Creditor-on-Creditor Violence*

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Abstract

Distressed firms often raise liquidity by stripping collateral from syndicated-loan creditors and pledging it to new lenders—an emerging practice known as "creditor-on-creditor violence." We model this phenomenon. An insolvent firm needs liquidity to avoid bankruptcy but cannot observe potential investors' required returns. This information asymmetry leads to inefficient bankruptcies in equilibrium. This inefficiency resembles debt overhang: equity holders negotiate selfishly, potentially blocking surplus-enhancing financing, to reduce wealth transfers to existing creditors. By diverting collateral from existing creditors, equity holders capture more surplus from financing, aligning incentives and improving efficiency. Our theory explains why new debt contracts still permit creditor-on-creditor violence.

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1 Introduction

Revlon, a leading beauty company, was struggling financially in 2020. Revlon had substantial secured term loan debt and declining sales. In May 2020, Revlon's problems led to an innovative debt restructuring. A coalition holding just over 50% of Revlon's secured term loan consented to amend their credit agreement. The amendment allowed Revlon to "dropdown" the term loan collateral into a new subsidiary that was not covered by the credit agreement. The coalition lent Revlon $910 million in new funds and rolled over $950 million of their old term loan, receiving a new debt instrument secured by the newly available collateral. Thus, Revlon raised $910 million in liquidity using this new financing and the participating coalition maintained their secured priority. Crucially, the nonparticipating term loan holders, who held just under 50% of the debt, lost their collateral rights in default. The excluded investors suffered a massive loss: the secondary-market price of the loan fell from 80% to 28% of par value (Mugford and Gulick, 2022). While the transaction was fervently disputed and litigated, it was ultimately upheld in court as valid (Ellis, Margolis, and Yeh, 2023).

Practitioners have divided views on these transactions, which have become increasingly common in the last decade (Credit Sights, 2025). Lenders and other practitioners call this "creditor-on-creditor violence." They point out that many of these barely legal transactions are quickly followed by bankruptcies (e.g., Revlon). These critics believe this trend is harmful, eroding trust in the corporate borrowing system. Firms like Revlon euphemistically call these restructurings liability management exercises ("LMEs"). They argue these restructurings are a beneficial way to increase liquidity and prevent bankruptcies. Supporting the proponents' view, Buccola and Nini (2024) study recent loan contracts and show that lenders knowingly enable Revlon-style dropdown LMEs: "Contracts could adjust to prevent dropdowns but did not." Both before and after dropdowns became common, roughly half of contracts enable them, suggesting that LMEs can create value.

To reconcile the conflicting views, we develop a model of how LMEs affect both ex-post and ex-ante firm value. Our analysis shows that both critics and proponents of LMEs are partially correct. On the one hand, LMEs increase the cost of secured debt, since creditors anticipate possible future mistreatment. On the other hand, LMEs can prevent some costly bankruptcies that traditional debt renegotiation could not avert. Thus, enabling LMEs in debt contracts improves total surplus ex ante, in spite of higher borrowing costs.

Intuitively, when an insolvent firm requires liquidity to avoid a costly bankruptcy, a standard debt-overhang problem (Myers, 1977) arises: equity holders do not internalize the benefit of averting bankruptcy, since bankruptcy costs fall primarily on debt holders. In principle, this problem could be resolved through efficient renegotiation between the firm and its creditors. However, even in settings where a concentrated group of hedge funds holds most of the term-loan debt, asymmetric information hinders such negotiations. Because equity holders cannot observe creditors' required returns, another form of debt overhang arises: equity holders offer only limited compensation for new liquidity, risking rejection when creditors' required returns are high, because equity holders fail to internalize the value destroyed by a breakdown in negotiations. We show that LMEs mitigate this problem. By enabling equity holders to transfer value from existing creditors to new liquidity providers, LMEs give equity holders more incentives to ensure successful financing. In this way, LMEs reduce debt overhang by aligning equity's interests more closely with firm value maximization.

Our main contribution is to provide a theoretical formalization of this intuition. At the outset of our three-period model, a firm issues a secured term loan. In the first period, the firm chooses whether to enable LMEs in the debt contract. Competitive bank lenders price the loan fairly, given rational expectations about the firm's future behavior. In the second period, there is a chance that a shock hits the firm, causing distress. A distressed firm is insolvent and must raise additional liquidity. Otherwise, the firm must immediately file for bankruptcy. If the firm raises liquidity, it gains a chance of avoiding bankruptcy; however, in the third period, some firms that obtain financing still fail and enter bankruptcy. While bankruptcy may create value in certain cases, we focus on firms for which bankruptcy entails significant deadweight costs.[1] Accordingly, in most of our analysis, we assume that raising liquidity is the efficient outcome, as it maximizes total ex-ante surplus.[2]

As in practice, we assume that the original bank lenders sell the distressed firm's debt to specialized investors such as hedge funds. While the firm could in principle approach outside investors for liquidity, we prove that it is optimal to seek financing from its existing debt holders. We make two key assumptions about these investors. First, providing liquidity to distressed firms requires specialized expertise. The set of distressed-debt investors is therefore different from the banks who invest in the first period, and they earn excess returns due to their expertise. Heterogeneity in outside options implies that some investors require high returns to supply liquidity. Second, each investor's required return is private information.[3] This informational asymmetry between investors and the distressed firm is the key friction that prevents efficient renegotiation.

To begin our analysis, we consider a firm that bans LMEs in its debt contract. In the resulting subgame-perfect equilibrium, equity holders make a take-it-or-leave-it offer to raise liquidity from the existing debt holders (e.g., hedge funds).[4] Each investor privately observes its own required return (or equivalently their funding costs) and forms rational expectations about the strategies and types of other investors. In a subgame, investors' simultaneous accept–reject decisions constitute a Nash equilibrium. With simultaneous moves, a free-rider problem arises: each investor has an incentive to withhold funding in the hope that another investor will supply liquidity to avert a bankruptcy. However, unlike in widely-held-bond restructurings, the small number of term-loan holders mitigates this problem: each

[1]For example, Antill (2022) shows that agency conflicts in Chapter 11 bankruptcy lead to inefficient liquidations and acquisitions that destroy 14% of firm value in expectation (see also Ayotte and Morrison (2009)). Antill and Hunter (Forthcoming) show that indirect costs from lost customers amount to 12%-15% of firm value in large public Chapter 11 cases. Antill (2024) shows that inefficiently high professional fees destroy 16% of value in smaller Chapter 7 liquidations.

[2]In Subsection 4.4, we instead assume that raising liquidity is inefficient. We show that LMEs worsen efficiency. This explains why 50% of firms choose to ban dropdowns (Buccola and Nini, 2024).

[3]In Internet Appendix G, we prove that our results hold under an alternative assumption: required returns are zero, but hedge funds privately observe signals about the probability that raising liquidity prevents bankruptcy.

[4]For tractability, we assume that equity holders have only a single opportunity to make an offer. This timing reflects a realistic assumption that liquidity must be raised within a short time window.

investor faces a substantial probability of being pivotal if its few competitors demand high returns. Moreover, while investors have a right to collateral in bankruptcy, they anticipate that they could be undersecured in a bad state and suffer losses in bankruptcy. This provides further incentives for ex post renegotiation.

Moving backward in time, equity holders anticipate how investors will respond to any potential offer. Because they do not observe investors' required returns, equity holders face a tradeoff: offering a high return increases the likelihood of raising liquidity, whereas offering a low return preserves more value for equity if investors' required returns turn out to be low. We show that equity holders optimally offer a low return ex post—risking deal failure—if and only if leverage exceeds a threshold. This behavior resembles debt overhang: equity holders underinvest in raising liquidity, setting a low interest rate and risking bankruptcy, because part of the surplus from avoiding bankruptcy accrues to debt holders. This underinvestment persists even when debt holders provide liquidity because of asymmetric information: equity holders cannot observe investors' required returns and therefore they cannot extract the entire surplus.

Stepping further back to the first period, lenders charge a high interest rate because the possibility of an ex-post inefficient bankruptcy depresses the price that hedge funds are willing to pay for distressed debt. In this sense, equity holders suffer ex-ante from their ex-post inability to raise liquidity. Our first result, therefore, is that asymmetric information prevents distressed firms from efficiently renegotiating with debt holders to avoid bankruptcy.[5]

Next, we introduce LMEs by modifying the model that we just analyzed: if a majority of investors agree to an LME, then investors who reject the offer forfeit their collateral rights. This loss of collateral is a defining feature of most LMEs (e.g., Revlon), as we describe in Section 2. Following the steps outlined above, we construct a subgame-perfect equilibrium with LMEs. For any fixed return that equity holders offer on new liquidity, investors are more willing to accept under an LME. The logic is straightforward: rejecting is costlier in an

[5] Note that this first result is similar to results in the literature (e.g., Giammarino (1989) and the "collateral hangover" modeled in Donaldson, Gromb, and Piacentino (2021)); we discuss this below.

LME, since a rejecting investor receives zero recovery if the firm subsequently goes bankrupt after the LME. Crucially, moving backward in time, we show that LMEs make equity holders more willing to offer a return that guarantees investor acceptance. Intuitively, LMEs enable equity holders to raise new liquidity by diverting collateral from the rejecting debt holders to the accepting debt holders. This diversion makes equity holders internalize more of the benefit of a successful deal with LMEs, so they are more willing to pay the necessary return to guarantee success. In this way, LMEs mitigate the debt-overhang problem in renegotiation. Stepping further back to period one, lenders charge a higher interest rate to compensate for future collateral diversion. Importantly, however, this is merely a transfer: the overall ex-ante surplus increases, as inefficient bankruptcies are avoided. For this reason, we prove that equity holders optimally enable LMEs ex ante.

To ease exposition, we develop our baseline model with LMEs in the simplest possible setting. Section 3 presents the assumptions and Section 4 presents the main results. Then, Section 5 shows the robustness of our main result and analyzes various extensions. Specifically, in Subsections 5.3 to 5.6, we show that the main result—LMEs mitigate the debt-overhang problem in renegotiation—is robust to a range of extensions: (i) allowing for any finite number of investors, (ii) accommodating a wide class of distributions for required returns, (iii) permitting optimally designed mechanisms to elicit investors' private information, (iv) introducing non–debt-holding investors who can provide liquidity, (v) introducing investors' heterogeneous beliefs, and (vi) incorporating unsecured debt issued ex ante to preserve collateral for raising liquidity in distress. Further, using various extensions of the baseline model in Section 3, we highlight three additional results. First, in Subsection 5.1, we show that firms benefit ex ante by banning a specific type of "non-pro-rata" LME, consistent with the empirical evidence in Buccola and Nini (2024). Second, in Subsection 5.2, we formalize how LMEs address a distinct friction from exit consents, a related device used in bond restructurings, and we prove that firms can benefit from LMEs even when exit consents are available. Third, in Subsection 5.7, using a dynamic tradeoff model with minimal features, we show that the availability of LMEs increases optimal leverage by reducing expected bankruptcy costs.

Finally, in Section 6, we incorporate LMEs into a continuous-time model of corporate investment and liquidity management. Comparing the model solution with LMEs to the solution with only traditional renegotiation, we show that LMEs reduce the risk of inefficient renegotiation failure. By reducing expected default costs, LMEs increase ex-ante firm value, leverage, and ex-post investment.

Contribution to the literature: The key novel results of our model are: (i) when asymmetric information impedes debt renegotiation, LMEs align the ex-ante and ex-post incentives of equity holders by transferring value to equity, thereby preventing inefficient bankruptcies; and (ii) because LMEs reduce expected bankruptcy costs ex post, equity holders improve ex-ante firm value by enabling LMEs ex ante, even when alternative debt renegotiation mechanisms are available.

We contribute to a long literature on distressed debt renegotiation.[6] None of these papers, however, study LMEs. From a theoretical perspective, our assumptions are closest to Giammarino (1989), Detragiache and Garella (1996), and Donaldson, Gromb, and Piacentino (2021), who show that distinct asymmetric-information frictions—regarding collateral values, firm fundamentals, or borrower creditworthiness—can impede renegotiation. In our institutional setting, asymmetric information is a more relevant friction than other frictions.[7] For example, term loans are typically held by only a few parties, making the

[6]See, for example, Bulow and Shoven (1978); White (1980); Roe (1987); Hart and Moore (1988); Giammarino (1989); Gertner and Scharfstein (1991); Aghion and Bolton (1992); Rajan (1992); Kahan and Tuckman (1993); Aghion, Dewatripont, and Rey (1994); Hart and Moore (1994); Berglöf and Von Thadden (1994); Chemmanur and Fulghieri (1994); Hart and Moore (1995); Bernardo and Talley (1996); Bolton and Scharfstein (1996); Detragiache and Garella (1996); Padilla and Pagano (1997); Mella-Barral and Perraudin (1997); Hart and Moore (1998); Bolton and Freixas (2000); Detragiache, Garella, and Guiso (2000); Fan and Sundaresan (2000); Hubert and Schäfer (2002); Carletti (2004); Hege and Mella-Barral (2005); Carletti, Cerasi, and Daltung (2007); Hackbarth, Hennessy, and Leland (2007); Garleanu and Zwiebel (2009); Von Thadden, Berglöf, and Roland (2010); Bannier (2010); Bolton and Oehmke (2011, 2015); Donaldson et al. (2020); Glode and Opp (2023).

[7]Asymmetric information is a relevant friction in many corporate finance settings: see, for example, Grenadier and Malenko (2011); Malenko and Tsoy (2025a).

holdout problem far less acute than in bond restructurings. Likewise, distressed term-loan borrowers avoid default by promising future cash flows to debt holders, which is inconsistent with a common incomplete-contracts assumption that future cash flows are not contractable (Bolton and Scharfstein, 1996; Hart and Moore, 1998). From a practical perspective, our setting is closely related to the literature on exit consents in bond restructurings (Gertner and Scharfstein, 1991; Kahan and Tuckman, 1993; Bernardo and Talley, 1996; Hege and Mella-Barral, 2005; Donaldson et al., 2020). Like LMEs, exit consents allow firms to transfer value away from creditors who refuse to participate in a restructuring. Yet there are two important differences. First, exit consents apply to bond restructurings, whereas LMEs are predominantly used to strip collateral from term loans through a novel legal innovation. Second, exit consents address a holdout problem relevant in bond markets, while LMEs mitigate a different friction that arises in term loans—asymmetric information about debt holders' required returns. In this sense, which we formalize in Subsection 5.2, our results on the role of LMEs in promoting efficiency are novel.

We also contribute to a long literature studying how debt contract features can mitigate debt overhang ex post.[8] Again, none of these papers study LMEs. We do not claim that LMEs are the only solution to the debt overhang problem that arises in debt renegotiation. Rather, we show that LMEs constitute an additional tool that firms have recently employed in practice to complement existing mechanisms—such as maturity choices, covenant design, unsecured debt, the choice between bonds and bank debt, credit lines, and DIP financing in bankruptcy. We formally compare LMEs to unsecured debt and bonds in Section 5,[9] and in Section 4, we discuss their relation to other similar contractual tools.

[8] See, for example, Detragiache, Garella, and Guiso (2000); Bris and Welch (2005); Hackbarth and Mauer (2012); Rampini and Viswanathan (2013); Brunnermeier and Oehmke (2013); Diamond and He (2014); Acharya, Almeida, Ippolito, and Perez (2014); Morellec, Valta, and Zhdanov (2015); Della Seta, Morellec, and Zucchi (2020); Donaldson, Gromb, and Piacentino (2020); Rampini and Viswanathan (2020); Malenko and Tsoy (2025b); Hu and Varas (2025); Donaldson, Koont, Piacentino, and Vanasco (2025); Donaldson, Gromb, and Piacentino (2025).

[9] Theories such as Donaldson, Gromb, and Piacentino (2020, 2025); Donaldson, Koont, Piacentino, and Vanasco (2025) emphasize the role of secured debt in preventing dilution and mitigating underinvestment. In Subsection 5.4, we model similar forces and show how secured debt with an LME option can create additional value by mitigating an asymmetric information friction that arises in debt renegotiation.

Our continuous-time model in Section 6 contributes to a theory literature studying how the resolution of financial distress ex post impacts ex ante firm policies.[10] We make a novel contribution to this literature by showing how LMEs impact capital structure, investment, and liquidity management.

Finally, we contribute to the recent law and finance literature on LMEs (Dick, 2021; Ayotte and Scully, 2021; Buccola, 2023a,b; Ayotte and Badawi, 2023; Baird, 2023; Buccola and Nini, 2024; Badawi, Buccola, and Nini, 2024; Buccola and Kahan, 2025; McKinley, 2024; Ivashina and Vallee, 2025). Several of these papers suggest informally that LMEs may alleviate debt overhang or mitigate holdout problems for the rare term loans that lack concentrated ownership.

However, none of these papers develop a formal model of how LMEs can create value ex ante, which is our key contribution.[11] More broadly, we are the first to formally analyze how aggressive debt-renegotiation tactics can generate value ex ante by resolving bargaining frictions in the renegotiation of a term loan held by a small number of creditors.[12]

[10]See, for example, Morellec (2001); Lambrecht (2001); François and Morellec (2004); Hackbarth, Hennessy, and Leland (2007); Geelen (2019); Antill and Grenadier (2019); Zhong (2021); Hu, Varas, and Ying (2021); Hartman-Glaser, Mayer, and Milbradt (2025); Antill and Clayton (2025).

[11]In an appendix of an undistributed contemporaneous working paper, McKinley (2024) extends Donaldson, Gromb, and Piacentino (2020) and models LMEs as a reduction in the initial debt outstanding due to a reduction in debt capacity. This reduction reduces expected deadweight losses from bankruptcy and improves efficiency. We differ in at least two important ways: (1) The assumption that atomistic creditors face a holdout problem makes McKinley (2024) closer to a model of exit consents in bond restructurings, as in Donaldson, Gromb, and Piacentino (2020). In contrast, we show how LMEs, which target term-loan debt, solve a different friction that arises even when a few hedge funds hold most of the term loan. (2) Rather than explicitly assuming that LMEs reduce initial debt outstanding, we model how LMEs endogenously solve an ex-post renegotiation friction and how this in turn changes the ex-ante size and split of firm value.

[12]Ayotte and Badawi (2023) conjecture that contracts are imperfect and evolving and contracting parties gradually learn how to permit more activity without worsening an underlying managerial agency conflict. They capture this with a genetic algorithm and show that even when LMEs are a value destroying flaw, they are a necessary part of learning to contract. We complement their work by showing how a more traditional economic model with rational expectations can explain the continued use of contracts that allow for LMEs. Huang, Lewellen, and Wang (2024) study creditor coalitions.

2 Institutional Details

In this section, we describe the institutional details that motivate our model assumptions. We provide further details in Internet Appendix J.

2.1 Liability management exercise mechanics

We model liability management exercises (LMEs), transactions defined by a legal innovation that transfers collateral from existing term loans to newly issued debt. LMEs alter the contractual rights of secured debt holders. Unlike unsecured debt, secured debt is explicitly backed by collateral, either a firm's specific asset or all of its assets. Outside of bankruptcy, secured lenders may seize collateral upon default (e.g., through foreclosure). If the collateral is sold, first-lien secured lenders are repaid from the sale proceeds before any other creditor. In bankruptcy, a plan can only be confirmed if secured lenders either receive full repayment or recover the value of their collateral. Thus, both inside and outside of bankruptcy, first-lien secured debt enjoys the highest priority in a firm's capital structure.[13]

The defining feature of an LME is that it strips existing secured debt of its highest-priority status (Dick, 2021; Ayotte and Scully, 2021; Buccola, 2023b). The firm issues a new security that "cuts in line" and gains priority over the existing first-lien secured debt, which previously held the strongest claim to collateral. This process is known as *priming*. Priming is typically accomplished through two novel legal maneuvers: the *dropdown* and the *uptier*. Both rely on creative interpretations of credit agreements—the contracts that govern the rights of debt holders.

