quad (40)$$

$$+ \frac{1}{2} \sigma^2 (1 - \tau_c)^2 E''(W) + \zeta (0 - E(W)).$$

The boundary conditions for $E(W)$ at the endogenous payout boundary $W = \bar{W}$ and the liquidation boundary $W = -C$ are identical to those in the baseline model, as given by equations (20) and (23). The fact that $E(W)$ is continuously differentiable at $W = 0$ and $W = W^*$ gives us four additional conditions. Finally, the same super-contact condition

(21) determines the optimal payout boundary \bar{W} .

Similarly, the value of debt $D(W)$ now satisfies the following ODE in the region $W^* \leq W \leq \bar{W}$:

$$(1 - \tau_i)rD(W) = (1 - \tau_i)b + [(1 - \tau_c)(\mu + (r - \lambda)W - \nu(C)C - b) + \tau_c\zeta I] D'(W) + \frac{1}{2}\sigma^2(1 - \tau_c)^2 D''(W) + \zeta(D(W - I) - D(W)). \quad (41)$$

In the region $0 \leq W < W^*$, the ODE for $D(W)$ is:

$$(1 - \tau_i)rD(W) = (1 - \tau_i)b + [(1 - \tau_c)(\mu + (r - \lambda)W - \nu(C)C - b) + \tau_c\zeta I] D'(W) + \frac{1}{2}\sigma^2(1 - \tau_c)^2 D''(W) + \zeta(L - I + W - D(W)). \quad (42)$$

The last term follows from the fact that when a depreciation shock arrives for $W < W^*$, the value of debt is equal to the full liquidation value L net of the depreciation capital expenditure I plus the remaining cash (or minus the credit line $-W$ if $W < 0$). Finally, when $W < 0$, the ODE for $D(W)$ is:

$$(1 - \tau_i)rD(W) = (1 - \tau_i)b + [(1 - \tau_c)(\mu + (r + \delta)W - \nu(C)(W + C) - b) + \tau_c\zeta I] D'(W) + \frac{1}{2}\sigma^2(1 - \tau_c)^2 D''(W) + \zeta(L - I + W - D(W)). \quad (43)$$

The boundary conditions for $D(W)$ at the liquidation boundary $W = -C$ and the payout boundary $W = \bar{W}$ are the same (26) and (27). The fact that $D(W)$ is continuously differentiable at $W = 0$ and $W = W^*$ also gives us four additional conditions.

We calibrate the model with depreciation shocks by setting the size of the shock to $I = 0.25$ and the shock intensity to $\zeta = 0.15$. These parameter choices imply that the large depreciation shocks happen on average once per 6.7 years, and the size of these shocks is 2.5 times the annualized volatility of cash flows σ in the baseline model. In order to make the model with depreciation shocks more comparable with the baseline model, we compensate for the impact of the depreciation shocks on firm value by increasing the mean ROA μ in the baseline model by the expected annual depreciation capital expenditure ζI .

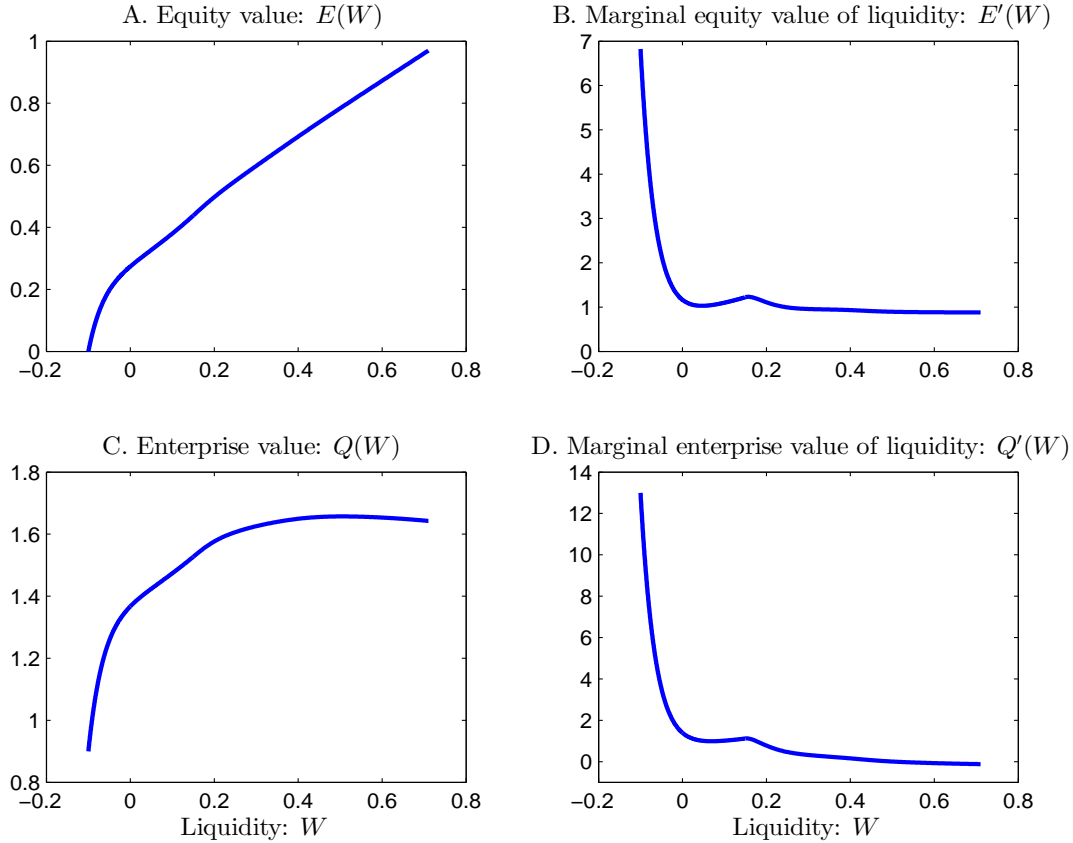


Figure 14: **Stochastic depreciation shocks.** This figure plots the equity value $E(W)$, the enterprise value $Q(W)$, and their derivatives for the model with depreciation shocks.

The remaining parameters take the baseline parameter values in Table 1.

The model solution is plotted in Figure 14. The main qualitative change induced by the depreciation shock is the appearance of *local convexity* of the equity value $E(W)$ around $W^* = 0.15$. As Panel B shows, $E'(W)$ is increasing to the left and decreasing to the right of W^* . This local convexity of $E(W)$ is due to the fact that the firm will be forced to liquidate and the equity value will drop to zero whenever the depreciation shock hits on the left side of W^* . Intuitively, it reflects the well-known benefit of *gambling for resurrection* for equity holders when W is close to W^* . If W is sufficiently far away from W^* , the precautionary motive for holding cash dominates again, either because the firm has no hope of surviving a large depreciation shock or because it is not concerned about surviving the shock, which makes $E(W)$ concave as in the baseline model.

Another model that generates a similar local convexity pattern is the model of a financially constrained firm with lumpy investment of [Hugonnier, Malamud, and Morellec \(2013\)](#). Such a non-concave region, of course, has important implications for the firm's optimal hedging policy. It is beyond the scope of this paper to provide a detailed analysis of optimal hedging as in [Bolton, Chen, and Wang \(2011, 2013\)](#). However, based on the insights of that analysis we can infer that the firm will optimally switch from a hedging policy seeking to reduce cash-flow volatility when W is close to $-C$ and when W is sufficiently larger than W^* , to a policy seeking to load up on cash-flow volatility when W is in a neighborhood of W^* .