2.1.1 Dropdowns

In a dropdown, the firm transfers collateral to a subsidiary that is not bound by the existing credit agreement. The subsidiary can then issue new debt secured by the transferred collateral, thereby priming the original secured debt.

[13] For further details, see §9-615 of the Uniform Commercial Code and 11 U.S.C. §1129(b)(2)(A).

J. Crew executed the first dropdown in 2017 (Ayotte and Scully, 2021; Buccola and Nini, 2024; Ivashina and Vallee, 2025). The transaction unfolded in three steps. First, J. Crew offered partial early repayment—a form of seniority—to any term loan holder willing to support an amendment to the credit agreement. According to the credit agreement, such an amendment required approval from 50% of term loan holders; 88% accepted the payment and voted in favor. Second, the amendment authorized J. Crew to transfer $347 million of collateral to a new subsidiary not bound by the original agreement. Finally, the new subsidiary issued debt secured by the transferred collateral, thereby priming the original loan.

While each new dropdown differs in its details, most follow the broad structure of J. Crew's LME (Buccola and Nini, 2024). Crucially, dropdowns typically require some level of consent from existing term loan holders.[14] Many firms—for example, Revlon—obtain this consent by allowing supportive lenders to exchange their old debt for the newly issued debt. If at least 50% of holders agree, the consenting lenders retain a claim on the firm's collateral, while the dissenting lenders lose theirs.

2.1.2 Uptiers

In an uptier, the credit agreement is amended to permit the issuance of new priming debt. Many agreements allow such an amendment if, and only if, at least 50% of existing debt holders consent (Buccola and Nini, 2024; Badawi, Buccola, and Nini, 2024).

Serta Simmons executed the first uptier in 2020 (Ayotte and Scully, 2021; Buccola, 2023a; Buccola and Nini, 2024). Subsequent transactions—such as those by Trimark and Boardriders—have closely followed this template. First, the firm assembles a coalition holding more than 50% of the existing secured debt. Second, it offers the coalition new debt that primes

[14] Some credit agreements like J. Crew's allow for limited collateral transfers without lender consent (Ivashina and Vallee, 2025). In theory, these firms could conduct a dropdown with no consent from term loan holders. However, in practice, many firms executing a dropdown follow J. Crew's strategy and coerce a majority of term loan holders to amend the agreement to allow for a larger collateral transfer. See, for example, Revlon's dropdown and Envision Healthcare's dropdown (Buccola, 2023a).

the existing debt and permits them to exchange their old claims for the new priming debt. In return, the coalition consents to amend the credit agreement, thereby authorizing the priming debt. As a result, the coalition retains the strongest claim on the collateral through ownership of the new debt, while nonparticipating lenders lose their priority.

2.1.3 Exclusive versus nonexclusive LMEs

In some LMEs, all term loan holders are offered the opportunity to participate. In these cases, e.g., Revlon and J. Crew, only those who decline are primed by the new secured debt without compensation.

In contrast, other LMEs extend participation only to a handpicked subset of lenders. This coalition is deliberately chosen to hold more than 50% of the outstanding term loan debt, giving it the power to amend the credit agreement without support from the excluded minority. Such transactions are known as "exclusive" or "non-pro-rata" LMEs. Market observers widely view uptiers as far more likely than dropdowns to be structured in this exclusive manner.[15]

2.2 Unlike exit consents, LMEs target term loans

Exit consents refer to a type of bond exchange offer in which holders who wish to swap their existing bonds for new ones must also consent to an amendment of the original indenture (Buccola, 2023a). If the amendment receives sufficient support, it strips away covenants that previously protected the old bonds. LMEs share similarities with exit consents, a technique used in bond restructurings for decades (Gertner and Scharfstein, 1991; Kahan and Tuckman, 1993; Donaldson et al., 2020). Importantly, unlike exit consents, LMEs transfer collateral away from term loans.

Some practitioners use the term LME specifically to refer to a transaction that transfers collateral away from a loan (S&P Global, 2024). Others adopt a broader definition that

[15] For example, Buccola and Nini (2024) refer to uptiers as "non-pro-rata uptiers" and write "the non-pro-rata character of uptiers, in contrast to dropdowns, might similarly produce distinctive efficiency properties."

also includes the priming of secured notes (Melveny, 2022; Buccola, 2023a). Even under this broader definition, however, the vast majority of LMEs target term loans (Credit Sights, 2025; Badawi, Buccola, and Nini, 2024). LMEs are therefore novel in that they transfer collateral away from secured term loans. According to Credit Sights (2025), the first LME occurred in 2014, with activity surging beginning in 2020.

2.2.1 In LMEs, a few hedge funds own most of the term loan

LMEs usually target a type of secured syndicated term loan known as an institutional term loan. Although these loans are often arranged by banks, they are quickly sold to other investors in an active secondary market (Badawi, Buccola, and Nini, 2024; Blickle et al., 2020). Even after trading in the secondary market, ownership of institutional term loans remains highly concentrated. Using a dataset of loan trades, Giannetti and Meisenzahl (2021) find that, on average, ten debtholders hold 80% of a loan. Ownership becomes even more concentrated when firms enter distress: it is common for a majority of a distressed loan to be held by just a handful of hedge funds. Giannetti and Meisenzahl (2021) show that ownership concentration spikes as the borrower's credit quality declines. This is because CLOs sell distressed loans and hedge funds buy them.

Consider, for example, the 2023 LME conducted by Robertshaw. At the time of its LME, four asset managers held 76% of its term loan (McColm, Bolin, and Mengden, 2023; U.S. Bankruptcy Court for S.D. Texas, 2024). Similarly, by the time Neiman Marcus filed for bankruptcy, three asset managers owned 74% of its term loan.[16] We provide other examples in Internet Appendix J.

2.2.2 Exit consents and LMEs address distinct economic frictions

A long theoretical literature shows that exit consents address the following free-rider problem in bond restructurings (Gertner and Scharfstein, 1991; Kahan and Tuckman, 1993; Donaldson

[16]See entry 146 of the docket for the Neiman Marcus bankruptcy: `https://cases.stretto.com/public/X064/10214/PLEADINGS/1021405082080000000011.pdf`.

et al., 2020). Consider a bond exchange in which each bondholder may swap an old bond for a new one at a haircut to (discount of) face value. If all bondholders accept, the reduction in face value reduces the firm's default probability. But when each holder owns only a tiny fraction of the total debt, any single decision has no meaningful effect on default risk. Thus, if a bondholder expects others to accept the offer, thereby reducing default risk, she has an incentive to free ride by keeping her full face value. When every bondholder shares this incentive, the restructuring is undermined. Exit consents address this problem by punishing free riders.

While the free-rider problem is relevant for bonds, where ownership is dispersed, it is far less significant for term loans, where ownership is concentrated, as discussed above. LMEs therefore address a friction distinct from the free-rider problem emphasized in earlier work on exit consents. This paper makes a novel contribution by showing that LMEs can facilitate renegotiation in the presence of information asymmetry—a friction that remains important even under concentrated ownership and would otherwise block efficient renegotiation. We formalize this distinction between exit consents and LMEs in Subsection 5.2.

2.3 Some distressed firms avert bankruptcy with LME liquidity

Practitioners generally agree that LMEs are primarily used for two purposes: raising new liquidity and reducing outstanding debt (Global Restructuring Review, 2025; Quinn Emanuel, 2025). Some LMEs reduce debt outstanding when participating borrowers exchange their primed debt for new debt at an exchange rate that implies a haircut. For example, in the Serta Simmons uptier, debtholders exchanged each dollar of old debt for 74 cents of new debt (King and Spalding, 2022). More commonly, the objective of an LME is to raise liquidity with participating creditors injecting new funds as part of the deal. Buccola (2023b) emphasizes this liquidity motive: "the aim is to secure liquidity for the company without having to resort to Chapter 11." Accordingly, we focus on the role of LMEs in providing liquidity, though in our model LMEs can also endogenously reduce outstanding debt.[17]

According to a study of 38 LMEs by S&P Global, 37% of firms subsequently filed for bankruptcy shortly after the transaction (S&P Global, 2024). This rate is far above the baseline frequency of bankruptcy filings, indicating that LMEs are typically undertaken by highly distressed firms. We therefore assume in our model that the distressed firm seeking liquidity is insolvent. At the same time, the same statistic implies that 63% of firms avoided bankruptcy following an LME. This motivates our assumption that many distressed firms can avert bankruptcy by raising liquidity through an LME, but not all of them succeed.

2.4 LMEs are a new and ongoing phenomenon

Before J. Crew's LME, most term loan holders likely did not believe such transactions were possible (Ayotte and Scully, 2021). In this sense, early LMEs occurred under debt contracts that did not intentionally permit them. These transactions often triggered litigation, but courts generally upheld their validity. As Buccola and Nini (2024) observe, "the legality of a generic dropdown has not been seriously questioned."[18]

However, once investors realized that LMEs were possible, they could have introduced contractual provisions to block them. As Ayotte and Scully (2021) note, "after the fact, it is obvious that J. Crew's lenders could have stopped this specific maneuver with a simple change to the contract. Indeed, some subsequent loan agreements did exactly that."

Surprisingly, however, roughly half of loan contracts still permit dropdowns. Buccola and Nini (2024) study a large sample of loans issued after LMEs became common to assess whether contract terms adapted. They find that while new loans generally continue to allow nonexclusive dropdowns, they explicitly prohibit exclusive uptiers:

> "Loans originated after the Serta transaction became much more likely to block uptier transactions, which suggests that loan contracts can adjust rapidly to cur-

[17] The extent of the endogenous debt reduction depends on the willingness of the debt holders to provide liquidity on subsidized terms that imply a haircut.

[18] Some courts have invalidated certain uptiers (e.g., Serta Simmons), see Internet Appendix J for details.

tail borrowers' flexibility. Conversely, the frequency of loans susceptible to a dropdown transaction changed little after the J. Crew transaction, which suggests that giving borrowers flexibility to repledge collateral may be valuable... Contracts could adjust to prevent dropdowns but did not."

Buccola and Nini (2024) conclude from this evidence that the possibility of a dropdown must create value ex ante.[19] This is precisely what we show in our model. Moreover, we prove in an extension that firms benefit from banning uptiers.

2.5 Motivating our model assumptions using institutional facts

Together, these facts motivate our main assumptions. First, we assume distressed, insolvent firms use LMEs to raise liquidity, which sometimes avert bankruptcy (Subsection 2.3). Second, we assume that conducting an LME requires majority consent from existing term loan holders to transfer collateral, so the firm approaches these lenders to raise liquidity (Subsection 2.1). Third, we assume that the firm negotiates with a small number of hedge funds that own the term loan debt (Subsection 2.2). Given this ownership concentration, we do not assume a classical holdout problem in which no individual lender is pivotal. Instead, we focus on a different friction–information asymmetry. Finally, our main results explain why contracts allow nonexclusive LMEs, while an extension shows why firms optimally ban exclusive LMEs (Subsection 2.4).

3 Baseline Model

In this section, we describe the assumptions of our baseline model.

[19] There are alternative explanations of these facts, which we discuss in Internet Appendix J.

3.1 Model setup

3.1.1 Timing, project payoffs, and debt financing

There are three periods: $t = 0, 1, 2$. There is no time discounting. At date 0, the firm pays a startup cost I_0 to launch a positive net present value (NPV) project. To raise I_0, the firm issues secured debt. At date 1, there are two possible states of the world: a good state and a distress state. With probability $1 - p_1$, the state is good and the project's payoff is $C^H > I_0$. With probability p_1, the firm is in distress. In distress, the firm requires a liquidity injection of I_1. Absent a liquidity injection, the firm defaults on its secured debt and files for a costly bankruptcy, producing a low payoff $C^L < I_0$. A liquidity injection allows the distressed firm to survive another period. However, its value at date 2 is still random. With probability $1 - p_2$, the project yields a high payoff C^H at date 2. With probability p_2, the firm files for bankruptcy in spite of the liquidity injection, producing a low payoff C^L. The parameters $I_0, C^H, C^L, p_1, p_2, I_1$ are exogenous. Figure 1 illustrates the model timeline.

The date-0 debt contract promises an endogenous payment K_0, which debt holders receive if the project's final payoff is C^H. As equity holders are protected by limited liability, debt holders receive only C^L in bankruptcy.[20]

The date-0 loan is secured and therefore senior to any new date-1 debt financing unless an LME occurs (Subsection 2.1).[21] At date 0, the firm also chooses whether to ban or allow LMEs (Subsection 2.4). The date-0 loan is priced by perfectly competitive banks. Formally, K_0 is set so that date-0 lenders earn zero expected profit.

3.1.2 Distress and insolvency

Once the firm is in distress at date 1, only specialized hedge funds are willing to purchase the firm's debt from original debt holders (banks). For simplicity, we assume that there are

[20]In this sense, the debt is undersecured. However, the debt could be interpreted as fully secured in the initial state, and in distress the collateral value falls.

[21]See Subsection 5.4 for an extension that allows for both secured and unsecured debt.

two hedge funds, each of which acquires half of the firm's debt.[22] To ease exposition, we assume that hedge funds purchase the distressed firm's debt before privately observing their funding costs, which we introduce later.

In our baseline model, to raise liquidity at date 1, equity holders must approach the same hedge funds that have acquired the debt.[23] In Subsection 5.3, we prove that this assumption is without loss of generality: equity holders optimally choose to raise liquidity from existing debt holders even when other investors are available.[24] Because the firm raises liquidity by negotiating with its existing debt holders, we refer to this process as a 'renegotiation.' Throughout, we use the terms 'debt holder,' 'hedge fund,' and 'investor' interchangeably.

Motivated by Subsection 2.3, we assume that the distressed firm is insolvent at date 1:

Condition 1. $p_2 C^L + (1 - p_2)C^H < I_0 + I_1$.

The left side is the date-1 value of the distressed firm's expected cash flow if it receives a liquidity injection. Throughout Sections 3 - 4, we assume that Condition 1 holds.

3.1.3 Renegotiation under asymmetric information about investor costs

At date 1, if the firm is in distress, equity holders make a take-it-or-leave-it renegotiation offer to the two hedge funds.[25] In this renegotiation, equity holders propose a total payment, K_1, in exchange for a total liquidity injection, I_1. We allow the endogenous offer K_1 to be less than I_1, reflecting either (i) a below-market interest rate on the injected funds I_1, (ii) a haircut on the previously promised debt payment, or (iii) both. If the renegotiation succeeds and the firm avoids bankruptcy, equity holders receive the residual value $C^H - K_1 - K_0$.[26]

[22] Subsection 5.5 shows that our results hold for any finite number of debt investors.

[23] This is a realistic assumption for LMEs: the firm must obtain consent from one of the two term-loan holders to conduct an LME (Subsection 2.1). Absent an LME, the firm could, in principle, raise liquidity from outside investors who do not hold the debt.

[24] Intuitively, injecting liquidity into an insolvent firm subsidizes existing debt holders by reducing the risk of a bankruptcy that would leave them impaired. For this reason, debt holders are willing to accept a lower return in exchange for providing new liquidity. Relatedly, an investor who injects liquidity would benefit from also purchasing the debt and thereby internalizing this subsidy.

[25] Note that if the firm is not in distress at date 1, equity holders receive $C^H - K_0$.

[26] Formally, we assume that $C^H \geq K_0 + K_1$ and $C^H \geq K_0$. Further, we impose a condition later that implies that $C^L < K_0$ and $C^L < K_0 + K_1$.

In a successful renegotiation, the participating investors jointly inject the funds I_1 needed to avoid bankruptcy. Both investors simultaneously decide whether to participate. If both participate, each injects $I_1/2$ and receives $K_1/2$. If only one participates, it injects the full I_1 and receives K_1. If neither participates, the firm files for bankruptcy. Our main results are robust to alternative deal structures. In Subsection 5.5, we demonstrate that the same results hold when liquidity is raised through an optimally designed mechanism.

We assume that distressed debt investing is not perfectly competitive: some hedge funds require a positive expected return.[27] Specifically, investor i incurs a 'proportional cost' θ_i for each dollar injected into the firm. This can be interpreted as either (i) the opportunity cost—the positive return the investor requires for forgoing an alternative deal—or (ii) the investor's cost of capital. Accordingly, if investor i provides all of the liquidity I_1, then investor i incurs a total cost of $I_1(1 + \theta_i)$; if both investors participate, investor i pays $I_1(1 + \theta_i)/2$ to inject $I_1/2$.

Crucially, investor i's realization of θ_i is private information. Each investor $i = a, b$ draws θ_i independently from the same binary distribution. For simplicity, we assume $\theta_i \in \{0, \overline{\theta}\}$ with $\overline{\theta} > 0$. All results continue to hold if θ_i is drawn from an arbitrary distribution subject to certain technical conditions (see Subsection 5.5). Let q denote the probability that investor i has a high cost: $q \equiv \mathbb{P}(\theta_i = \overline{\theta})$, which implies $\mathbb{P}(\theta_i = 0) = 1 - q$.

The key friction in our model is the asymmetric information regarding investors' funding costs. Equity holders make their renegotiation offer K_1 without observing $\{\theta_a, \theta_b\}$. Importantly, we show that this information asymmetry leads to renegotiation failure in equilibrium, even when renegotiation increases surplus, and is thus efficient, in all states of the world.

The assumptions about funding costs and information asymmetry are consistent with empirical evidence that *expected* returns vary across hedge funds by at least 11 percentage points annually (Bali, Brown, and Caglayan, 2012). If expected returns vary, required returns should vary as well. Moreover, it is reasonable to view required returns as private

[27]See Bali, Brown, and Caglayan (2012); Eckbo, Li, and Wang (2023).

information.[28] Accordingly, we view our assumptions regarding θ_i as realistic. Nonetheless, Internet Appendix G shows that our results also hold under an alternative assumption: hedge funds privately observe signals about the probability p_2 that a successful renegotiation is followed by bankruptcy. Thus, our results are robust as long as hedge funds are privately informed about their interest in the transaction, even when required returns are zero.

3.1.4 Initial contract choice: traditional renegotiation vs LMEs

At date 0, equity holders decide whether to enable LMEs in the debt contract. Without LMEs, only "traditional" renegotiation is possible. In a traditional renegotiation, an investor who rejects the renegotiation offer receives $C^L/2$ if the firm files for bankruptcy or $K_0/2$ otherwise under the original contract. An investor who accepts the renegotiation offer pays $I_1(1+\theta_i)/n$ to inject I_1/n, where $n \in \{1, 2\}$ is the number of investors accepting the deal, and then receives $C^L/2$ in bankruptcy or $K_0/2 + K_1/n$ otherwise.

Alternatively, equity holders may enable LMEs. An LME works exactly the same as a traditional renegotiation with one key exception: if one investor accepts the offer while the other rejects ($n = 1$), the rejecting investor forfeits their right to any bankruptcy payment. Specifically, if one and only one investor accepts the offer and thus injects liquidity, the accepting investor receives C^L while the rejecting investor receives 0 in bankruptcy. If both investors accept the offer, both receive $C^L/2$ in bankruptcy.

3.2 Investor payoffs and subgame-perfect renegotiation strategies

We now formalize the payoffs that investors receive from accepting or rejecting a renegotiation offer at date 1. We assume that investor strategies are *subgame perfect*: in any subgame following an offer K_1, the investor responses comprise a Nash equilibrium. We now describe this post-offer Nash equilibrium under traditional renegotiation and LMEs. In Section 4, we

[28] Each hedge fund i has an incentive to pretend its θ_i is high to justify demanding a high return. However, θ_i cannot be communicated credibly. For example, even if a hedge fund could reveal a term sheet for an alternative potential deal, there is no way to communicate the probability of successful closing. Thus, it is cheap talk for a hedge fund to claim that its θ_i is high.

formalize our equilibrium concept for the entire game, including the equity holders' choice of K_1 that anticipates these investor responses.