Table 6 reports how the firm's financing policy is affected by the presence of depreciation shocks and depreciation tax allowances. The implications of the presence of depreciation tax shields for the static tradeoff theory have been explored by [DeAngelo and Masulis \(1980\)](#). Their main conclusion is that depreciation tax shields are a *substitute* for debt tax shields and that higher depreciation tax allowances should induce firms to issue less debt. The reason is that the tax shield benefits of debt are less fully realized when depreciation tax allowances increase, and since debt is tax disadvantaged at the personal level the firm optimally responds by reducing debt. This substitutability of debt and depreciation tax shields is also true in our dynamic model under the Miller benchmark for financially unconstrained firms.

However, as can be seen in Table 6, for a financially constrained firm, depreciation tax shields can become a *complement* to debt tax shields. By comparing the optimal coupon b when there is no depreciation tax allowance in the third row ($b = 0.071$) to the optimal coupon when there is depreciation tax allowance in the second row ($b = 0.087$), one immediately observes that the firm increases debt (and reduces outside equity) when it can take advantage of a depreciation tax allowance. This surprising result is again explained by the effects of the depreciation tax allowance on liquidity and the servicing costs of debt. Thanks to the depreciation tax allowance, the firm can retain a higher fraction of after-tax earnings and therefore can afford to pay higher coupon without increasing the risk of liquidation. This in turn induces the firm to issue more debt (and less outside equity).

Table 6: **Capital depreciation shocks.** This table reports the results from adding capital depreciation shocks.

	credit line	coupon rate	outside equity	payout bound	initial cash	project value	debt value	interest coverage	market leverage
	C	b	a	\bar{W}	W_0	J_0	D_0	μ/b	L_0
no shock	0.100	0.080	0	0.326	0.303	0.763	1.326	1.497	0.635
with allowance	0.100	0.087	0.129	0.711	0.462	0.662	1.393	1.377	0.647
no allowance	0.102	0.071	0.469	0.671	0.397	0.345	1.132	1.689	0.635

Table 6 also shows how large liquidity shocks (in the form of real depreciation shocks) affect the firm's financial policy. Comparing rows one and two we observe that the firm responds to the threat of large liquidity shocks by: *i*) raising more initial funds (W_0 increases from 0.303 to 0.462); *ii*) holding more cash (\bar{W} increases from 0.362 to 0.711), and *iii*) relying more on outside equity (a increases from 0 to 0.129). Interestingly, the firm's market leverage and book leverage (as measured by the interest coverage) actually increase in the presence of large liquidity shocks (L_0 increases from 0.635 to 0.647 and interest coverage drops from 1.497 to 1.377). The reason again is to be found in the adjustment in the firm's cash policy, which both reduces the servicing costs of debt and increases the market value of debt, so that the firm is induced to issue more term debt.

9 Recurrent External Equity Financing

When the project's expected productivity μ is high and the fixed cost of external financing Φ is low, the firm will want to raise fresh external funds rather than force the project into early liquidation when it runs out of cash. We now analyze this situation by giving the firm the option to raise new funds via a subscription rights offering whenever it runs out of cash. Under the rights offering new equity is allocated to existing shareholders in proportion to their ownership, so that the entrepreneur's initial ownership stake $(1 - a)$

remains unchanged.¹⁴

Let $M > 0$ denote the total amount of liquidity after the new equity issuance. We shall refer to M as the liquidity return point after equity issuance. This amount is chosen to maximize the total value of equity $E(W)$. Given that equity value is continuous before and after the rights offering, the following boundary condition for $E(W)$ must hold at the boundary $W = -C$:

$$E(-C) = E(M) - \Phi - \frac{M + C}{1 - \gamma_E}. \quad (44)$$

Note that an equity issuance only occurs when the firm exhausts its credit line, since the firm has to pay a commitment fee on any unused credit line, and there is no benefit of replenishing the liquidity reserve early. As M represents the post-issue amount of liquidity, the total (net) amount issued is $M + C$. The right-hand side of (44) represents the post rights-issue equity value minus both the fixed and the proportional costs of equity issuance. Second, since the return point M is optimally chosen, the marginal value of the last dollar raised must be equal to the marginal cost of funds $1/(1 - \gamma_E)$. This gives the following smooth-pasting boundary condition at M :

$$E'(M) = \frac{1}{1 - \gamma_E}. \quad (45)$$

The firm's equityholders will only choose the refinancing option if

$$E(-C) = E(M^*) - \Phi - \frac{M^* + C}{1 - \gamma_E} \geq 0, \quad (46)$$

where M^* satisfies the optimality condition given in (45). Given that $E(M^*)$ is a decreasing function of the coupon payment b , condition (46) puts an upper bound on how much term debt the firm can issue at time 0 while credibly committing to permanently servicing this debt. Any coupon below this upper bound will always be serviced, as the firm will always prefer to raise new equity when it runs out of cash. Thus, any such term debt will be safe

¹⁴This assumption is mainly for tractability, as it does not cause future equity dilution. Allowing for equity issuance towards new investors will generate qualitatively the same results.

Table 7: **Comparative statics: Refinancing case.** This table reports the results from comparative statics on the financing cost parameters in the refinancing case.

	credit line	coupon rate	payout bound	return point	initial cash	project value	debt value	interest coverage	market leverage
	C	b	\bar{W}	M	W_0	J_0	D_0	μ/b	L_0
baseline	0.101	0.107	0.271	0.000	0.762	0.803	1.790	1.117	0.690
$\lambda = 2\%$	0.243	0.098	0.069	-0.146	0.602	0.788	1.629	1.228	0.674
$\Phi = 5\%$	0.099	0.103	0.324	0.079	0.652	0.757	1.720	1.163	0.694
$\sigma = 20\%$	0.088	0.103	0.620	0.062	0.692	0.754	1.719	1.163	0.695
$\gamma_D = 6\%$	0.150	0.077	0.131	-0.078	0.190	0.733	1.277	1.566	0.635

and will be valued at $D = b/r$.

Table 7 describes the firm’s optimal financial policy and market value under five different parameterizations. The first row is the baseline case for which we have in particular: $\lambda = 0.5\%$, $\Phi = 1\%$, $\sigma = 10\%$ and $\gamma_D = 1\%$. In the other four parameterizations we change one parameter value at a time: in the second row we increase the cash carry cost to $\lambda = 2\%$; in the third row we increase the fixed issuance cost to $\Phi = 5\%$; in the fourth row we increase volatility to $\sigma = 20\%$, and in the fifth row we increase the marginal debt issuance cost to $\gamma_D = 6\%$.

Under the first four parameterizations the firm chooses the maximum feasible coupon b that satisfies the constraint (46). That is, the firm prefers to issue only safe debt that it can credibly service forever. The debt is low enough that the firm’s equityholders (weakly) prefer to raise new funds through an equity issue rather than liquidate the firm whenever the firm runs out of cash. Thus, issuance of only safe debt is a possible prediction of the dynamic tradeoff theory when the firm faces external financing costs.

When marginal debt issuance costs are high (e.g, when $\gamma_D = 6\%$ in our final parametrization), it becomes optimal for the firm not to exhaust even its safe debt capacity. In this case, the firm’s optimal coupon b is such that the constraint (46) is slack. A major critique of the tradeoff theory has been that it must be false since: “If the theory is right, a value-maximizing firm should never pass up interest tax shields when the probability of

financial distress is remotely low.” (Myers (2001)). But our analysis illustrates that this critique only applies to financially unconstrained firms. When firms face external financing costs, the optimal tradeoff between tax shield benefits of debt and servicing costs may well be obtained with safe debt.

In the second row of Table 7, we examine the case when cash-carrying costs are significantly higher than under the baseline parametrization ($\lambda = 2\%$). Other things equal, an increase in λ results in a lower equity value so that the firm’s riskfree debt capacity as implied by constraint (46) drops. But the main effect of an increase in λ is an overall reduction in cash holdings as illustrated by: *i*) the sharp reduction in \bar{W} ; *ii*) the increase in the credit line limit C , and *iii*) the reduction in the amount of funds raised M when the firm runs out of cash.