3.2.1 Post-offer date-1 subgame with traditional renegotiation

Consider a subgame in which equity holders promised K_0^* at $t = 0$, the firm entered distress at $t = 1$, and equity holders made a renegotiation offer K_1. Investor i observes their cost θ_i and the renegotiation offer K_1, and forms a rational expectation $\widetilde{P}_j^{\text{Trad}}$ of the probability that the other investor, j, accepts the offer. Investor i's decision to accept or reject the offer K_1 must be optimal in any such subgame, given the expectation $\widetilde{P}_j^{\text{Trad}}$. Conditional on being in distress, investor i's expected payoff from accepting the offer is

$$R_{\text{accept}}^{\text{Trad}}(\theta_i) = \frac{C^L}{2} + \underbrace{(1-p_2)\frac{K_0^* - C^L}{2}}_{\text{time-0 debt value increase}} + \underbrace{\left(1 - \frac{\widetilde{P}_j^{\text{Trad}}}{2}\right)\left[(1-p_2)K_1 - I_1(1+\theta_i)\right]}_{\text{expected capital gains from new debt}}. \tag{1}$$

The first term captures investor i's share of the firm's value in bankruptcy, which is the value of their debt absent a liquidity injection. By injecting liquidity, investor i captures an expected appreciation of their date-0 debt holding (the first underbraced term) and a return associated with their new liquidity provision (the second underbraced term). The latter (underbraced) term accounts for the possibility that investor i is the sole liquidity provider, as well as the possibility of two liquidity providers.[29]

Similarly, investor i's expected payoff from rejecting an offer is

$$R_{\text{reject}}^{\text{Trad}} = \frac{C^L}{2} + \underbrace{\widetilde{P}_j^{\text{Trad}}(1-p_2)\frac{K_0^* - C^L}{2}}_{\text{time-0 debt value increase}}, \tag{2}$$

where the underbraced term captures the (expected) capital gains from their date-0 debt holdings if the other investor accepts the offer. Investor i accepts the offer if and only if

$$R_{\text{accept}}^{\text{Trad}}(\theta_i) \geq R_{\text{reject}}^{\text{Trad}}. \tag{3}$$

[29]If the other investor provides no liquidity, which occurs with probability $(1-\widetilde{P}_j^{\text{Trad}})$, investor i's expected net payoff from injecting liquidity is $[(1-p_2)K_1 - I_1(1+\theta_i)]$. If the other investor also provides liquidity, which occurs with probability $\widetilde{P}_j^{\text{Trad}}$, investor i's net payoff from injecting liquidity is $[(1-p_2)K_1 - I_1(1+\theta_i)]/2$. Summing these two terms yields the second underbraced term.

From the perspective of investor i, the probability that investor j injects liquidity, $\widetilde{P}_j^{\text{Trad}}$, is

$$\widetilde{P}_j^{\text{Trad}} = \mathbb{P}\bigg(R_{\text{accept}}^{\text{Trad}}(\theta_j) \geq R_{\text{reject}}^{\text{Trad}}\bigg). \tag{4}$$

In words, $\widetilde{P}_j^{\text{Trad}}$ is the probability that investor j's privately observed cost θ_j leads them to accept the offer. By symmetry, we have $\widetilde{P}_i^{\text{Trad}} = \widetilde{P}_j^{\text{Trad}}$, which we denote by $\widetilde{P}^{\text{Trad}}(K_1)$.

Finally, for a given offer K_1 and associated acceptance probability $\widetilde{P}^{\text{Trad}}(K_1)$ defined by (1) to (4), we can calculate the probability of a successful renegotiation as:[30]

$$A^{\text{Trad}}(K_1) = 1 - (1 - \widetilde{P}^{\text{Trad}}(K_1))^2. \tag{5}$$

In summary, for a given date-1 traditional renegotiation offer K_1, the subgame-perfect investor responses are described by (1) to (5).

3.2.2 Post-offer date-1 subgame with LMEs

We define investor payoffs with LMEs in the same way as before, with one exception: if one investor accepts an LME and the other rejects, and the firm subsequently files for bankruptcy, then the accepting investor has seniority over the rejecting investor, thus receiving the entire payoff C^L in bankruptcy. This assumption directly captures the definition of an LME: nonparticipating secured debt holders lose their claim to collateral in bankruptcy (Subsection 2.1).

Let $\widetilde{P}_j^{\text{LME}}$ denote the probability that investor j with a privately observed funding cost θ_j accepts the liquidity-injection LME offer. Formally, with an LME offer, an investor's payoff

[30] The equality follows from the fact that a renegotiation succeeds unless both investors reject, which occurs with probability $(1 - \widetilde{P}^{\text{Trad}})^2$ since θ_a and θ_b are independent.

from accepting the offer becomes

$$R^{\text{LME}}_{\text{accept}}(\theta_i) = \frac{C^L}{2} + \underbrace{(1-p_2)\frac{K_0^* - C^L}{2}}_{\text{time-0 debt value increase}} + \underbrace{\left(1 - \frac{\widetilde{P}_j^{\text{LME}}}{2}\right)\left[(1-p_2)K_1 - I_1(1+\theta_i)\right]}_{\text{expected capital gains from new debt}}$$
$$+ \underbrace{(1-\widetilde{P}_j^{\text{LME}})p_2\frac{C^L}{2}}_{\substack{\text{benefit of priming} \\ \text{non-participating investor}}} . \tag{6}$$

Equation (6) is the same as (1) under traditional renegotiation, except that investor i receives the entire expected bankruptcy payoff $p_2 C^L$ if the other investor rejects. Investor i's expected payoff from rejecting an offer is:

$$R^{\text{LME}}_{\text{reject}} = \frac{C^L}{2} - \underbrace{\widetilde{P}_j^{\text{LME}} p_2 \frac{C^L}{2}}_{\text{cost of not participating}} + \underbrace{\widetilde{P}_j^{\text{LME}}(1-p_2)\frac{K_0^* - C^L}{2}}_{\text{benefit of not participating}} . \tag{7}$$

This is the same as equation (2) under traditional renegotiation, except that investor i loses any bankruptcy payoff if they reject and the other investor accepts. As before, investor i accepts if and only if:

$$R^{\text{LME}}_{\text{accept}}(\theta_i) \geq R^{\text{LME}}_{\text{reject}}. \tag{8}$$

As in the traditional renegotiation setting, investor i forms a rational expectation that investor j follows the same strategy, so investor i expects that investor j accepts the LME with the following probability:

$$\widetilde{P}_j^{\text{LME}} = \mathbb{P}\left(R^{\text{LME}}_{\text{accept}}(\theta_j) \geq R^{\text{LME}}_{\text{reject}} \right) . \tag{9}$$

As before, by symmetry, $\widetilde{P}_i^{\text{LME}} = \widetilde{P}_j^{\text{LME}} := \widetilde{P}^{\text{LME}}(K_1)$. Therefore, conditional on being in the distress state, the probability that the LME successfully raises liquidity is given by

$$A^{\text{LME}}(K_1) = 1 - (1 - \widetilde{P}^{\text{LME}}(K_1))^2. \tag{10}$$

3.3 Date-0 debt pricing and equity holders' limited liability

3.3.1 Date-0 debt pricing

At date 0, bank lenders earn zero profit on their investment I_0. To determine the promised repayment at date 1, K_0^*, banks anticipate the future equilibrium strategies: the equilibrium offer K_1^* chosen by equity holders and hedge funds' equilibrium response strategies in the subgame when the firm is in distress at date 1.

As liquidity injection will occur when at least one hedge fund agrees to renegotiate, the *ex post* funding cost per unit of liquidity injected, denoted by Θ, equals: (a) θ_i if only investor i participates; (b) $(\theta_a+\theta_b)/2$ if both investors participate; or (c) 0 if neither investor participates. We can summarize all these scenarios by expressing Θ as follows:

$$\Theta \equiv \frac{\sum_i \theta_i \mathbf{1}_{\text{accept}}(K_1^*, \theta_i)}{1+\prod_i \mathbf{1}_{\text{accept}}(K_1^*, \theta_i)}. \tag{11}$$

In this equation, $\mathbf{1}_{\text{accept}}(K_1^*, \theta_i)$ is an indicator equal to one if a hedge fund with cost θ_i accepts the offer K_1^* in the subgame equilibrium described in Subsection 3.2.

Banks anticipate that they will sell the debt to hedge funds if the firm enters distress at date 1. Thus, the banks' payoff in distress is equal to the price that the hedge funds pay for the debt.[31] Accordingly, whether banks anticipate a future LME or traditional renegotiation, we can write the banks' future payoffs as the sum of: (i) $(1-p_1)K_0$, the banks' payoff in the good state K_0 multiplied by probability of the good state, $(1-p_1)$; and (ii) the product of p_1 and the hedge funds' expected future payoff. Formally, for any fixed K_1^*, Θ, and the probability of a successful liquidity injection $A(K_1^*)$, the banks' date-0 break-even condition that determines K_0^* is:

$$I_0 = \underbrace{(1-p_1)K_0^* + p_1 C^L}_{\substack{\text{expected debt repayment} \\ \text{absent liquidity injection}}} + p_1 \Big[\underbrace{A(K_1^*)(1-p_2)(K_0^*+K_1^*-C^L)}_{\text{benefits of liquidity injection}} - \underbrace{I_1\big(A(K_1^*)+\mathbb{E}[\Theta]\big)}_{\text{costs of liquidity injection}} \Big]. \tag{12}$$

The costs of liquidity injection include two terms: (i) the required liquidity I_1 multiplied by the probability of a successful liquidity injection $A(K_1^*)$ and (ii) the expected liquidity cost

[31] Recall that this price is determined before hedge funds observe θ_i.

paid by participating investors: $I_1\mathbb{E}[\Theta]$.

Note that (12) holds whether banks anticipate a future LME or traditional renegotiation. However, the inputs in these two scenarios are different: under traditional renegotiation, $K_1^* = K_1^{\text{Trad}}$, $A(\cdot) = A^{\text{Trad}}(\cdot)$, and Θ are determined by (1) - (5), while under LMEs, $K_1^* = K_1^{\text{LME}}$, $A(\cdot) = A^{\text{LME}}(\cdot)$ and Θ are determined by (6) - (10).

3.3.2 Equity holders' limited liability

We impose the important and realistic assumption that equity holders have limited liability. To ensure that equity value is positive, we impose a parameter restriction that prevents this limited liability constraint from binding when the firm value is C^H:

Condition 2. $(1-p_2)(C^H - C^L) > I_1\left(1 + \overline{\theta}\left(q + \frac{1-p_1}{1-p_1p_2}(1-q)\right)\right) + (I_0 - C^L)\frac{1-p_2}{1-p_1p_2}$.

Condition 2 is equivalent to requiring that $C^H > K_0^* + K_1^*$ for the equilibrium values of (K_0^*, K_1^*). The intuition for this condition is as follows: when equity holders choose a low K_1 that risks renegotiation failure, the choice is driven by the core equilibrium incentives that we describe in the following section, not a mechanical constraint.

Additionally, we impose the following condition:

Condition 3. $I_1 > p_2C^L$.

Condition 3 ensures that, when the firm realizes a low value C^L, the limited liability constraint binds and the equity holders file for bankruptcy: $K_0^* > C^L, K_0^* + K_1^* > C^L$. Throughout Sections 3 - 4, we assume that Conditions 2 - 3 hold.

4 Main Results

Section 3 described the entire model except for one key decision: the equity holders' choice of K_1. Given K_1, investors respond using the strategies described in Subsection 3.2 and the date-0 debt price is given by (12). In this section, we characterize the equity holders' equilibrium renegotiation strategy K_1 and present our main results: LMEs can create value ex-ante by preventing costly bankruptcies ex-post that could not have been prevented by "traditional" term-loan renegotiation.

To demonstrate our main results, we compare K_1 under three equilibrium concepts. First, Subsection 4.1 studies a hypothetical 'full commitment' benchmark in which equity holders commit ex-ante (before issuing debt) to a future date-1 offer K_1. By definition, the equilibrium outcome is *constrained efficient:* the offer K_1 maximizes ex-ante firm value given asymmetric information. While this equilibrium setting is highly unrealistic, it provides a useful benchmark for evaluating efficiency.

Second, Subsection 4.2 analyzes a more realistic equilibrium setting: equity holders choose a *traditional renegotiation* offer K_1 *after* they enter distress at date 1. We focus on the empirically relevant case in which any renegotiation failure is inefficient. We prove that inefficient renegotiation failure can nonetheless occur in this equilibrium. Inefficiency arises because information asymmetry distorts equity holders' choice of K_1 at date 1 more than in the 'full commitment' setting where equity holders commit to choosing K_1 at date 0. Under traditional renegotiation, equity holders' choice of K_1 at date 1 is lower than under commitment, causing excessive bankruptcies relative to the 'full commitment' benchmark.

Third, Subsection 4.3 introduces our final equilibrium concept: equity holders choose K_1 at date 1 and decide between an LME and traditional renegotiation. We establish the following main result: if renegotiation failure is inefficient, then introducing LMEs mitigates inefficiency. LMEs outperform traditional renegotiation by mitigating equity holders' incentive misalignment at date 1 and date 0. As a result, the date-1 offer of K_1 avoids inefficient bankruptcy for a wider range of parameters when LMEs are available. Accordingly, before issuing debt at date 0, if firms can choose between an equilibrium with LMEs and an equilibrium without LMEs, they prefer the equilibrium with LMEs. This result explains why many debt contracts enable LMEs.

Finally, in Subsection 4.4, we consider the case in which the constrained efficient solution features *renegotiation failure* with positive probability. In this case, we show that LMEs can promote excessive renegotiation, relative to a traditional renegotiation mechanism, making the equilibrium outcome less efficient. This result explains why some firms ban LMEs in debt contracts.

4.1 Equilibrium under commitment: Constrained efficiency

To begin, we define a hypothetical 'full commitment' equilibrium in which equity holders commit to a date-1 offer K_1 when they issue debt at date 0. Equity holders choose K_1 to maximize *ex-ante equity value*: the value of equity before the firm issues debt. This is equivalent to ex-ante firm value, a common efficiency metric in the literature. By definition, the full commitment equilibrium outcome is thus *constrained efficient:* K_1 achieves the highest feasible ex-ante firm value given asymmetric information.

Investor payoffs and responses are determined by equations (1) to (5). Importantly, since K_1 is chosen at date 0, it is not a "renegotiation" in the sense often used in the literature: equity holders commit to K_1 before they issue debt. This timing assumption, while highly unrealistic, provides a useful efficiency benchmark: K_1 maximizes ex-ante firm value.

To calculate ex-ante firm value, we note that it is equal to ex-ante equity value. This ex-ante equity value must equal the equity value immediately after the firm issues debt and uses the proceeds to pay the start up cost I_0. For fixed $K_0, K_1, A(K_1)$, this post-debt-issuance-date-0 equity value is:

$$E_0 = \underbrace{(1-p_1)(C^H - K_0)}_{\substack{\text{expected date-1 payoff}\\ \text{to equity holders}}} + \underbrace{p_1 A(K_1)(1-p_2)(C^H - K_0 - K_1)}_{\text{expected date-2 payoff to equity holders}} . \tag{13}$$

The first term in (13) is the equity holders' expected payoff absent liquidity injection. The second term captures the net benefits of liquidity injection.

Using banks' date-0 break-even condition (12), we can equivalently express (13) as follows.

Lemma 1. *Fix any value of K_1. Let the associated values $K_0, A(K_1), \Theta$ be jointly determined* *by the break-even condition (12) and the investor responses (1) to (5). Then ex-ante firm value E_0 is equal to:*

$$E_0 = \underbrace{(1-p_1)C^H + p_1 C^L - I_0}_{\substack{\text{project's NPV} \\ \text{absent liquidity injection}}} + p_1 \Big[\underbrace{A(K_1)(1-p_2)(C^H - C^L)}_{\text{benefits of liquidity injection}} - \underbrace{I_1\big(A(K_1) + \mathbb{E}[\Theta]\big)}_{\text{costs of liquidity injection}} \Big]. \quad (14)$$

All proofs appear in Internet Appendix A. This lemma establishes that the offer K_1 affects ex-ante firm value only through its impact on liquidity injection outcomes: (i) the probability $A(K_1)$ of successfully raising money and (ii) the expected average funding cost $\mathbb{E}[\Theta]$ of investors who agree to inject liquidity. This result follows from the fact that banks break even: a higher date-1 transfer K_1 from equity to debt must be offset by a lower date-0 transfer K_0, so transfers leave ex-ante firm value unchanged. Using (14), we introduce our formal equilibrium definition.

Full commitment equilibrium definition. A full commitment equilibrium is a pair (K_0^*, K_1^*) satisfying the following conditions: (a) the offer K_1^* maximizes ex-ante firm value (14), given K_0^* and investor responses (1) to (5); and (b) the promised debt repayment K_0^* satisfies the break-even condition (12), given K_1^* and the investor responses (1) to (5). Note that equity holders internalize the effect of K_1^* on K_0^* through date-0 pricing.

Under full commitment, equity holders and investors play a Stackelberg game in which equity holders move first as the leader by committing at date 0 to K_1^*, which determines K_0^* through rational debt pricing and investors' equilibrium responses. As we show later, K_1^* is 'time-inconsistent': if equity holders could change K_1^* at date 1 in the distress state, they would do so. Once we establish this efficiency benchmark, we move to more realistic timing assumptions.

4.1.1 Characterizing the full-commitment equilibrium

Examining the full commitment objective (14), equity holders trade off the benefits of liquidity injection (the second underbraced term) against the costs (the third underbraced term).

Under the following condition, the benefits of liquidity injection outweigh the costs:

Condition 4. $(1 - p_2)(C^H - C^L) > I_1\Big(1 + \frac{\bar{\theta}}{q}\Big)$.

In Condition 4, the left side is the gross benefit of liquidity injection. The right side arises from the cost of inducing a high-liquidity-cost creditor to provide liquidity, given the investors' private information. In the following lemma, we summarize how Condition 4 shapes the key results under full commitment.

Lemma 2. *Let K_1^* denote the optimal offer in the full-commitment equilibrium. By definition, K_1^* is constrained efficient as it optimizes ex-ante firm value (14).*

1. *If Condition 4 holds, then K_1^* ensures successful renegotiation: $A(K_1^*) = 1$ and $\Theta = (\theta_a + \theta_b)/2$.*

2. *If Condition 4 does not hold, then renegotiation fails with positive probability because high-cost investors reject K_1^*: $A(K_1^*) < 1$ and $\Theta = 0$.*

Asymmetric information, information rents, and constrained efficiency. Information asymmetry "constrains" the equity holders' choice of K_1: the offer can not depend on the realizations of θ_a, θ_b. This asymmetric information constraint introduces *information rents:* to achieve $A(K_1^*) = 1$, K_1^* must strictly exceed the payoff that a low-cost investor requires. To see this, note that $A(K_1^*) = 1$ only if high-cost investors accept the offer.[32] However, if high-cost investors accept K_1^*, then (1) implies that low-cost investors would have accepted less. Thus, if $A(K_1^*) = 1$, asymmetric information forces equity holders to "overpay" investors in renegotiation whenever a low-cost investor is present.

By the same logic, asymmetric information increases the expected funding cost $\mathbb{E}[\Theta]$. Formally, (1) implies that high-cost investors require a higher K_1^* than low-cost investors. Thus, were the firm informed about investors' funding costs, it would raise liquidity from only the lowest-cost investor: $\Theta = \min_{i=a,b} \theta_i$. However, given asymmetric information,

[32] Otherwise, renegotiation would fail in the situation in which both investors have high costs.

achieving $A(K_1^*) = 1$ requires an offer K_1^* that all investors accept. Thus, in Lemma 2, Θ is the average of investors' funding costs, not the minimum.[33]

By increasing $\mathbb{E}[\Theta]$, asymmetric information makes renegotiation more costly. Specifically, Condition 4 requires that the benefits of renegotiation justify (i) compensating a high-cost investors for their funding costs and (ii) paying the information rent to low-cost investors. In this sense, Condition 4 ensures that paying information rents is a (constrained) efficient investment.

In summary, the constrained efficient outcome is the one that equity holders would choose if they (i) could commit to an offer K_1 at date 0 but (ii) faced information asymmetry regarding θ_a, θ_b. Lemma 2 shows that guaranteed renegotiation ($A(K_1) = 1$) is the constrained-efficient outcome if and only if Condition 4 holds.

4.1.2 The importance of commitment problems

In Subsection 4.2, we move to an equilibrium definition based on a more realistic assumption: equity holders must wait until date 1 to choose a renegotiation strategy. We show that this equilibrium is not constrained efficient due to interactions between commitment problems and asymmetric information. Intuitively, information asymmetry introduces a wedge between equityholders' equilibrium objectives at date 1 and date 0: with information asymmetry, ensuring renegotiation requires equity holders to pay an information rent, which they are reluctant to do once debt has been issued. Since information asymmetry distorts the date-1 objective and a lack of commitment prevents equity holders from choosing K_1^* at date 0, the equilibrium K_1^* chosen at date 1 is inefficient. Our main result shows that introducing LMEs as a renegotiation mechanism aligns date 1 and date 0 incentives, improving ex-ante firm value. In this sense, LMEs can improve ex ante firm value by mitigating limited-commitment problems that arise when renegotiating debt under asymmetric information.

[33] In Subsection 5.5, we show that our main results hold even when equity holders can design an optimal mechanism to raise liquidity from only the investor with the lowest reported cost.

We do not claim that LMEs are the only solution to this lack of commitment. However, there is no obvious universal solution to this problem. Accordingly, we believe that our positive explanation for why debt contracts enable LMEs–they are an additional tool to mitigate this commitment problem under asymmetric information–is reasonable.

One reason that this commitment problem arises is the market segmentation described in Subsection 2.2. To ensure the constrained-efficient outcome in equilibrium, at date 0 equity holders would have to jointly commit to one promised payoff K_0 for date-0 lenders and another promised payoff K_1 to date-1 lenders. However, these lenders are different institutions: banks make loans to solvent firms while hedge funds buy distressed loans and extend liquidity in LMEs.

Other contracting devices could similarly attain constrained efficiency. For example, a firm could issue unsecured debt ex ante to preserve collateral for raising liquidity in distress. However, in Subsection 5.4, we prove that LMEs can sometimes improve efficiency even when firms are able to issue unsecured debt at date 0. Credit lines are another potential contracting solution–firms and borrowers often include revolving lines of credit in term-loan agreements, which are another way of ensuring that firms raise liquidity in distress. However, a deeply distressed and insolvent firm might not be able to access a credit line. For example, administrative agents for syndicated loans are required to issue a "stop funding notice" and close a revolving credit line if the borrower's financial conditions deteriorate.[34] Sufi (2009) shows that a firm loses access to roughly 25% of its credit lines after a covenant violation. Secured debt with an LME option can thus be another useful tool for the contingency when an insolvent firm is in need of liquidity, but ineligible to draw on a credit line.

Similarly, the LMEs that we model are similar in some ways to the use of debtor-in-possession (DIP) financing in bankruptcy. Firms issue new secured DIP loans in bankruptcy that are senior to existing secured debt, potentially mitigating debt overhang (Ayotte and

[34] See, for example, the case of Fontainebleau Las Vegas LLC, in which Fontainebleau exceed its projected costs and the term lenders closed the revolving line of credit: https://media.ca11.uscourts.gov/opinions/pub/files/201211815.pdf.

Skeel Jr, 2013). A key difference between LMEs and DIP loans is that LMEs occur *before* a bankruptcy filing. Thus, unlike DIP loans, LMEs have the potential to avoid deadweight losses that occur as soon as a firm files for bankruptcy.[35]

It is worth noting that there are other corporate-finance theory papers analyzing a tradeoff between commitment and flexibility, see, e.g., Donaldson, Gromb, and Piacentino (2025) and Hu and Varas (2025).[36] While these papers show that other contractual devices can mitigate limited-commitment problems, the prevalence of limited-commitment issues in the literature indicates that there is no universal solution available to all firms.[37] Indeed, in the 2019 American Finance Association Presidential Address, Demarzo (2019) asserts that commitment problems are the fundamental friction in capital structure choice.[38] Accordingly, since commitment problems lack a straightforward solution, we show that LMEs are an additional tool that firms use to solve commitment problems stemming from asymmetric information.

In summary, we provide a novel positive explanation for the empirical fact that firms use LMEs and lenders enable them; we do not claim that LMEs are the sole solution to the frictions that we study.

4.2 Equilibrium with ex-post traditional renegotiation

Having characterized the full-commitment equilibrium, we turn to a more realistic equilibrium concept in which equity holders cannot credibly commit at date 0 to a future renegotiation strategy. Instead, if distress occurs at date 1, equity holders re-optimize by choosing a new K_1, anticipating hedge funds' optimal strategies.

[35] For example, Antill and Hunter (Forthcoming) show that deadweight losses due to lost customers can occur the moment a firm's bankruptcy filing becomes publicized. Thus, a firm with large potential indirect bankruptcy costs might enable LMEs in its contracts, while a firm that benefits from bankruptcy might simply achieve the same outcome with a DIP loan.

[36] Amador, Werning, and Angeletos (2006) analyze a commitment-versus-flexibility tradeoff in savings problems with time-inconsistent preferences.

[37] If limited-commitment problems had a simple universal solution, there would not be a long literature proposing contractual solutions in specific settings.

[38] The first sentence is "Optimal dynamic capital structure choice is fundamentally a problem of commitment."

Specifically, if the firm enters distress at date 1, equity holders take K_0^* as given and choose K_1 to maximize:

$$\max_{K_1} \; E_1(K_1) \; = \underbrace{(1-p_2)(C^H - K_0^* - K_1)}_{\substack{\text{expected equity payoff} \\ \text{conditional on liquidity injection}}} \cdot A^{\text{Trad}}(K_1). \tag{15}$$

Equilibrium definition with ex-post traditional renegotiation. An equilibrium with ex-post traditional renegotiation consists of a pair (K_0^*, K_1^*) satisfying the following conditions: (a) the renegotiation offer K_1^* solves the equity holders' ex-post problem (15), taking as given K_0^* and the optimal investor responses characterized by (1) to (5); and (b) the promised payment K_0^* satisfies lenders' break-even condition (12), given K_1^* and (1) to (5). This definition presumes that LMEs are contractually prohibited at date 0. Subsection 4.3 relaxes this assumption and analyzes equilibria when LMEs are permitted.

Equilibrium with traditional restructuring. Recall that Lemma 1 characterizes ex-ante firm value for any K_1, whether it is chosen at date 0 or date 1. Under Condition 4, Lemma 2 implies that ex ante firm value is maximized when K_1^* guarantees liquidity injection. However, in an equilibrium with ex-post traditional renegotiation, equity holders choose K_1^* at date 1 to maximize the *ex-post* value (15), rather than *ex-ante* value (14). We now show that this more realistic timing can lead to a constrained inefficient K_1^*. Formally, the equilibrium features renegotiation failure even when Condition 4 holds, reducing ex ante value.

Proposition 1. *Let*

$$\mathbb{B}^{\text{Trad}} \equiv I_1\left(1 + \frac{\overline{\theta}}{q^2}\right) + \underbrace{\frac{1-p_2}{q}\left(\frac{I_0 - C^L - p_1 I_1 (1-q)\overline{\theta}}{1 - p_1 p_2}\right)}_{\text{greater than } 0} > \underbrace{I_1\left(1 + \frac{\overline{\theta}}{q}\right)}_{\text{Condition 4}}. \tag{16}$$

1. *If $(1-p_2)(C^H - C^L) \geq \mathbb{B}^{\text{Trad}}$, there exists an equilibrium $(K_0^{\text{Trad}}, K_1^{\text{Trad}})$ in which renegotiations always succeed: $A^{\text{Trad}}(K_1^{\text{Trad}}) = 1$.*

2. *If* $(1-p_2)(C^H - C^L) < \mathbb{B}^{\text{Trad}}$*, there is no equilibrium* (K_0^*, K_1^*) *in which* $A^{\text{Trad}}(K_1^*) = 1$*: renegotiations fail when both investors have high funding costs* ($\theta_a = \theta_b = \overline{\theta}$)*, but may succeed otherwise. Therefore,* $A^{\text{Trad}}(K_1^*) \leq 1 - q^2$*.*

Information asymmetry inefficiently distorts date 1 incentives. Proposition 1 and Lemma 2 together imply that the equilibrium with ex-post traditional renegotiation is constrained inefficient under the following condition:

$$I_1 \left(1 + \frac{\overline{\theta}}{q} \right) < (1 - p_2)(C^H - C^L) < \mathbb{B}^{\text{Trad}}. \tag{17}$$

Under this condition, asymmetric information creates a commitment problem that leads to inefficient renegotiation failure in equilibrium. To see this, first note that (17) implies Condition 4. Thus, paying information rents is an efficient investment: equity holders would increase ex-ante firm value if they could commit to pay rents and ensure renegotiation success. However, under (17), Proposition 1 shows that equity holders refuse to pay information rents once debt is priced at date 0. Before K_0 is set, the break-even condition implies that a higher K_1 is offset by a lower K_0. Thus, in the full-commitment equilibrium, equity holders internalize that paying information rents reduces K_0. However, once K_0^* is fixed at date 0, equity holders gain less from paying information rents. Thus, when equity holders choose K_1 at date 1, they require a higher benefit $(1 - p_2)(C^H - C^L)$ in order to pay the information rent and ensure renegotiation success: $\mathbb{B}^{\text{Trad}}$ is higher than the analogous full commitment threshold $I_1(1 + \overline{\theta}/q)$.

Absent commitment, renegotiation failure arises from a debt-overhang problem. Paying the information rent at date 1 to ensure a successful renegotiation is an efficient investment, but equity holders refuse to pay it because doing so subsidizes existing debt holders. Formally, as the initial debt I_0 increases, equity holders capture less of the benefit from a successful renegotiation. This can be seen from the property that $\mathbb{B}^{\text{Trad}}$ increases with I_0. While making a high K_1 offer that is always accepted is the constrained efficient choice, equity holders do not fully capture the surplus from renegotiation. As a result, when I_0 increases, equity holders have stronger incentives to underinvest by offering a low K_1 that risks renegotiation failure. The following corollary formalizes this result.

Corollary 1. *Suppose that* $(1-p_2)(C^H - C^L) > I_1\left(1+\frac{\overline{\theta}}{q^2}\right)$*. Then the equilibrium features renegotiation failure, which is inefficient, if and only if* I_0 *exceeds an exogenous cutoff* I_0^* *defined by:*

$$I_1\left(1+\frac{\overline{\theta}}{q^2}\right)+\frac{1-p_2}{q}\left(\frac{I_0^* - C^L - p_1 I_1(1-q)\overline{\theta}}{1-p_1 p_2}\right)= (1-p_2)(C^H - C^L).$$

In summary, under the condition given in (17), renegotiation may inefficiently fail in equilibrium when only traditional renegotiation mechanisms are available: equity holders are only willing to renegotiate with a low-funding-cost investor but not a high-funding-cost investor. In the following subsection, we show that LMEs can help address this debt-overhang problem by increasing equity holders' profits from ex-post renegotiation.

4.3 Equilibrium with LMEs

Equilibrium definition with ex-post LMEs. We define an equilibrium with ex-post LME offers by making one change to our prior equilibrium definition: investor payoffs and responses are given by (6)-(10), which characterize LME payoffs. Formally, an equilibrium with LMEs consists of a pair (K_0^*, K_1^*) satisfying the following conditions: (a) equity holders choose K_1^* to maximize the ex-post problem (15), taking K_0^* and the optimal investor responses characterized by (6)-(10) as given, replacing $A^{\text{Trad}}(K_1^*)$ with $A^{\text{LME}}(K_1^*)$; and (b) the promised payment K_0^* satisfies lenders' break-even condition (12), replacing $A(K_1^*)$ with $A^{\text{LME}}(K_1^*)$, given K_1^* and (6)-(10). This definition presumes that LMEs are the only renegotiation mechanism available at date 1, which we conjecture and verify is without loss of generality: if equity holders can choose between LMEs or traditional renegotiation at date 1, they always choose LMEs. At the end of this section, we use the properties of this equilibrium to characterize the decision to enable LMEs or not.

Importantly, enabling LMEs modifies the mapping from K_1 to an acceptance probability:

$A^{\text{LME}}(K_1) \neq A^{\text{Trad}}(K_1)$. The following proposition shows that LMEs allow equity holders to ensure $A^{\text{LME}}(K_1) = 1$ with a lower K_1 offer. This change in equity holders' date 1 objective motivates a different K_1 that ensures $A^{\text{LME}}(K_1) = 1$. In this sense, when renegotiation success is efficient, LMEs align date 0 and date 1 incentives and improve ex ante firm value. The following proposition states this main result, as well as the impact of LMEs on K_0.

Proposition 2. *Let*

$$\mathbb{B}^{\text{LME}} \equiv \mathbb{B}^{\text{Trad}} - \underbrace{\frac{C^L}{q}\left(1 - \frac{1-p_2}{1-p_1p_2}\right)}_{\text{greater than } 0} < \mathbb{B}^{\text{Trad}}. \tag{18}$$

1. *If* $(1-p_2)(C^H - C^L) \geq \mathbb{B}^{\text{LME}}$, *there exists an equilibrium* $(K_0^{\text{LME}}, K_1^{\text{LME}})$ *in which LME renegotiations always succeed:* $A^{\text{LME}}(K_1^{\text{LME}}) = 1$. *Compared to the corresponding equilibrium under traditional renegotiation and when* $A^{\text{Trad}}(K_1^{\text{Trad}}) = 1$ *holds (see part 1 in Proposition 1), LMEs increase* K_0^* : $K_0^{\text{LME}} > K_0^{\text{Trad}}$ *and reduce* K_1^*: $K_1^{\text{LME}} < K_1^{\text{Trad}}$.

2. *If equity holders can choose at date 1 between an LME and a traditional renegotiation offer, they optimally choose an LME.*

LMEs can improve ex-ante firm value. Combining Lemma 2, Proposition 1, and Proposition 2, we see that LMEs improve ex-ante firm value under the following condition:

$$\max\left\{ I_1\left(1 + \frac{\overline{\theta}}{q}\right), \mathbb{B}^{\text{LME}} \right\} < (1-p_2)(C^H - C^L) < \mathbb{B}^{\text{Trad}}. \tag{19}$$

Specifically, whenever $I_1 \left(1 + \frac{\overline{\theta}}{q}\right) < (1-p_2)(C^H - C^L)$, the constrained efficient offer ensures renegotiation success (Lemma 2). When $(1-p_2)(C^H - C^L) < \mathbb{B}^{\text{Trad}}$, Proposition 1 shows that ex-post traditional renegotiation fails to ensure renegotiation success: K_1^* is inefficiently low and K_0^* is inefficiently high. As discussed earlier, this inefficiency arises because equity holders are unwilling to pay information rents ex post. However, LMEs change equity holders' date 1 objective: $A^{\text{LME}}(K_1)$ is weakly greater than $A^{\text{Trad}}(K_1)$ since debt holders fear losing collateral if they reject. Equivalently, the necessary information rent to achieve $A^{\text{LME}}(K_1) = 1$ is smaller with LMEs. In this sense, LMEs align date 0 and date 1 incentives by motivating equity holders to ensure $A^{\mathrm{LME}}(K_1) = 1$. When $(1-p_2)(C^H - C^L) > \mathbb{B}^{\mathrm{LME}}$, this beneficial impact of LMEs on incentives is sufficient to ensure renegotiation success, improving ex-ante firm value.[39] Thus, under (19), our results imply that firms optimally choose to enable LMEs at date 0 because doing so improves ex-ante equity value. This aligns with the empirical finding of Buccola and Nini (2024) that new loan contracts continue to enable dropdowns (Subsection 2.4). Notably, there is no need to ban traditional renegotiation: part 2 of Proposition 2 shows that equity holders choose LMEs ex post, improving ex-ante value, even if traditional renegotiation is also an option.

Intuition for main results. Proposition 1 shows that equity holders have a debt overhang problem. When the firm needs to raise substantial debt at date 0, it owes a great deal at date 1. This leaves little value for equity, even if the project is successful. Thus, when the date-0 debt grows, equity holders have less to gain from ensuring that a renegotiation succeeds. As a result, while renegotiation is an efficient investment, equity holders do not internalize this, so they offer a low return to investors and risk deal failure. This debt-overhang problem grows with the outstanding debt: the lower bound $\mathbb{B}^{\mathrm{Trad}}$ increases with I_0.

Figure 2 illustrates the ex-ante efficiency of the equilibrium under traditional renegotiation and the equilibrium with an LME option by comparing them to the 'full-commitment" equilibrium. On the y axis is the gross benefit of renegotiation $(1 - p_2)(C^H - C^L)$ and on the x axis is the amount I_0 raised via debt financing at date 0. The flat dashed black line at the bottom corresponds to $I_1(1 + \overline{\theta}/q)$. Thus, in the region above this line, renegotiation is constrained efficient by Lemma 2. However, Figure 2 shows there is a large region above this line in which (i) renegotiation is constrained efficient but (ii) renegotiation can fail in the ex-post traditional renegotiation equilibrium. As I_0 increases, the boundary $\mathbb{B}^{\mathrm{Trad}}$ (the blue dashed line) increases. In the area between the flat black dashed line and the blue dashed line, the equilibrium with ex-post traditional renegotiation is constrained inefficient because of the debt-overhang problem described above. As one would expect, this debt overhang

[39]For a given K_0^*, K_1^*, Θ, ex-ante firm value under LMEs is given by (14), replacing $A(\cdot)$ with $A^{\mathrm{LME}}(\cdot)$.

problem becomes worse as I_0 increases: Figure 2 shows that, as I_0 increases, the range in which renegotiation fails in equilibrium expands.

Finally, LMEs help solve this inefficient debt-overhang problem. Intuitively, giving equity holders the ability to reallocate value between cooperating and noncooperating debt holders allows them to achieve a renegotiation with a lower payoff to existing debt holders. In this sense, LMEs are a transfer from debt holders to equity holders. The entire debt-overhang problem arises because equity holders do not internalize enough of the benefit of a successful renegotiation. Since LMEs increase equity holders' share of the renegotiation surplus, LMEs help mitigate this debt-overhang problem: equity holders have more to lose from a failed renegotiation. Figure 2 demonstrates this result: the range of values for which efficient renegotiation occurs in equilibrium is larger when LMEs are available. Specifically, the solid black line plots the boundary $\mathbb{B}^{\text{LME}}$ above which the equilibrium with LMEs is efficient. The solid black line is lower than the dashed blue line, so LMEs enable efficient equilibria for a wider range of parameters.

4.4 When LMEs are less efficient than traditional renegotiation

In our preceding analysis in Subsections 4.1-4.3, as Condition 4 is assumed to hold, renegotiation is always constrained efficient. However, Condition 4 may not hold for some firms. Many insolvent firms can benefit from bankruptcy (Ayotte, 2015). Even when bankruptcy is efficient, equity holders might still have an incentive to inefficiently gamble for resurrection, raising liquidity to avoid an immediate bankruptcy that wipes out equity but improves social surplus. As we show below, when raising liquidity in distress is inefficient, LMEs reduce ex-ante firm value by facilitating inefficient gambling for resurrection.