The third row of Table 7 considers the effects of an increase in the fixed cost of external funding to $\Phi = 5\%$. The main effect of this change is to push the firm to avoid reliance on external funding even more by: *i*) raising more funds when it is forced to return to capital markets (a higher M), and; *ii*) holding on to cash for longer (a higher \bar{W}). Finally, an increase in cash-flow volatility to $\sigma = 20\%$ has similar effects, as reported in the fourth row of Table 7.

In sum, when the firm is sufficiently profitable that raising new funds (when it runs out of cash) is preferable to liquidation for the founders of the firm at time 0, then the firm optimally chooses to issue only safe debt at time 0 and restricts its debt obligations so as to avoid a *debt overhang* situation that could induce equityholders to prefer liquidation ex post.

10 Conclusion

Although the tax-advantage of debt has long been recognized as an important consideration for corporate financial policy, the tradeoff theory has had an uncertain standing, with many empirical studies concluding that it is flat-out rejected by the data. We have shown that one reason why the tradeoff theory performs poorly empirically is that it only applies to

financially unconstrained firms. In the presence of external financing costs, firms' financial policy is more complex and involves both a liability and asset management dimension. Thus, when there is a change in tax policy, for example, financially constrained firms generally have two margins along which they can respond: they can either adjust their debt policy or their cash policy (or both).

As we have shown, the cash management dimension of corporate financial policy radically modifies the classical tradeoff theory. So much so that the theory for unconstrained firms provides very misleading predictions for corporate financial policies of constrained firms. For example, an important new cost of debt financing for financially constrained firms is the *debt servicing cost*: interest payments drain the firm's valuable precautionary cash holdings and thus impose higher expected external financing costs on the firm. Interest payments may help shield earnings from corporate taxation, but they potentially induce inefficient liquidation or costly external financing. Because liquidity/cash is valuable for a financially constrained firm, the firm thus has an optimal portfolio choice among external equity, debt, and liquidity, which includes both cash and a credit line. We have shown that *cash is not negative debt* and offered a novel explanation for the "*debt conservatism puzzle*" highlighted in the empirical literature testing the static tradeoff theory by showing that financially constrained firms choose to limit their debt and interest expenses in order to preserve their cash holdings and to avoid a debt overhang situation which would reduce equityholders' incentives to raise new funds ex post.

Another important change introduced by external financing costs is that realized earnings are generally separated in time from payouts to shareholders, implying that the classical Miller-formula for the net tax benefits of debt no longer holds. We show that the standard Miller effective tax rate calculation only applies at the payout boundary. In the interior region the tax calculation is very different given that the firm defers its payment to shareholders, so that the standard Miller formula does not apply. We further show that the firm may not even exhaust its risk-free debt capacity because servicing debt can be expensive. Finally, our model is a valuation model for debt and equity in the presence of taxes and external financing costs. It shows how the classical *adjusted present value*

methodology breaks down for financially constrained firms, as it does not account for the value of cash for both debt and equity.

Finally, our analysis of the tax implications for corporate financial policies of financially constrained firms is also relevant to the fiscal policy debate on changing the corporate tax rate. We have shown that the main effect of a reduction of the corporate income tax rate from 35% to 25% is to substantially increase corporate savings and the value of equity, but to leave corporate debt policies almost unchanged. Our analysis suggests that the lower corporate tax rate would not only lower corporate tax revenues but also potentially personal tax revenues from investment income in light of the stronger incentives to save inside the firm.

Two major simplifications of our baseline analysis are that: *i*) the firm only faces i.i.d. cash-flow shocks; and *ii*) there is only one fixed productive asset. These assumptions mainly help make the analysis very transparent. These can be relaxed, as the analysis in Section 8 introducing a jump liquidity risk in the form of a real depreciation shock à la [Holmstrom and Tirole \(1997\)](#) illustrates. In future research, we plan to extend our framework further to incorporate both persistent and potentially permanent productivity shocks and also ongoing corporate investment.