Proposition 3. *Suppose that Condition 4 is not satisfied. Also, suppose that LMEs guarantee successful renegotiation but traditional renegotiation does not:*

$$\mathbb{B}^{\text{LME}} < (1 - p_2)(C^H - C^L) < \mathbb{B}^{\text{Trad}}. \tag{20}$$

Under additional technical conditions stated in Internet Appendix A.7, the following results hold.

1. *In any constrained efficient full-commitment equilibrium, $A(K_1^*) = 1 - q^2$.*

2. *When LMEs are banned, an equilibrium exists under ex-post traditional renegotiation and this equilibrium is also constrained efficient.*

3. *When LMEs are allowed, the firm always chooses the LME option and $A^{\mathrm{LME}}(K_1^{\mathrm{LME}}) = 1$ in any equilibrium with LMEs. Ex-ante firm value, given by (14) with $A^{\mathrm{LME}}(K_1^{\mathrm{LME}})$ replacing $A(K_1)$, is less than the corresponding value under traditional renegotiation.*

Therefore, the firm benefits from banning LMEs ex ante.

Proposition 3 states that equity holders can create value at date 0 by banning LMEs if they anticipate that they themselves inefficiently gamble for resurrection in the future. Thus, whether a firm optimally enables or bans LMEs depends on whether raising liquidity in a future distress state will create or destroy value. This prediction aligns with the evidence in Buccola and Nini (2024), which shows that roughly half of firms enable dropdowns in debt contracts while the other half of firms ban them.

5 Extensions

In this section, we extend our baseline model from the preceding section. In Subsection 5.1, we show why firms ban exclusive LMEs but enable nonexclusive LMEs (Buccola and Nini, 2024). In Subsection 5.2, we show that exit consents and LMEs target different frictions, creating a role for LMEs to improve efficiency even when firms have an option to issue bonds and use exit consents. In Subsections 5.3-5.6, we show how our core results remain robust when we relax key assumptions—allowing for outside financing, unsecured debt, richer investor distributions, mechanism design, and heterogeneous beliefs. Finally, in Subsection 5.7, we introduce a dynamic version of our model in which a firm chooses its capital structure at date 0 by trading off tax benefits against default costs. In this setting, we prove that LMEs can increase optimal leverage by mitigating default costs.

5.1 Exclusive LMEs

In our first extension, we introduce exclusive "non-pro-rata" LMEs (Subsection 2.1) by making two changes to the baseline model of Section 3. First, we introduce a new form of date-1 renegotiation. Instead of traditional renegotiation or a nonexclusive LME, equity holders can propose an exclusive LME by randomly selecting a single debt holder i and making a take-it-or-leave-it offer that only i can accept. If the selected investor accepts, they receive the entire LME payoff; if they reject, the firm immediately files for bankruptcy.[40]

Second, we assume that investors' required returns are convex in the amount of liquidity they supply. This convexity reflects that hedge funds must rely on increasingly costly sources of financing to fund larger transactions. It also captures the higher return a hedge fund requires when committing substantial capital to a single deal, thereby forgoing diversification benefits.

Other than these two modifications, all assumptions remain as in our baseline model. In this extended setup, we show that some firms can create value ex ante by (i) enabling nonexclusive LMEs but (ii) banning exclusive LMEs. Internet Appendix B provides the formal statement and proof of this result.

Exclusive LMEs are inefficient ex ante because investor funding costs are convex. Under convexity, the aggregate cost of liquidity provision is strictly higher when only one investor participates than when both do. By guaranteeing that at most one investor supplies liquidity, exclusive LMEs raise the total cost of renegotiation and reduce overall surplus. Nevertheless, exclusive LMEs can be attractive to equity holders ex post. A participating investor is willing to accept a very low repayment K_1 in an exclusive deal because (i) they know they can steal the excluded investor's collateral and (ii) they know the firm will go bankrupt if they reject.

[40] This assumption provides a simple reduced-form way to capture how one group of investors may interpret a failed deal as evidence that the other group received a negative signal during negotiation or due diligence.

Since exclusive LMEs are inefficient ex ante but attractive ex post, firms optimally ban them.

This result is consistent with the empirical findings of Buccola and Nini (2024). In practice, while recent debt contracts continue to permit nonexclusive dropdown LMEs, they increasingly include provisions that explicitly prohibit exclusive uptier LMEs. Internet Appendix B provides further details.

5.2 Exit consents

As discussed in Subsection 2.2, LMEs bear some resemblance to exit consents: both mechanisms encourage participation in distressed restructurings by threatening nonparticipants with a loss of value. The contexts, however, differ in important ways. Exit consents are designed to mitigate holdout problems in dispersed bondholder restructurings. By contrast, LMEs operate in the setting of syndicated term loans, which are typically held by a small number of concentrated investors, making classical holdout concerns less salient. Accordingly, in Section 4 we show that LMEs mitigate a different friction.

Nonetheless, it is theoretically informative to compare exit consents and LMEs. In Internet Appendix C, we develop a model extension to compare them. In the extension, a firm issues debt and later may seek to reduce debt outstanding if it experiences distress. If the firm issues bonds, it can restructure through exit consents, which partially mitigate the holdout problem. Alternatively, if the firm issues term loans, it can renegotiate through an LME. While LMEs are typically used to raise liquidity, they also sometimes feature debt reduction (Subsection 2.3), which is the focus of this extension. We establish an intuitive result: when the information-asymmetry friction is less severe than the holdout problem, LMEs dominate exit consents by promoting more efficient debt reduction. This result helps rationalize why some firms issue bonds and rely on exit consents to restructure, while others issue term loans and restructure through LMEs. We provide further details in Internet Appendix C.

5.3 Raising liquidity from outside investors

In our baseline model, we assume that the firm must raise liquidity from the investors who hold its existing debt. This assumption is natural in the context of LMEs, since the transaction requires the consent of at least 50% of debt holders (Subsection 2.1). Absent an LME, however, equity holders could in principle raise liquidity from any investor. In Internet Appendix D, we show formally that equity holders optimally choose to raise liquidity from incumbent debt holders, even when outside investors are available for a non-LME transaction.

This result follows directly from the assumption that the firm is insolvent. Under insolvency, existing debt holders benefit when the firm raises liquidity, since doing so can potentially avert bankruptcy costs. As a result, existing debt holders are willing to accept a lower return than other investors to provide financing. Consequently, equity holders optimally raise liquidity from existing debt holders.[41] As we show formally in Internet Appendix D, this implies that excluding outside investors in our baseline model is without loss of generality.

5.4 The choice between secured and unsecured debt

In our baseline model, the firm issues secured debt and anticipates a future debt-overhang problem when raising liquidity. We show that secured debt with an embedded option to conduct an LME can mitigate this problem. Of course, other natural contractual solutions are possible. For example, the firm could issue unsecured debt at date 0, thereby preserving collateral capacity for future pledging. This reduces the cost of capital at date 1 and alleviates debt overhang. In Internet Appendix E, we compare unsecured debt with LME-enabled secured debt as alternative solutions to date-1 debt overhang. Specifically, we extend our baseline model to allow for three financing options at date 0: (1) unsecured debt, (2) secured debt banning LMEs, or (3) secured debt enabling LMEs.

For some parameterizations, we find that date-0 unsecured debt is the most effective

[41] Conversely, an investor who plans to provide liquidity would benefit by first purchasing existing debt.

cure for date-1 debt overhang: it preserves the ability to issue senior secured debt at date 1 (Stulz and Johnson, 1985). For other parameterizations, however, LME-enabled secured debt raises ex ante firm value even relative to unsecured debt. The intuition is that, at date 1, incumbent secured lenders are willing to provide especially cheap financing in an LME, because refusing to participate entails a severe loss of seniority: (i) the competing investor receives a new priming claim on the new liquidity, and (ii) the competing investor's existing claim leaps ahead of the non-participant's claim. While (i) also arises when a new secured loan primes existing unsecured debt, (ii) is unique to LMEs. As a result, LMEs can offer cheaper financing than issuing new senior secured debt, and they give equity holders a stronger incentive to ensure successful renegotiation. Internet Appendix E shows formally that, under these conditions, LME-enabled secured debt can improve ex ante efficiency by resolving debt overhang, even relative to the more traditional strategy of preserving collateral.

While this theoretical comparison is informative, it is important to note that many firms cannot issue unsecured debt in practice. Lenders frequently require collateral, particularly during recessions (Benmelech, Kumar, and Rajan, 2022, 2024), and unrated firms are often reliant on bank financing that is collateralized (Nini and Smith, 2024). For such firms, which lack the option to preserve collateral, LME-enabled secured debt provides a mechanism to mitigate debt overhang.

5.5 Optimal mechanism design and robustness of main results

In Internet Appendix F, we relax our assumptions to allow for: (i) any finite number of investors, (ii) a continuous distribution for θ_i with any well behaved probability density function,[42] and (iii) the use of a mechanism that equity holders optimally design to induce the revelation of the $\{\theta_i\}$ and raise financing from the investor with the lowest θ_i. We model

[42] We require the distribution to have full support over the interval $[0, \overline{\theta}]$ and satisfy a monotone likelihood ratio property.

traditional renegotiation by assuming that, off the equilibrium path, an investor who declines to participate in the mechanism keeps their original debt claim. For LMEs, we assume that an investor who refuses to participate is primed by those who do. Importantly, we show that the optimal mechanism features renegotiation failure on the equilibrium path. If all investors report high θ_i values, the mechanism prescribes bankruptcy, since this threat reduces the information rents that equity must pay to induce truthful reporting. We further prove that LMEs reduce the incidence of bankruptcy and thereby improve efficiency, precisely because they lower the information rents required for incentive compatibility. In this sense, our result holds in a general setting.

5.6 Differences in beliefs

In our baseline model, we assume that investors differ in their required returns θ_i, which can be positive. In Internet Appendix G, we consider an alternative specification in which required returns are normalized to zero, but investors hold heterogeneous beliefs about p_2. We show that LMEs can still enhance ex ante firm value in this setting. Hence, our results do not hinge on heterogeneity in required returns. Rather, the key assumption is that investors are privately informed about their willingness to provide liquidity.

5.7 A simple dynamic model of capital structure with LMEs

How do LMEs change a firm's initial capital structure choice? We develop a continuous-time model with the fewest necessary ingredients to answer this question. All agents are risk neutral with a common discount rate r. At $t = 0$, a firm can incur a startup cost I_0 to launch a project, which generates pre-tax cash flow $\mu_0 > 0$ per unit time. Let π denote the corporate income tax rate.

At $t = 0$, the firm can issue secured debt to raise I_0 to fund the project. Alternatively, the firm can forgo any debt issuance and instead issue equity to finance I_0. If the firm issues debt, the debt pays a perpetual coupon m_0 per unit time until the firm defaults. This debt issuance allows the firm to collect a tax shield of πm_0 per unit time.

The firm decides whether to issue debt based on a standard trade off between tax benefits and bankruptcy costs. The realization of tax benefits or bankruptcy costs depends on the realization of a jump shock, which arrives at the first arrival time T of a Poisson process with constant arrival rate λ.

When the shock arrives at time T, the firm must raise liquidity to cover an expense I_T. If the firm fails to raise I_T, the pre-tax cash flow permanently falls to μ_L per unit time. Even if the firm successfully raises I_T, the size of the jump shock at time T is still random. Conditional on raising I_T, with probability $1-p$, the shock is mild so that the firm continues to produce μ_0 per unit time. With probability p, the shock is severe, causing the pre-tax cash flow to fall to μ_L.

If the firm issues debt at date zero, the jump shock is compounded by bankruptcy costs: if the cash flow falls to μ_L, then the firm must file for bankruptcy, destroying a fraction α of firm value. Thus, if the cash flow falls at date T, it falls to μ_L if the firm has no debt or $\mu_L(1-\alpha)$ if the firm has debt, where $\alpha \in (0,1)$ is a measure of the distress cost as in Leland (1994) and the dynamic contingent capital-structure literature.

When the firm raises liquidity at time T, its options depend on its date-0 capital structure. If the firm forgoes debt at date 0, then at time T it makes a take-it-or-leave-it offer to raise I_T from equity investors. If the firm issues debt at time 0, then the debt is sold at time T to two debt investors. As in the baseline model, each investor has a funding cost $\overline{\theta}$ with probability q. If the firm issues debt, equity holders can make a take-it-or-leave-it renegotiation offer to the two investors who own the debt. Renegotiation occurs after the jump has arrived but before the severity of the jump is realized. The new offer pays a perpetual coupon m_T per unit time, such that debt holders receive total coupons $m_0 + m_T$ in perpetuity if they accept and the firm avoids bankruptcy at time T. If the firm avoids bankruptcy, it collects a tax shield of $\pi(m_0 + m_T)$ per unit time.

In summary, applying the perpetuity formula $\int_0^\infty e^{-rs} ds = r^{-1}$, the combined value of debt and equity at the end of time T takes one of four values: (1) $(1-\pi)\mu_0/r$ if the firm issues equity and the cash flow is high; (2) $[(1-\pi)\mu_0+\pi(m_0+m_T)]/r$ if the firm issues debt and the cash flow is high; (3) $(1-\pi)\mu_L/r$ if the firm issues equity and the cash flow falls; or (4) $(1-\pi)(1-\alpha)\mu_L/r$ if the firm issues debt and the cash flow falls. In this sense, the time T firm value depends on tax benefits, bankruptcy costs, and the likelihood of raising liquidity to maintain high cash flows. At time 0, the firm chooses a capital structure, and decides whether to enable LMEs, based on these potential time T firm values. It also considers the tax benefits that it receives before time T by issuing debt.

As in our baseline model, we solve the model backwards. At time T, equity holders choose an offer to raise liquidity and investors decide whether to accept. We assume that raising liquidity is efficient. At time 0, the firm chooses (i) whether to issue debt or not, and (ii) whether to enable LMEs or rely solely on traditional renegotiation. If the firm issues debt, the initial coupon m_0 is determined by a break even condition.

In Internet Appendix H, we derive the equilibrium in closed form for the three potential date 0 choices: (i) the firm with no debt, (ii) the firm with debt banning LMEs, and (iii) the firm with debt enabling LMEs. We derive a sufficient condition under which LMEs improve firm value and shift the firm's capital structure toward higher leverage, relative to a world with only traditional renegotiation. Intuitively, when LMEs are possible, the firm is more likely to avoid bankruptcy costs α. This reduces the cost of debt, causing the firm to be more inclined to use debt at date 0. In this sense, our model predicts that firms should rely on more secured debt as LMEs become more common.

6 LMEs and Liquidity Management

In Section 4, we show that the availability of LMEs can increase firm value by facilitating debt renegotiation when traditional renegotiation would fail. Building on this result, we ask how LMEs might also influence other corporate policies. To address this question, we now extend the analysis of Section 4 to a dynamic setting. Note that the simple dynamic model in Subsection 5.7 is a special case of the model developed in this section.

6.1 Assumptions

Consider a continuous-time setting. At $t = 0$, shareholders can incur a startup cost I_0 to launch a project, which generates independently and identically distributed cash flows with mean $\mu_0 > 0$ and volatility σ per unit time.

After $t = 0$, the firm can potentially increase the mean of its cash flow through investment. Specifically, over a small time interval $(t, t + dt)$, the firm can pay a convex investment cost $\phi i_t^2 dt$ to introduce a probability $i_t dt$ that its expected cash flow increases, where ϕ is the investment cost parameter. We let T_G denote the time at which this increase in expected cash flow is realized. At time T_G, the expected cash flow increases from μ_0 to $\mu_G > \mu_0$, where μ_0 and μ_G are constants. After time T_G, the firm has no more investment opportunities, so $i_t = 0$ for $t \geq T_G$. We thus can write the firm's incremental cash flow, net of its investment cost, over a small time interval $(t, t + dt)$ as:

$$dY_t = \mu_t dt + \sigma dB_t - \phi i_t^2 dt \,, \tag{21}$$

where B_t is a standard Brownian motion and $\mu_t = \mu_0 \mathbf{1}_{t<T_G} + \mu_G \mathbf{1}_{t\geq T_G}$.

At $t = 0$, the firm finances I_0 by issuing perpetual interest-only debt at par with a face value of $D_0 \geq I_0$, which implies initial cash holdings of $W_0 = D_0 - I_0$. Debtholders receive coupon payments continuously at a rate of m_0 per unit time, where m_0 is endogenously determined by debtholders' break-even condition at issuance $t = 0$. As we show later, in general it is optimal for the firm to hold cash.

Cash dynamics. In each increment of time dt, the firm collects operating profits net of investment costs dY_t, which are defined in (21). Additionally, the firm earns interests on its savings W_t at the risk-free rate r, pays coupons $m_0 dt$ to debtholders, and pays taxes at the rate of π on its profits.

The firm also incurs a carry cost ξ per unit time for each unit of cash it holds, a standard assumption in the literature, to reflect a lower return that firms receive on savings. The firm can also raise equity financing at any time t: issuance $dN_t > 0$ requires an external financing cost $h_0 + h_1 dN_t$, where h_0 is the fixed equity issuance cost and $h_1 > 0$ is the marginal cost of equity issuance as in the dynamic corporate finance literature.[43] The firm can also make a payout $dU_t \geq 0$ to equity holders at any time t.

Given these assumptions, the cash holdings W_t is a state variable that evolves as follows:

$$dW_t = (1-\pi)\left(dY_t + rW_t dt - \underbrace{m_0 dt}_{\text{coupon}}\right) - \underbrace{\xi W_t dt}_{\text{cash carry cost}} - \underbrace{dU_t}_{\text{dividends}} + \underbrace{dN_t}_{\text{issuance}}, \tag{22}$$

where dY_t is given in (21).

Liquidity shock, distress, and debt renegotiation. The firm is also subject to a (downward) jump shock, which arrives at a constant rate $\lambda > 0$. When this shock hits at stochastic time T, the firm must inject funds of I_T to avoid an immediate bankruptcy. If the firm files for bankruptcy, its debt holders share the firm's bankruptcy value $\widehat{C}^L \equiv (1-\pi)(1-\alpha)\mu_L/r$, where $\alpha \in (0,1)$ is a measure of the distress cost.

If $W_T \geq I_T$, the firm has sufficient cash holdings to survive the liquidity shock, but its cash holdings immediately decrease to $W_T - I_T$ at time T. If the firm does not have sufficient savings to meet the liquidity shock, in that $W_T < I_T$, it must raise $I_T - W_T$ so that it can pay the expense and survive at T. In this case, its post-T cash holdings drop to zero.

If the firm pays the expense, it survives the liquidity shock and avoids bankruptcy with probability $1-p$, producing a terminal value of $(1-\pi)\mu_G/r$.[44] However, with probability p, the firm must file for bankruptcy even after meeting the liquidity shock with a total cash

[43] The firm is financially constrained in the sense that it must pay a cost to issue equity, though it is allowed to issue equity an unlimited number of times. This notion of a financially constrained firm has a precedent in the literature: see, for example, Froot, Scharfstein, and Stein (1993), Kaplan and Zingales (1997), Hennessy and Whited (2007), Bolton, Chen, and Wang (2011), and Hugonnier, Malamud, and Morellec (2015). Nonetheless, we acknowledge that some scholars consider a different notion of financial constraints–a firm cannot issue equity, as in our baseline model of Section 3–to be more natural. This no equity-issuance case is a special case of our formulation in which external equity is costly to issue.

[44] For simplicity, we assume that the terminal value is based on μ_G, regardless of whether $T_G \leq T$ or $T_G > T$. Thus, there is no incentive to invest after time T.

payment of I_T. In this case, firm value drops to the same level $\widehat{C}^L$ that the firm would have realized had it failed to meet the liquidity shock with an injection of I_T.