References

- Acharya, V., S. A. Davydenko, and I. A. Strebulaev, 2012, “Cash Holdings and Credit Risk,” *Review of Financial Studies*, 25, 3572–3609.
- Acharya, V. V., H. Almeida, and M. Campello, 2007, “Is cash negative debt? A hedging perspective on corporate financial policies,” *Journal of Financial Intermediation*, 16, 515–554.
- Acharya, V. V., H. Almeida, and M. Campello, 2013, “Aggregate Risk and the Choice between Cash and Lines of Credit,” *The Journal of Finance*, 68, 2059–2116.
- Altinkilic, O., and R. S. Hansen, 2000, “Are There Economies of Scale in Underwriting Fees? Evidence of Rising External Financing Costs,” *Review of Financial Studies*, 13, 191–218.
- Bates, T. W., K. M. Kahle, and R. M. Stulz, 2009, “Why Do U.S. Firms Hold So Much More Cash than They Used To?,” *Journal of Finance*, 64, 1985–2021.
- Bolton, P., H. Chen, and N. Wang, 2011, “A Unified Theory of Tobin’s q , Corporate Investment, Financing, and Risk Management,” *Journal of Finance*, 66, 1545–1578.
- Bolton, P., H. Chen, and N. Wang, 2013, “Market Timing, Investment, and Risk Management,” *Journal of Financial Economics*, 109, 40–62.
- Bolton, P., and D. S. Scharfstein, 1990, “A Theory of Predation Based on Agency Problems in Financial Contracting,” *American Economic Review*, 80, 93–106.
- Collins, J. H., and D. Kemsley, 2000, “Capital Gains and Dividend Taxes in Firm Valuation: Evidence of Triple Taxation,” *The Accounting Review*, 75, 405–427.
- DeAngelo, H., L. DeAngelo, and T. Whited, 2011, “Capital Structure Dynamics and Transitory Debt,” *Journal of Financial Economics*, 99, 235–261.
- DeAngelo, H., and R. W. Masulis, 1980, “Optimal capital structure under corporate and personal taxation,” *Journal of Financial Economics*, 8, 3–29.

- Decamps, J., T. Mariotti, J. Rochet, and S. Villeneuve, 2011, “Free Cash Flow, Issuance Costs, and Stock Prices,” *Journal of Finance*, 66, 1501–1544.
- DeMarzo, P. M., and M. J. Fishman, 2007, “Optimal Long-Term Financial Contracting,” *Review of Financial Studies*, 20, 2079–2128.
- DeMarzo, P. M., and Y. Sannikov, 2006, “Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model,” *Journal of Finance*, 61, 2681–2724.
- Fama, E. F., and K. R. French, 2012, “Capital Structure Choices,” *Critical Finance Review*, 1, 59–101.
- Fischer, E. O., R. Heinkel, and J. Zechner, 1989, “Dynamic Capital Structure Choice: Theory and Tests,” *Journal of Finance*, 44, 19–40.
- Frank, M. Z., R. Singh, and T. Y. Wang, 2010, “Personal Income Taxes and Corporate Investment,” Working Paper, University of Minnesota.
- Gamba, A., and A. Triantis, 2008, “The Value of Financial Flexibility,” *Journal of Finance*, 63, 2263–2296.
- Goldstein, R., N. Ju, and H. Leland, 2001, “An EBIT-Based Model of Dynamic Capital Structure,” *Journal of Business*, 74, 483–512.
- Graham, J. R., 1996, “Debt and the marginal tax rate,” *Journal of Financial Economics*, 41, 41–73.
- Graham, J. R., 2000, “How Big Are the Tax Benefits of Debt?,” *Journal of Finance*, 55, 1901–1941.
- Graham, J. R., 2003, “Taxes and Corporate Finance: A Review,” *Review of Financial Studies*, 16, 1075–1129.
- Harris, T.-S., and D. Kemsley, 1999, “Dividend Taxation in Firm Valuation : New Evidence,” *Journal of Accounting Research*, 37, 275–291.