Now we discuss how a financially constrained firm with insufficient cash at time T (i.e., $W_T < I_T$) comes up with funding $I_T - W_T$ to meet the liquidity shock. In this case, the firm attempts to raise liquidity at time T by making a take-it-or-leave-it offer, promising an additional perpetual stream of coupons $m_T dt$ in exchange for a liquidity injection of $I_T - W_T$. As in our baseline model of Section 3, there are two debt holders who potentially provide liquidity, each of whom has an independent, privately observed funding cost equal to $\overline{\theta}$ with probability q and zero with probability $1-q$. LMEs and traditional renegotiation payoffs are modeled in the same way as in the baseline model.

6.2 Solving the firm's problem

We characterize the problem in three steps. First, we solve the problem at T when the liquidity shock hits. Second, we analyze the firm's problem and equilibrium debt pricing for time $t \in (0, T)$. Finally, we complete the model solution by solving the firm's time-0 problem including a choice between LMEs and traditional renegotiation as in Sections 3 - 4.

At time T, if $W_T < I_T$, the firm has to raise $I_T - W_T$ by issuing perpetual debt with a constant continuous coupon payment m_T to avoid a costly bankruptcy at time T. Debtholders decide simultaneously whether to accept or reject the offer m_T.

At time $t \in (0, T)$, the firm chooses its investment, equity payouts, equity issuance, and default timing to maximize its equity value:

$$E(W_t; m_0) = \mathbb{E}_t\left[\int_t^T e^{-r(s-t)}(dU_s - dN_s - dH_s) + e^{-r(T-t)} E_T^*(W_T, m_0)\right], \tag{23}$$

where H_s denotes the cumulative (undiscounted) equity issuance costs up to time s and $E_T^*(W_T, m_0)$ is the firm's equity value at T under the firm's time-T optimal strategy.[45] We

[45] We choose parameters to focus on the economically interesting case where equity financing costs h_0, h_1 are moderate so that shareholders will always choose to raise external equity rather than default before time T.

define $E_T^*(W_T, m_0)$ and the optimal time$-T$ offer m_T in Internet Appendix I.1.

Debt pricing. Banks provide funds D_0 at $t = 0$ and receive coupons at a rate of m_0 until time T. At time T, the date-0 lenders sell the debt to hedge funds at an equilibrium price $D_T^*(W_T, m_0)$ that reflects the expected hedge fund payoff at time T, given m_0, W_T, the firm's equilibrium time-T strategy (i.e., the coupon rate for the newly issued debt $m_T^*(W_T, m_0)$), and investors' responses. We define $D_T^*(W_T, m_0)$ in Internet Appendix I.1. For $t < T$, the debt price is:

$$D(W_t; m_0) = \mathbb{E}_t\left[\int_t^T e^{-r(s-t)} m_0 ds + e^{-r(T-t)} D_T^*(W_T, m_0)\right]. \tag{24}$$

The coupon $m_0 = m_0^*(D_0)$ is endogenously determined to make lenders break even: $D_0 = D(W_0; m_0)$, where $W_0 = D_0 - I_0$ is the firm's initial cash holdings. Finally, at time 0, equity holders choose $D_0 \geq I_0$ to maximize ex-ante firm value: $E(D_0 - I_0; m_0^*(D_0))$.

6.3 Results

Comparing the model solution with LMEs to the solution with only traditional renegotiation, we find the following predictions. First, enabling time-T LMEs increases ex-ante firm value. This happens because: (a) LMEs prevent some bankruptcies at time T, in the sense that $A(W_T, m_T^*(W_T)) = 1$ for a larger region of W_T values under LMEs than under traditional renegotiation, and (b) by reducing expected default costs, LMEs enable firms to choose higher leverage, increasing the debt coupon m_0 and associated tax benefits. Second, LMEs mitigate underinvestment: for any w, $i^*(w)$ is higher under LMEs than under traditional renegotiation. Investment rises because LMEs increase the firm's continuation value, increasing the marginal value of investment, and because the firm has a stronger desire to increase cash flows to support the higher coupon m_0 that LMEs motivate.

Finally, LMEs have an ambiguous effect on cash holdings, which are characterized by an endogenous upper bound $\overline{W}$ on cash holdings: $\overline{W}$ is optimally determined by the firm's desire to pay dividends and avoid the carry cost ξ once cash is high enough. We find that

$\overline{W}$ can be higher or lower with LMEs than under traditional restructuring, depending on parameters. On one hand, LMEs make it easier to raise funds at time T, so having a high cash balance at T is less important. On the other hand, LMEs promote high leverage and a high coupon, so the firm must conserve more cash to support a higher debt coupon payment. In our preferred parameterization, the second effect dominates and $\overline{W}$ is higher with LMEs.

Figure 3 shows the results described above, based on a numerical solution of the model using the parameter values listed in Table 1. In Internet Appendix I, we prove that our numerical solution converges to a variational-inequality-based model solution and we provide a verification theorem proving that the associated policy is optimal.

7 Conclusion

Distressed firms have recently used aggressive tactics to raise liquidity, stripping collateral from existing secured lenders unless they inject new funds. Critics of this "creditor-on-creditor violence" argue that this trend is harmful, eroding trust among corporate lenders. Proponents argue that these "LMEs" help firms raise liquidity and avoid bankruptcy. In line with this view, Buccola and Nini (2024) show that borrowers and lenders continue to include clauses in loan contracts that enable dropdown LMEs.

We provide a formal model showing that (1) in the presence of information asymmetry, equity holders "underinvest" in debt renegotiation by offering low returns to investors and risking deal failure, because part of the benefit of avoiding bankruptcy accrues to debt holders, (2) LMEs solve this inefficiency because equity holders internalize more of the benefit of a successful deal when they can steal collateral from dissenting investors, and (3) as a result, firms create value ex ante by enabling LMEs in contracts.

Notably, it is possible that some liquidity injections destroy surplus: equity holders might gamble for resurrection at the expense of debt holders.[46] In this scenario, we prove that LMEs

[46]Likewise, LMEs might prevent bankruptcies that create value (Ayotte, 2015) because equity holders do not internalize the bankruptcy benefits.

destroy value by worsening equity holders' "overinvestment" in value-destroying opportunities. Thus, we provide a positive explanation for why *some* firms enable LMEs in contracts while others ban them.

Finally, the LMEs that we study share some features with "pay-to-play provisions" that venture capital firms use in their contracts with portfolio companies (Weiss, 2025). Biotechnology firms regularly raise financing through contracts with these provisions, which subordinate the claims of any investor who refuses to provide further capital in future financing rounds (Sparks, 2019). In this sense, our model explains a phenomenon that is broader than the use of LMEs by distressed syndicated-loan borrowers.

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Figure 1: Model Timeline

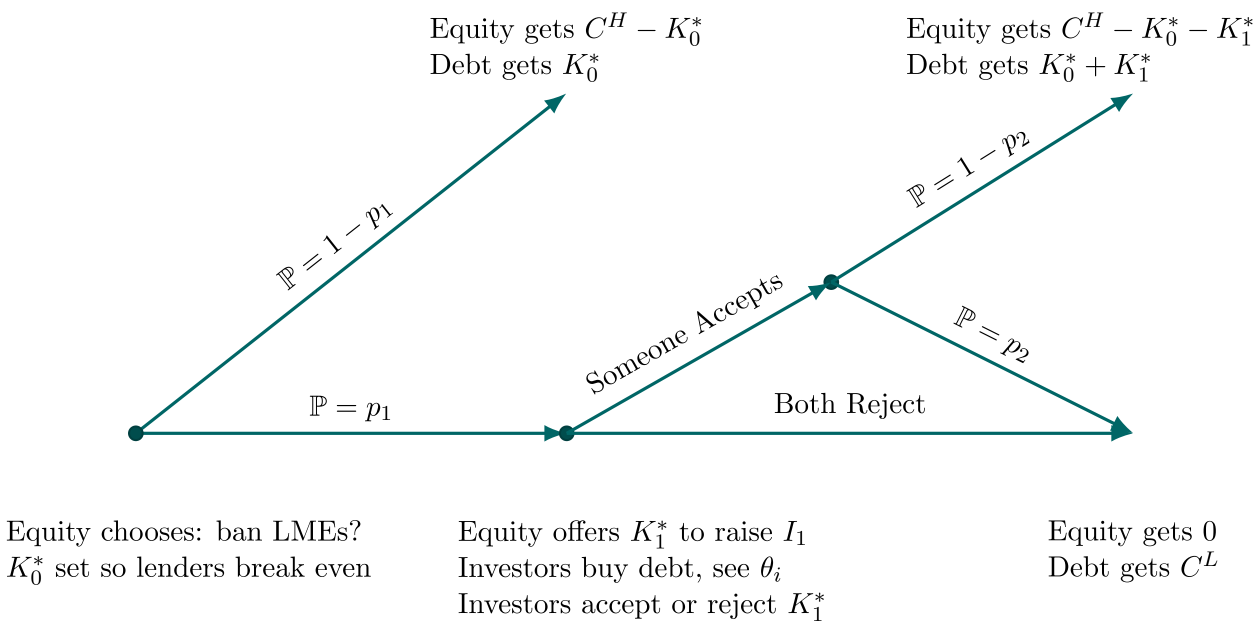

Figure 2: Ex-Ante Efficiency: Full Commitment vs Ex-Post Renegotiation

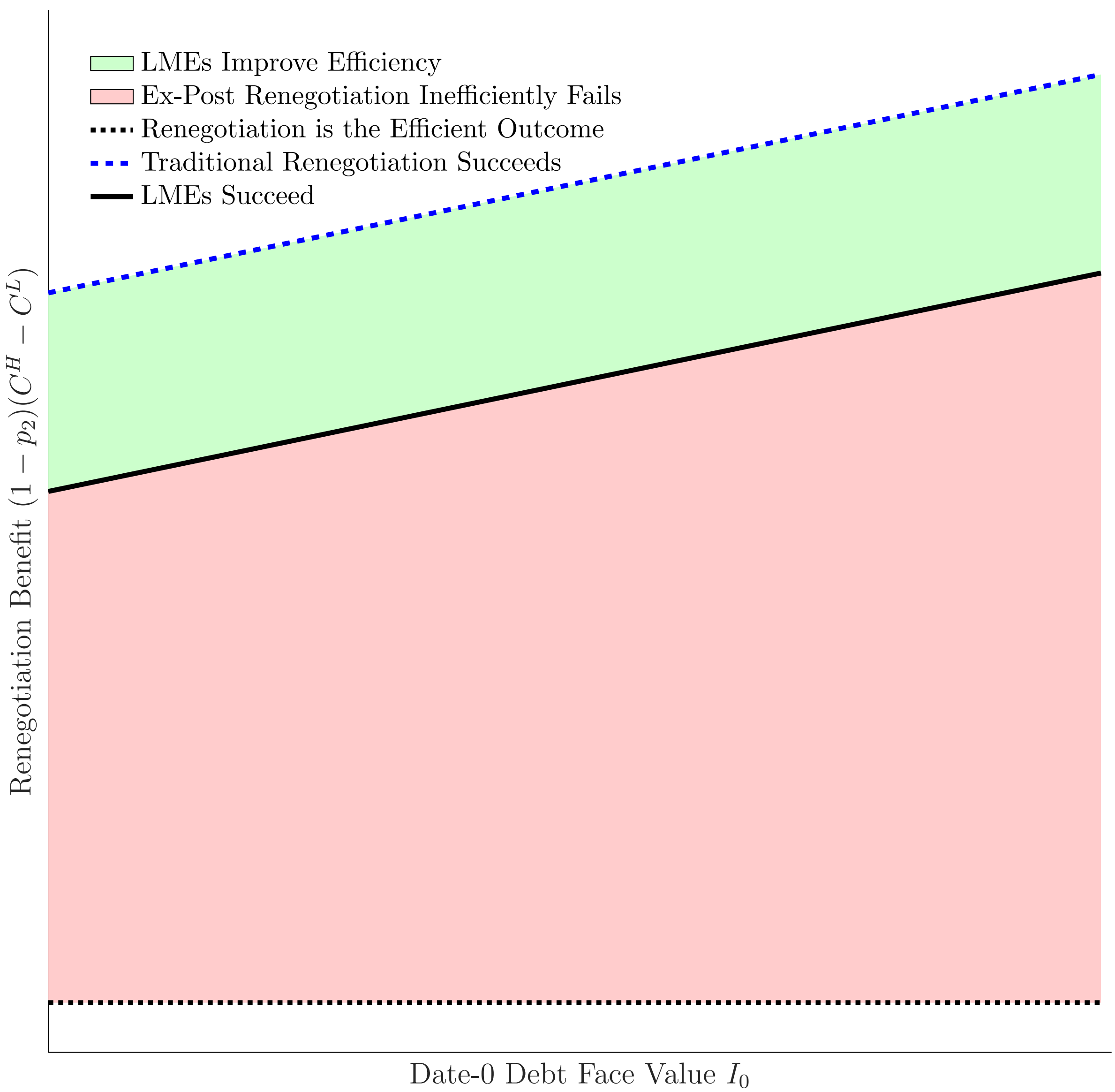

Figure 3: The Impacts of Enabling LMEs in a Dynamic Model

Solution: $W_0^{\text{Trad}} = 0.037$, $W_0^{\text{LME}} = 0.302$, $m_0^{\text{Trad}} = 0.595$, $m_0^{\text{LME}} = 0.878$, $\overline{W}^{\text{Trad}} = 3.986$, and $\overline{W}^{\text{LME}} = 3.997$. Parameter values are listed in Table 1.

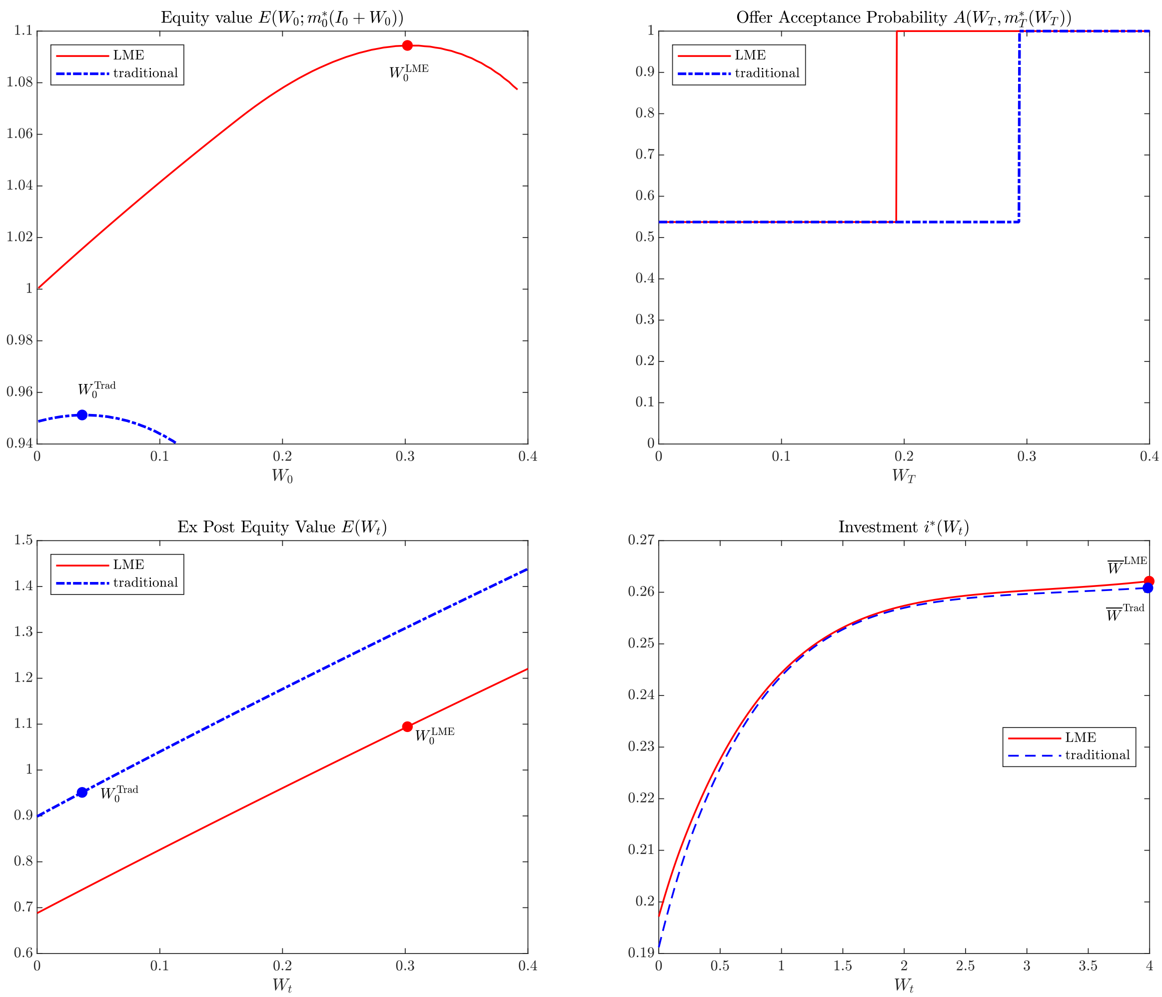

Table 1: Parameter Values for the Dynamic Model Extension

This table reports the parameter values that we assume in Section 6.

r	Risk-free rate	0.06
μ_0	Cashflow drift for $t < \min\{T, T_G\}$	1
μ_G	Cashflow drift in good state ($t \geq T$) and $t \in [T_G, T)$	1.51
μ_L	Cashflow drift in bad state ($t \geq T$)	0.23
σ	Cashflow volatility	1.5
α	Bankruptcy cost parameter	0.2
π	Corporate income tax rate	0.21
ξ	Cash carrying cost	0.01
λ	Liquidity shock arrival rate	0.8
$\overline{\theta}$	Investors' fundung cost	0.02
I_0	Project startup cost	5.2
I_T	Liquidity shock caused expense	4
p	Probability of failure after injection	0.67
q	Probability that an investor has a high cost	0.68
ϕ	Cost of investment	1
h_0	Equity issue fixed cost	0.2
h_1	Equity issue proportional cost	0.08

Internet Appendices

A Proofs

This Appendix provides proofs for all of the results in Section 3 and Section 4.

A.1 Limited liability condition

To begin, we formalize the sense in which Condition 2 (limited liability) ensures that $K_0+K_1 < C^H$. The following lemma shows this for our most relevant equilibria: those in which $A(K_1) = 1$ and K_0 is the equilibrium value implied by (12). Since K_0^{Trad} is the relevant equilibrium value under traditional renegotiation or full commitment, and $K_0^{\text{Trad}} < K_0^{\text{LME}}$, this lemma covers the relevant equilibria for Subsections 4.1 to 4.3.

Lemma 3. *Fix an equilibrium in which $A(K_1) = 1$ and $K_0 \geq K_0^{\text{Trad}}$, where*

$$K_0^{\text{Trad}} = \frac{I_0 - p_1\left[p_2 C^L + I_1 \overline{\theta}(1-q)\right]}{1 - p_1 p_2}. \tag{A.1}$$

Suppose that the break-even condition (12) holds. Then Condition 2 implies that $C^H > K_1 + K_0$.