- Hart, O., and J. Moore, 1994, “A theory of debt based on the inalienability of human capital,” *Quarterly Journal of Economics*, 109, 841–879.
- Hart, O., and J. Moore, 1998, “Default And Renegotiation: A Dynamic Model Of Debt,” *The Quarterly Journal of Economics*, 113, 1–41.
- Hennessy, C. A., and T. M. Whited, 2005, “Debt Dynamics,” *Journal of Finance*, 60, 1129–1165.
- Hennessy, C. A., and T. M. Whited, 2007, “How Costly Is External Financing? Evidence from a Structural Estimation,” *Journal of Finance*, 62, 1705–1745.
- Holmstrom, B., and J. Tirole, 1997, “Financial Intermediation, Loanable Funds, and the Real Sector,” *The Quarterly Journal of Economics*, 112, 663–91.
- Hugonnier, J., S. Malamud, and E. Morellec, 2013, “Capital Supply Uncertainty, Cash Holdings, and Investment,” Working Paper, Swiss Finance Institute.
- Jensen, M. C., and W. H. Meckling, 1976, “Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure,” *Journal of Financial Economics*, 3, 305–360.
- Karatzas, I., and S. E. Shreve, 1991, *Brownian Motion and Stochastic Calculus*, Springer-Verlag, New York, 2nd edn.
- Kim, C.-S., D. C. Mauer, and A. E. Sherman, 1998, “The Determinants of Corporate Liquidity: Theory and Evidence,” *Journal of Financial and Quantitative Analysis*, 33, 335–359.
- Kimball, M., and N. Mankiw, 1989, “Precautionary saving and the timing of taxes,” *Journal of Political Economy*, 97, 863–879.
- Leary, M. T., and M. R. Roberts, 2005, “Do Firms Rebalance Their Capital Structures?,” *Journal of Finance*, 60, 2575–2619.
- Leland, H., 1994, “Corporate Debt Value, bond covenants, and Optimal Capital Structure,” *Journal of Finance*, 49, 1213–1252.

- Leland, H., 1998, "Agency Costs, Risk Management, and Capital Structure," *Journal of Finance*, 53, 1213–1243.
- Lemmon, M. L., M. R. Roberts, and J. F. Zender, 2008, "Back to the Beginning: Persistence and the Cross-Section of Corporate Capital Structure," *Journal of Finance*, 63, 1575–1608.
- Merton, R. C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, 449–470.
- Miller, M. H., 1977, "Debt and Taxes," *Journal of Finance*, 32, 261–275.
- Miller, M. H., and M. S. Scholes, 1978, "Dividends and taxes," *Journal of Financial Economics*, 6, 333–364.
- Myers, S. C., 1974, "Interactions of Corporate Financing and Investment Decisions—Implications for Capital Budgeting," *Journal of Finance*, 29, 1–25.
- Myers, S. C., 1984, "The Capital Structure Puzzle," *Journal of Finance*, 39, 574–592.
- Myers, S. C., 2001, "Capital Structure," *Journal of Economic Perspectives*, 15, 81–102.
- Rauh, J. D., 2009, "Risk Shifting versus Risk Management: Investment Policy in Corporate Pension Plans," *Review of Financial Studies*, 22, 2487–2533.
- Riddick, L. A., and T. M. Whited, 2009, "The Corporate Propensity to Save," *Journal of Finance*, 64, 1729–1766.
- Ross, S. A., 2005, "Capital Structure and the Cost of Capital," *Journal of Applied Finance*, 15, 5–23.
- Strebulaev, I. A., 2007, "Do Tests of Capital Structure Theory Mean What They Say?," *Journal of Finance*, 62, 1747–1787.
- Sufi, A., 2009, "Bank Lines of Credit in Corporate Finance: An Empirical Analysis," *Review of Financial Studies*, 22, 1057–1088.