Proof of Lemma 3: Suppose that the break-even condition (12) holds and $A(K_1) = 1$. Then plugging $A(K_1) = 1$ into (12), we see that

$$\begin{aligned} & C^H > K_0 + K_1 && \text{(A.2)} \\ \iff \quad & I_0 < (1-p_1)K_0 + p_1 C^L + p_1[(1-p_2)(C^H - C^L) - I_1(1+\mathbb{E}[\Theta])], && \text{(A.3)} \end{aligned}$$

which is equivalent to

$$(1-p_2)(C^H - C^L) > I_1(1+\mathbb{E}[\Theta]) + \frac{I_0 - [(1-p_1)K_0 + p_1 C^L]}{p_1}. \tag{A.4}$$

Further, if $A(K_1) = 1$, then the expected funding cost for participating lenders is

$$I_1\mathbb{E}[\Theta] = I_1\mathbb{E}\left[\frac{\sum_i \theta_i \mathbf{1}_{\text{accept}}(K_1,\theta_i)}{1+\prod_i \mathbf{1}_{\text{accept}}(K_1,\theta_i)}\right] = I_1\left[q^2\overline{\theta} + (1-q)^2 0 + 2q(1-q)\frac{\overline{\theta}+0}{2}\right] = I_1\overline{\theta}q. \tag{A.5}$$

Plugging $\mathbb{E}[\Theta] = q\overline{\theta}$ into (A.4), we obtain

$$(1-p_2)(C^H - C^L) > I_1(1+q\overline{\theta}) + \frac{I_0 - [(1-p_1)K_0 + p_1 C^L]}{p_1}. \tag{A.6}$$

Thus, we have shown that (A.6) is equivalent to $C^H > K_1 + K_0$ under the conditions that $A(K_1) = 1$ and (12) holds. We see that the right side of (A.6) is decreasing in K_0, so if (A.6) holds for K_0^{Trad} then it holds for $K_0 \geq K_0^{\text{Trad}}$.

Plugging K_0^{Trad} into (A.6), a sufficient condition for $C^H > K_0^{\text{Trad}} + K_1^{\text{Trad}}$ is

$$\begin{aligned}
&(1-p_2)(C^H - C^L) > I_1(1+q\overline{\theta}) + \frac{I_0 - p_1C^L}{p_1} - \frac{1-p_1}{p_1}\frac{I_0 - p_1\left[p_2C^L + I_1\overline{\theta}(1-q)\right]}{1-p_1p_2} \\
&= I_1(1+\overline{\theta}q) + \frac{1-p_1}{1-p_1p_2}(1-q)I_1\overline{\theta} + \frac{I_0}{p_1}\left[1 - \frac{1-p_1}{1-p_1p_2}\right] - C^L\left[1 - \frac{1-p_1}{p_1}\frac{p_1p_2}{1-p_1p_2}\right] \\
&= I_1\left(1 + \overline{\theta}\left(q + \frac{1-p_1}{1-p_1p_2}(1-q)\right)\right) + (I_0 - C^L)\frac{1-p_2}{1-p_1p_2}.
\end{aligned} \tag{A.7}$$

The final line is the right side of Condition 2. Thus, under the conditions of the lemma, Condition 2 ensures that $C^H > K_0 + K_1$.

A.2 Proof that creditors are impaired in bankruptcy

Next, we verify that an implicit assumption in our setup is valid. Specifically, since debt holders receive the entire firm value in bankruptcy and equity holders receive nothing in bankruptcy, we implicitly assume that $K_0 > C^L$.

We now show that this implicit assumption is without loss of generality: it is implied by our explicit assumptions. Recall that we assume that equity holders are constrained to choose an offer K_1 satisfying $K_0 + K_1 \leq C^H$: equity cannot promise a higher payoff to debt holders than the firm could possibly produce. We also assume that $C^L < I_0$ and that the firm is insolvent in distress: Condition 1 holds. Together, these assumptions imply that $K_0 > C^L$.

To see this, note that at date 0, debt holders expect the following future payoffs:

$$
\begin{aligned}
&(1-p_1)\min\{C^H, K_0\} + p_1[1 - A(K_1)]\min\{C^L, K_0\} \\
&+p_1\Big[A(K_1)p_2\min\{C^L, K_0+K_1\} + A(K_1)(1-p_2)\min\{C^H, K_0+K_1\} - I_1\,\big(\,A(K_1) + \mathbb{E}[\Theta]\,\big)\Big].
\end{aligned}
$$

By the break even condition, this entire expression must equal I_0. Since $\mathbb{E}[\Theta] \geq 0$, by replacing $\min\{C^H, K_0\}$ with K_0, replacing $\min\{C^H, K_0+K_1\}$ with K_0+K_1, and replacing both $\min\{C^L, K_0\}$ and $\min\{C^L, K_0+K_1\}$ with C^L, we obtain

$$
\begin{aligned}
I_0 &\leq (1-p_1)K_0 + p_1\Big[C^L + A(K_1)\,\big(\,(1-p_2)(K_0+K_1-C^L) - I_1\,\big)\Big] \\
&\leq (1-p_1)K_0 + p_1\Big[C^L + A(K_1)\,\big(\,(1-p_2)(C^H-C^L) - I_1\,\big)\Big] \\
&\leq (1-p_1)K_0 + p_1\Big[C^L + A(K_1)\,\big(\,I_0 - C^L\,\big)\Big] \\
&\leq (1-p_1)K_0 + p_1\Big[C^L + \big(\,I_0 - C^L\,\big)\Big] = (1-p_1)K_0 + p_1 I_0,
\end{aligned}
$$

where the second inequality uses $K_0 + K_1 \leq C^H$ and the third inequality uses Condition 1. This implies $K_0 \geq I_0$. Combined with the assumption that $C^L < I_0$, we have $C^L < K_0$. Thus, if the firm is insolvent in distress and the break even condition is satisfied, then $K_0 > C^L$.

A.3 Proof of Lemma 1

We prove (14) for any K_0 and K_1 satisfying the following break even condition:

$$\begin{aligned} I_0 =&(1-p_1)\min\{C^H, K_0\} + p_1[1-A(K_1)]\min\{C^L, K_0\} \\ &+p_1\Big[A(K_1)p_2\min\{C^L, K_0+K_1\} + A(K_1)(1-p_2)\min\{C^H, K_0+K_1\} - I_1\ \big(\ A(K_1)+\mathbb{E}[\Theta]\ \big)\Big]. \end{aligned} \tag{A.8}$$

Then the result in Lemma 1 is obtained under the constraints $C^L \le K_0 + K_1 \le C^H$ and $C^L \le K_0 \le C^H$.

The date-0 value of the equity E_0 that accounts for all possible scenarios is

$$\begin{aligned} E_0 =&(1-p_1)(C^H-K_0)^+ + p_1[1-A(K_1)](C^L-K_0)^+ \\ &+ p_1A(K_1)\Big[p_2(C^L-K_0-K_1)^+ + (1-p_2)(C^H-K_0-K_1)^+\Big], \end{aligned} \tag{A.9}$$

where $x^+ = \max\{x, 0\}$. Since $(x-y)^+ + \min\{x,y\} = x$, by summing equations (A.8) and (A.9), we obtain

$$\begin{aligned} E_0 &= E_0 + I_0 - I_0 \\ =&(1-p_1)C^H + p_1[1-A(K_1)]C^L \\ &+ p_1\Big[A(K_1)p_2C^L + A(K_1)(1-p_2)C^H - I_1\ \big(\ A(K_1)+\mathbb{E}[\Theta]\ \big)\Big] - I_0 \\ =&(1-p_1)C^H + p_1C^L - I_0 + p_1\Big[A(K_1)\ \big(\ (1-p_2)(C^H-C^L) - I_1\ \big)\ -I_1\mathbb{E}[\Theta]\Big]. \end{aligned} \tag{A.10}$$

Then we arrive at (14).

Note that switching from (1)-(5) to (6)-(10) changes nothing in this argument, it only changes the definitions of $A(\cdot)$ and $\mathbf{1}_{\text{accept}}(K_1, \theta_i)$. In this sense, the above argument holds with or without LMEs; introducing LMEs only changes the definitions of $A(\cdot)$ and $\mathbf{1}_{\text{accept}}(K_1, \theta_i)$.

A.4 Proof of Lemma 2

Lemma 1 implies that ex-ante firm value takes one of three values. First, if $A(K_1) = 0$, then $\mathbf{1}_{\text{accept}}(K_1, \theta_i) = 0$ has to hold and ex-ante firm value is

$$(1 - p_1)C^H + p_1 C^L - I_0. \tag{A.11}$$

Second, suppose K_1 is such that $\mathbf{1}_{\text{accept}}(K_1, \theta_i) = 1$ for $\theta_i = 0$ but $\mathbf{1}_{\text{accept}}(K_1, \theta_i) = 0$ for $\theta_i = \overline{\theta}$. Then $\widetilde{P}(K_1) = 1 - q$, $A(K_1) = 1 - q^2$, and the expected funding cost for participating lenders is

$$I_1 \mathbb{E}[\Theta] = I_1 \mathbb{E}\left[\frac{\sum_i \theta_i \mathbf{1}_{\text{accept}}(K_1, \theta_i)}{1 + \prod_i \mathbf{1}_{\text{accept}}(K_1, \theta_i)}\right] = I_1(1 - q^2)0 = 0. \tag{A.12}$$

Using Lemma 1, ex-ante firm value for such a strategy is then

$$(1 - p_1)C^H + p_1 C^L - I_0 + p_1(1 - q^2)\Big[(1 - p_2)(C^H - C^L) - I_1\Big]. \tag{A.13}$$

Finally, suppose K_1 is such that $\mathbf{1}_{\text{accept}}(K_1, \theta_i) = 1$ for both high and low θ_i. Then $\widetilde{P}(K_1) = A(K_1) = 1$, and the expected funding cost for participating lenders is

$$I_1 \mathbb{E}\left[\frac{\sum_i \theta_i \mathbf{1}_{\text{accept}}(K_1, \theta_i)}{1 + \prod_i \mathbf{1}_{\text{accept}}(K_1, \theta_i)}\right] = I_1\left[q^2\overline{\theta} + (1 - q)^2 0 + 2q(1 - q)\frac{\overline{\theta} + 0}{2}\right] = I_1 \overline{\theta} q. \tag{A.14}$$

Again using Lemma 1, we obtain the following expression for ex-ante firm value:

$$(1 - p_1)C^H + p_1 C^L - I_0 + p_1\Big[(1 - p_2)(C^H - C^L) - I_1(1 + \overline{\theta} q)\Big]. \tag{A.15}$$

This value (when both hedge funds participate regardless of their funding costs) is higher than the value (when a hedge fund participates if and only if its fund cost is zero) given in (A.13) if and only if

$$
\begin{aligned}
& (1-p_2)(C^H - C^L) - I_1(1+\overline{\theta}q) > (1-q^2)\Big[(1-p_2)(C^H - C^L) - I_1\Big] \\
\iff\ & q^2\big[(1-p_2)(C^H - C^L) - I_1\big] > I_1\overline{\theta}q \\
\iff\ & (1-p_2)(C^H - C^L) > I_1\Big(1+\frac{\overline{\theta}}{q}\Big).
\end{aligned}
$$

This is Condition 4. Finally, note that if this condition holds, then $(1-p_2)(C^H - C^L) > I_1(1+\overline{\theta}/q) > I_1$. Thus, if this holds, (A.15) is higher than (A.13) and (A.13) is higher than (A.11), completing our proof. That is, we have shown that $A(K_1) = 1$ maximizes ex-ante value if and only if Condition 4 holds. Since K_1 is chosen to optimize ex-ante value in the full commitment equilibrium, it follows that $A(K_1) = 1$ in the full commitment equilibrium if and only if Condition 4 holds.

Again, switching from (1) - (5) to (6) - (10) changes nothing in this argument, it only changes the definitions of $A(\,\cdot\,)$ and $\mathbf{1}_{\text{accept}}(K_1, \theta_i)$. In this sense, the above argument holds with or without LMEs; introducing LMEs only changes the definitions of $A(\,\cdot\,)$ and $\mathbf{1}_{\text{accept}}(K_1, \theta_i)$.

A.5 Proof of Proposition 1

We first introduce a lemma. The purpose of this lemma is to show that we can use the subgame-perfect investor responses described by (1) to (5) to evaluate the profitability of the relevant off-equilibrium-path deviations. This lemma is necessary because our derivation of (1) to (5) assumed that $K_0 + K_1 > C^L$, which might not necessarily hold for off-equilibrium-path deviations.

Lemma 4. *Fix K_0^* satisfying $C^H \geq K_0^* \geq C^L$. Then for any date-1 traditional renegotiation offer K_1 such that $K_0^* + K_1 \leq C^H$ and $A^{\text{Trad}}(K_1) > 0$, the subgame-perfect investor responses are described by (1) to (5).*

The proof of this lemma appears at the end of this proof. Because we assume $K_0 \leq C^H$ and $K_0 + K_1 \leq C^H$, then we derive from the result in Internet Appendix A.2 that $K_0 \geq C^L$. Using Lemma 4, we can thus use investor responses described by (1) to (5) to evaluate the profitability of any relevant K_1, on or off the equilibrium path. Taking the lemma as given, we now prove Proposition 1.

Fix an equilibrium in which investors and bank lenders anticipate $A^{\text{Trad}}(K_1^*) = \widetilde{P}^{\text{Trad}}(K_1^*) = 1$. Then (3) must hold for all θ_i, including $\theta_i = \overline{\theta}$, so

$$\begin{aligned}
&\frac{(1-p_2)K_0^* + p_2 C^L}{2} + \frac{1}{2}\big[-I_1(1+\overline{\theta}) + (1-p_2)K_1^*\big] \geq p_2\frac{C^L}{2} + \frac{(1-p_2)K_0^*}{2} \\
\iff\ & (1-p_2)K_1^* \geq I_1(1+\overline{\theta}) \iff K_1^* \geq \frac{I_1(1+\overline{\theta})}{1-p_2}.
\end{aligned}$$

If $C^H - K_0^* < \frac{I_1(1+\overline{\theta})}{1-p_2}$, then any K_1^* satisfying $A^{\text{Trad}}(K_1^*) = 1$ can not be an equilibrium because limited liability would be violated. Thus, we must have $C^H - K_0^* \geq \frac{I_1(1+\overline{\theta})}{1-p_2}$.

If $K_1^* > \frac{I_1(1+\overline{\theta})}{1-p_2}$, equity holders could reduce K_1^* without changing $A^{\text{Trad}}(K_1^*) = 1$, improving their objective (15). Thus, in any equilibrium in which $A^{\text{Trad}}(K_1^*) = 1$, we must have

$$K_1^* = K_1^{\text{Trad}} := \frac{I_1(1+\overline{\theta})}{1-p_2}. \tag{A.16}$$

When lenders anticipate (A.16) at $t = 0$, we have $A^{\text{Trad}}(K_1^*) = 1$, and all investors participate in restructurings. So by (A.14) the expected investor funding cost is $I_1\mathbb{E}[\Theta] \equiv I_1\overline{\theta}q$. Recall that we assume $C^H \geq K_0^* + K_1^* > C^L$ and $C^H \geq K_0^* > C^L$ and verify these inequalities later. Then (12) implies that

$$\begin{aligned}
I_0 &= (1-p_1)K_0^* + p_1\Big[C^L + (1-p_2)(K_0^* + K_1^* - C^L) - I_1 - I_1\mathbb{E}[\Theta]\Big] \\
&= (1-p_1p_2)K_0^* + p_1\Big[p_2C^L + I_1\overline{\theta} - I_1\overline{\theta}q\Big].
\end{aligned}$$

Thus, in any equilibrium in which lenders expect $A^{\text{Trad}}(K_1^*) = 1$,

$$K_0^* = K_0^{\text{Trad}} := \frac{I_0 - p_1\big[p_2C^L + I_1\overline{\theta}(1-q)\big]}{1-p_1p_2}. \tag{A.17}$$

By the argument of Internet Appendix A.1, we have $C^H > K_0^{\text{Trad}} + K_1^{\text{Trad}}$. Then, according to the result proved in Internet Appendix A.2, we have $K_0^{\text{Trad}} > C^L$. Since $K_1^* = K_1^{\text{Trad}} > 0$, we have $C^H > K_0^{\text{Trad}} + K_1^{\text{Trad}} > C^L$ and $C^H > K_0^{\text{Trad}} > C^L$.

Finally, we need to verify that $K_1^* = K_1^{\text{Trad}}$ solves equity's date one problem (15). Consider the alternative. Equity could set a low K_1 ensuring restructuring fails, which yields a value of zero for the objective. Alternatively, equity could set the lowest K_1 that ensures a type 0 investor accepts. To calculate this, note that if $\mathbf{1}_{\text{accept}}(K_1, \theta_i) = 0$ when $\theta_i = \overline{\theta}$ and $\mathbf{1}_{\text{accept}}(K_1, \theta_i) = 1$ when $\theta_i = 0$, then $\widetilde{P}^{\text{Trad}}(K_1) = 1 - q$. Off the equilibrium path, investors would accept such an offer if (3) holds with $\widetilde{P}^{\text{Trad}}(K_1) = 1 - q$:

$$\begin{aligned}
&\frac{(1-p_2)K_0^* + p_2 C^L}{2} + \left(1 - \frac{\widetilde{P}^{\text{Trad}}(K_1)}{2}\right)\big[(1-p_2)K_1 - I_1(1+\theta_i)\big] \\
&\geq (1 - \widetilde{P}^{\text{Trad}}(K_1) + p_2 \widetilde{P}^{\text{Trad}}(K_1))\frac{C^L}{2} + \widetilde{P}^{\text{Trad}}(K_1)\frac{(1-p_2)K_0^*}{2} \\
\iff\ &\frac{(1-p_2)K_0^* + p_2 C^L}{2} + \frac{1+q}{2}\big[(1-p_2)K_1 - I_1(1+\theta_i)\big] \\
&\geq [q + (1-q)p_2]\frac{C^L}{2} + \frac{(1-q)(1-p_2)K_0^*}{2} \\
\iff\ & q\frac{(1-p_2)K_0^* + p_2 C^L}{2} + \frac{1+q}{2}\big[(1-p_2)K_1 - I_1(1+\theta_i)\big] \geq \frac{qC^L}{2}.
\end{aligned}$$

Plugging in $\theta_i = 0$, dividing by $(1+q)(1-p_2)/2$, and rearranging, a type 0 investor accepts if and only if

$$K_1 \geq \frac{I_1}{1-p_2} + \frac{2}{(1+q)(1-p_2)}\left[\frac{qC^L}{2} - q\frac{(1-p_2)K_0^* + p_2 C^L}{2}\right]. \tag{A.18}$$

As before, if equity holders deviate to set $A^{\text{Trad}}(K_1) = 1 - q^2$, then they optimally choose the lowest K_1 that achieves this. Replacing the inequality with equality and simplifying,

$$K_1^{\text{Alt}} = \frac{I_1}{1-p_2} - \frac{q}{1+q}(K_0^* - C^L) = \frac{I_1}{1-p_2} - \frac{q}{1+q}(K_0^{\text{Trad}} - C^L). \tag{A.19}$$

We have $K_0^{\text{Trad}} + K_1^{\text{Alt}} - C^L = \frac{I_1}{1-p_2} + \frac{1}{1+q}(K_0^{\text{Trad}} - C^L) > 0$ using $K_0^{\text{Trad}} > C^L$.

Next, we compare the value of the date 1 equity objective under these two strategies. If $C^H - K_0^{\text{Trad}} - K_1^{\text{Trad}} \geq (1-q^2)\big(C^H - K_0^{\text{Trad}} - K_1^{\text{Alt}}\big)$, then it is not profitable to deviate to K_1^{Alt} when $C^H - K_0^{\text{Trad}} - K_1^{\text{Alt}} \geq 0$, and it is not feasible to deviate to K_1^{Alt} when $C^H - K_0^{\text{Trad}} - K_1^{\text{Alt}} < 0$. If $C^H - K_0^{\text{Trad}} - K_1^{\text{Trad}} < (1-q^2)\big(C^H - K_0^{\text{Trad}} - K_1^{\text{Alt}}\big)$, using $C^H - K_0^{\text{Trad}} - K_1^{\text{Trad}} > 0$, we conclude that $C^H - K_0^{\text{Trad}} - K_1^{\text{Alt}} > 0$, and deviating to K_1^{Alt} is feasible and profitable.

Thus, it is optimal to follow the equilibrium strategy $K_1^* = K_1^{\text{Trad}}$ rather than deviating to K_1^{Alt}

given in (A.19) if and only if

$$
\begin{aligned}
& C^H - K_0^{\text{Trad}} - K_1^{\text{Trad}} \geq (1-q^2)\big(C^H - K_0^{\text{Trad}} - K_1^{\text{Alt}}\big) \\
\iff\ & C^H - K_0^{\text{Trad}} - K_1^{\text{Trad}} \geq (1-q^2)\left[C^H - \frac{K_0^{\text{Trad}}}{1+q} - \left(\frac{I_1}{1-p_2} + \frac{qC^L}{1+q}\right)\right] \\
\iff\ & C^H - K_0^{\text{Trad}} - K_1^{\text{Trad}} + (1-q)K_0^{\text{Trad}} \geq (1-q^2)\left[C^H - \frac{I_1}{1-p_2}\right] - q(1-q)C^L \\
\iff\ & q^2(C^H - C^L) \geq K_1^{\text{Trad}} - (1-q^2)\frac{I_1}{1-p_2} + q(K_0^{\text{Trad}} - C^L) \\
\iff\ & q^2(C^H - C^L) \geq \frac{I_1(1+\overline{\theta})}{1-p_2} - (1-q^2)\frac{I_1}{1-p_2} + q(K_0^{\text{Trad}} - C^L) \\
\iff\ & (1-p_2)(C^H - C^L) \geq \frac{1}{q^2}\left[I_1\left(q^2 + \overline{\theta}\right)\right] + \frac{1-p_2}{q}(K_0^{\text{Trad}} - C^L).
\end{aligned} \tag{A.20}
$$

The right side is

$$
\mathbb{B}^{\text{Trad}} \equiv I_1\left(1 + \frac{\overline{\theta}}{q^2}\right) + \frac{1-p_2}{q}(K_0^{\text{Trad}} - C^L). \tag{A.21}
$$

Then we have shown that $A^{\text{Trad}}(K_1^*) = 1$ is an equilibrium if and only if $(1-p_2)(C^H - C^L) \geq \mathbb{B}^{\text{Trad}}$.

Further, since we have shown that $K_0^{\text{Trad}} > C^L$, we have

$$
\mathbb{B}^{\text{Trad}} = I_1\left(1 + \frac{\overline{\theta}}{q^2}\right) + \frac{1-p_2}{q}(K_0^{\text{Trad}} - C^L) \geq I_1\left(1 + \frac{\overline{\theta}}{q^2}\right) > I_1\left(1 + \frac{\overline{\theta}}{q}\right). \tag{A.22}
$$

Plugging the expression for $K_0^* = K_0^{\text{Trad}}$ into (A.21), we obtain

$$
\begin{aligned}
\mathbb{B}^{\text{Trad}} &= I_1\left(1 + \frac{\overline{\theta}}{q^2}\right) + \frac{1-p_2}{q}(K_0^{\text{Trad}} - C^L) \\
&= I_1\left(1 + \frac{\overline{\theta}}{q^2}\right) + \frac{1-p_2}{q}\left(\frac{I_0 - p_1\big[p_2C^L + I_1\overline{\theta}(1-q)\big]}{1-p_1p_2} - C^L\right).
\end{aligned}
$$

Note that

$$
\frac{I_0 - p_1p_2C^L}{1-p_1p_2} - C^L = \frac{I_0 - C^L}{1-p_1p_2}. \tag{A.23}
$$

Thus,

$$
\mathbb{B}^{\text{Trad}} = I_1\left(1 + \frac{\overline{\theta}}{q^2}\right) + \frac{1-p_2}{q}\,\frac{I_0 - C^L - p_1I_1\overline{\theta}(1-q)}{1-p_1p_2}. \tag{A.24}
$$

Combining (A.22) and (A.24), we obtain (16).

Finally, we prove that $(1-p_2)(C^H - C^L) \geq \mathbb{B}^{\text{Trad}}$ implies Condition 2. We only need to show that

$$\mathbb{B}^{\text{Trad}} > I_1(1+\overline{\theta}) + (I_0 - C^L)\frac{1-p_2}{1-p_1p_2}, \tag{A.25}$$

since $I_1(1+\overline{\theta})$ is larger than the first expression on the right side of Condition 2.

Using (A.24), we have

$$\begin{aligned}
&\mathbb{B}^{\text{Trad}} - \Big[I_1(1+\overline{\theta}) + (I_0 - C^L)\frac{1-p_2}{1-p_1p_2}\Big] \\
=&I_1\overline{\theta}\left(\frac{1}{q^2} - \frac{1-p_2}{q}\frac{p_1(1-q)}{1-p_1p_2} - 1\right) + (I_0 - C^L)\frac{1-p_2}{1-p_1p_2}\Big(\frac{1}{q} - 1\Big) \\
>&I_1\overline{\theta}\left(\frac{1}{q^2} - \Big(\frac{1}{q} - 1\Big)\frac{(1-p_2)p_1}{1-p_1p_2} - 1\right) > 0,
\end{aligned} \tag{A.26}$$

where the first inequality uses $\frac{1}{q} - 1 > 0$, the second inequality uses the fact that for any $\epsilon \in (0,1)$, $x^2 - (x-1)\epsilon - 1$ is increasing in $x > 1$. Then we finish the proof of Proposition 1.

Proof of Lemma 4: Since $C^H \geq K_0^* \geq C^L$, then for any feasible offer K_1 such that $C^H \geq K_0^* + K_1$, investor i's expected payoff from accepting the offer is

$$\begin{aligned}
\underline{R}_{\text{accept}}^{\text{Trad}}(\theta_i) =&(1-p_2)\frac{K_0^*}{2} + \left(1 - \frac{\widetilde{P}^{\text{Trad}}}{2}\right)\big[(1-p_2)K_1 - I_1(1+\theta_i)\big] \\
&+ p_2\left[\widetilde{P}^{\text{Trad}} \min\Big\{\frac{C^L}{2}, \frac{K_0^* + K_1}{2}\Big\} + (1 - \widetilde{P}^{\text{Trad}}) \min\Big\{\frac{C^L}{2}, \frac{K_0^*}{2} + K_1\Big\}\right],
\end{aligned} \tag{A.27}$$

and the expected payoff from rejecting the offer is still $R_{\text{reject}}^{\text{Trad}}$ given by (2).

Compared with $R_{\text{accept}}^{\text{Trad}}(\theta_i)$ given in (1), we have $\underline{R}_{\text{accept}}^{\text{Trad}}(\theta_i) \leq R_{\text{accept}}^{\text{Trad}}(\theta_i)$. The investor i accepts the offer K_1 if and only if $\underline{R}_{\text{accept}}^{\text{Trad}}(\theta_i) \geq R_{\text{reject}}^{\text{Trad}}$. Then we have $R_{\text{accept}}^{\text{Trad}}(\theta_i) \geq \underline{R}_{\text{accept}}^{\text{Trad}}(\theta_i) \geq R_{\text{reject}}^{\text{Trad}}$.

For the case $A^{\text{Trad}}(K_1) = 1$, we have $\widetilde{P}^{\text{Trad}} = 1$. Following the proof in Proposition 1, we derive from $R_{\text{accept}}^{\text{Trad}}(\overline{\theta}) \geq R_{\text{reject}}^{\text{Trad}}$ that $K_1 \geq \frac{I_1(1+\overline{\theta})}{1-p_2}$. Since $K_1 > 0$ and $K_0^* \geq C^L$, we have $K_0^* + K_1 > C^L$ and $\frac{K_0^*}{2} + K_1 > \frac{C^L}{2}$. Thus, $R_{\text{accept}}^{\text{Trad}}(\theta_i) = \underline{R}_{\text{accept}}^{\text{Trad}}(\theta_i)$.

For the case $A^{\text{Trad}}(K_1) = 1 - q^2$, we have $\widetilde{P}^{\text{Trad}} = 1 - q$. Following the proof in Proposition 1, we derive from $R^{\text{Trad}}_{\text{accept}}(0) \geq R^{\text{Trad}}_{\text{reject}}$ that

$$K_1 \geq \frac{I_1}{1-p_2} - \frac{q}{1+q}(K_0^* - C^L). \tag{A.28}$$

Using $K_0^* \geq C^L$, we have

$$K_0^* + K_1 - C^L \geq \frac{I_1}{1-p_2} + \frac{1}{1+q}(K_0^* - C^L) > 0, \tag{A.29}$$

and

$$\frac{K_0^* - C^L}{2} + K_1 \geq \frac{I_1}{1-p_2} + \Big(\frac{1}{2} - \frac{q}{1+q}\Big)(K_0^* - C^L) > \frac{1-q}{2(1+q)}(K_0^* - C^L) \geq 0. \tag{A.30}$$

Thus, $R^{\text{Trad}}_{\text{accept}}(\theta_i) = \underline{R}^{\text{Trad}}_{\text{accept}}(\theta_i)$.

A.6 Proof of Proposition 2

Recall that, under LMEs, the subgame-perfect investor responses are given by (6) to (10). The derivation of these equations implicitly assumed that $K_0 + K_1 > C^L$. In this proof, we assume that it is without loss of generality to restrict attention to offers K_1 such that investors respond according to (6) to (10). We then verify this assumption.

By (6) - (8), investor i accepts an offer K_1 if and only if

$$\begin{aligned}&\frac{(1-p_2)K_0^*}{2} + \Big(1 - \frac{\widetilde{P}^{\text{LME}}(K_1)}{2}\Big)\big[p_2 C^L - I_1(1+\theta_i) + (1-p_2)K_1\big] \\ \geq &(1 - \widetilde{P}^{\text{LME}}(K_1))\frac{C^L}{2} + \widetilde{P}^{\text{LME}}(K_1)\frac{(1-p_2)K_0^*}{2}.\end{aligned} \tag{A.31}$$

We follow the same steps as in the proof of Proposition 1. Fix an equilibrium in which investors and bank lenders anticipate $A^{\text{LME}}(K_1^*) = \widetilde{P}^{\text{LME}}(K_1^*) = 1$. Setting $\theta_i = \overline{\theta}$ in (A.31), we obtain

$$\begin{aligned}&\frac{(1-p_2)K_0^*}{2} + \frac{1}{2}\big[p_2 C^L - I_1(1+\overline{\theta}) + (1-p_2)K_1\big] \geq \frac{(1-p_2)K_0^*}{2} \\ \iff\ & K_1 \geq \frac{I_1(1+\overline{\theta}) - p_2 C^L}{1-p_2}.\end{aligned} \tag{A.32}$$

If $C^H - K_0^* < \frac{I_1(1+\overline{\theta})-p_2 C^L}{1-p_2}$, then any K_1^* satisfying $A^{\text{LME}}(K_1^*) = 1$ can not be an equilibrium because limited liability is violated. Thus, we must have $C^H - K_0^* \geq \frac{I_1(1+\overline{\theta})-p_2 C^L}{1-p_2}$.

If $K_1^* > \frac{I_1(1+\overline{\theta})-p_2 C^L}{1-p_2}$, equity holders could reduce K_1^* without changing $A^{\text{LME}}(K_1^*) = 1$, improving their objective at time 1. So, we must have

$$K_1^* = K_1^{\text{LME}} := \frac{I_1(1+\overline{\theta}) - p_2 C^L}{1-p_2}. \tag{A.33}$$

We can see that K_1^{LME} is smaller than K_1^{Trad}, which is given in (A.16). Additionally, under Condition 3, we have $K_1^{\text{LME}} > 0$.

When lenders anticipate (A.33) at $t = 0$, we have $A^{\text{LME}}(K_1^*) = 1$, and all investors participate in restructurings. So by (A.14) the expected investor funding cost is $I_1\mathbb{E}[\Theta] \equiv I_1\overline{\theta}q$. Recall that we assume $C^H \geq K_0^* + K_1^* > C^L$ and $C^H > K_0^* > C^L$ and verify these inequalities later. Then (12) and (A.33) imply that

$$\begin{aligned} I_0 &= (1-p_1)K_0^* + p_1\Big[C^L + (1-p_2)(K_0^* + K_1^* - C^L) - I_1 - I_1\mathbb{E}[\Theta]\Big] \\ &= (1-p_1p_2)K_0^* + p_1\Big[I_1\overline{\theta} - I_1\overline{\theta}q\Big]. \end{aligned} \tag{A.34}$$

Thus, in any equilibrium in which lenders expect $A^{\text{LME}}(K_1^*) = 1$,

$$K_0^* = K_0^{\text{LME}} := \frac{I_0 - p_1 I_1 \overline{\theta}(1-q)}{1-p_1p_2}. \tag{A.35}$$

We can see that K_0^{LME} is larger than K_0^{Trad}, which is given in (A.17). Further, the argument in Internet Appendix A.1 shows that $C^H > K_0^{\text{LME}} + K_1^{\text{LME}}$. Then, according to the result proved in Internet Appendix A.2, we have $K_0^{\text{LME}} > C^L$. Since $K_1^* = K_1^{\text{LME}} > 0$, we have $C^H > K_0^{\text{LME}} + K_1^{\text{LME}} > C^L$ and $C^H > K_0^{\text{LME}} > C^L$.

We need to verify that $K_1^* = K_1^{\text{LME}}$ solves equity's date one problem (15). Consider the alternative. Equity could set a low K_1 ensuring restructuring fails, which yields a value of zero for the objective. Alternatively, equity could set the lowest K_1 that ensures a type 0 investor accepts. To calculate this, note that if $\mathbf{1}_{\text{accept}}(K_1, \theta_i) = 0$ when $\theta_i = \overline{\theta}$ and $\mathbf{1}_{\text{accept}}(K_1, \theta_i) = 1$ when $\theta_i = 0$, then $\widetilde{P}^{\text{LME}}(K_1) = 1 - q$. Off the equilibrium path, investors would accept such an offer if (8) holds with $\widetilde{P}^{\text{LME}}(K_1) = 1 - q$:

$$\begin{aligned}
&\frac{(1-p_2)K_0^*}{2} + \left(1 - \frac{\widetilde{P}^{\text{LME}}(K_1)}{2}\right)\left[p_2 C^L - I_1(1+\theta_i) + (1-p_2)K_1\right] \\
&\geq (1 - \widetilde{P}^{\text{LME}}(K_1))\frac{C^L}{2} + \widetilde{P}\frac{(1-p_2)K_0^*}{2} \\
\Longleftrightarrow\ & \frac{(1-p_2)K_0^*}{2} + \frac{1+q}{2}\left[p_2 C^L - I_1(1+\theta_i) + (1-p_2)K_1\right] \geq q\frac{C^L}{2} + \frac{(1-q)(1-p_2)K_0^*}{2} \\
\Longleftrightarrow\ & \frac{q(1-p_2)K_0^*}{2} + \frac{1+q}{2}\left[-I_1(1+\theta_i) + (1-p_2)K_1\right] \geq \frac{C^L}{2}\left[q(1-p_2) - p_2\right].
\end{aligned}$$

Plugging in $\theta_i = 0$, dividing by $(1+q)(1-p_2)/2$, and rearranging, a type 0 investor accepts if and only if

$$\begin{aligned}
K_1 &\geq \frac{I_1}{1-p_2} + \frac{2}{(1-p_2)(1+q)}\left[\frac{C^L}{2}\left[q(1-p_2) - p_2\right] - \frac{q(1-p_2)K_0^*}{2}\right] \\
&= \frac{I_1}{1-p_2} + \frac{1}{1+q}\left[C^L\left[q - \frac{p_2}{1-p_2}\right] - K_0^* q\right] \\
&= \frac{I_1}{1-p_2} + \frac{q}{1+q}(C^L - K_0^*) - \frac{p_2 C^L}{(1+q)(1-p_2)}.
\end{aligned} \tag{A.36}$$

Then we obtain

$$K_1^{\text{Alt}} = \frac{I_1}{1-p_2} - \frac{q}{1+q}(K_0^* - C^L) - \frac{p_2 C^L}{(1+q)(1-p_2)}. \tag{A.37}$$

We have

$$\begin{aligned}
&K_0^* + K_1^{\text{Alt}} - C^L = \frac{I_1}{1-p_2} + \frac{1}{1+q}(K_0^* - C^L) - \frac{p_2 C^L}{(1+q)(1-p_2)} \\
&> \frac{I_1}{1-p_2} - \frac{p_2 C^L}{(1+q)(1-p_2)} > 0,
\end{aligned} \tag{A.38}$$

where the first inequality uses $K_0^* > C^L$, and the second inequality uses Condition 3.

Next, we compare the value of the date-1 equity objective under these two strategies. Similar to the proof of Proposition 1, it is optimal to follow the equilibrium strategy $K_1^* = K_1^{\text{LME}}$ rather than deviating to K_1^{Alt} given in (A.37) if and only if

$$\begin{aligned}
& C^H - K_0^* - K_1^* \geq (1-q^2)\Big[C^H - K_0^* - K_1^{\text{Alt}}\Big] \\
\iff\ & C^H - K_0^* - K_1^* \geq (1-q^2)\Big[C^H - K_0^* - \Big(\frac{I_1}{1-p_2} - \frac{q}{1+q}(K_0^* - C^L) - \frac{p_2 C^L}{(1+q)(1-p_2)}\Big)\Big] \\
\iff\ & C^H - K_0^* - K_1^* \geq (1-q^2)\Big[C^H - \frac{I_1}{1-p_2}\Big] - (1-q)K_0^* - q(1-q)C^L + \frac{p_2 C^L(1-q)}{(1-p_2)} \\
\iff\ & q^2(C^H - C^L) \geq K_1^* - (1-q^2)\frac{I_1}{1-p_2} + q(K_0^* - C^L) + \frac{p_2 C^L(1-q)}{(1-p_2)}. \qquad \text{(A.39)}
\end{aligned}$$

Multiplying both sides of (A.39) by $(1-p_2)/q^2$, we have

$$(1-p_2)(C^H - C^L) \geq \frac{1-p_2}{q^2}\Big[K_1^* - (1-q^2)\frac{I_1}{1-p_2} + q(K_0^* - C^L) + \frac{p_2 C^L(1-q)}{(1-p_2)}\Big]. \qquad \text{(A.40)}$$

The right side is

$$\begin{aligned}
\mathbb{B}^{\text{LME}} &\equiv \frac{1-p_2}{q^2}\Big[K_1^{\text{LME}} - (1-q^2)\frac{I_1}{1-p_2} + q(K_0^* - C^L) + \frac{p_2 C^L(1-q)}{(1-p_2)}\Big] \\
&= \frac{I_1(1+\overline{\theta}) - p_2 C^L}{q^2} - \frac{1-q^2}{q^2}I_1 + \frac{1-p_2}{q}(K_0^* - C^L) + \frac{p_2 C^L(1-q)}{q^2} \\
&= I_1\Big(1 + \frac{\overline{\theta}}{q^2}\Big) + \frac{1-p_2}{q}K_0^* - \frac{C^L}{q}, \qquad \text{(A.41)}
\end{aligned}$$

where the first equality uses $K_1^{\text{LME}} = \frac{I_1(1+\overline{\theta}) - p_2 C^L}{1-p_2}$. Substituting $K_0^* = K_0^{\text{LME}} = \frac{I_0 - p_1 I_1 \overline{\theta}(1-q)}{1-p_1 p_2}$ into (A.41), we obtain

$$\mathbb{B}^{\text{LME}} = I_1\Big(1 + \frac{\overline{\theta}}{q^2}\Big) + \frac{1-p_2}{q(1-p_1 p_2)}\big[I_0 - p_1 I_1 \overline{\theta}(1-q)\big] - \frac{C^L}{q}.$$

Comparing this to

$$\mathbb{B} = I_1\Big(1 + \frac{\overline{\theta}}{q^2}\Big) + \frac{1-p_2}{q}\frac{I_0 - C^L - p_1 I_1 \overline{\theta}(1-q)}{1-p_1 p_2}, \qquad \text{(A.42)}$$

we see that

$$\mathbb{B}^{\text{LME}} = \mathbb{B} - \frac{C^L}{q}\Big[1 - \frac{1-p_2}{1-p_1 p_2}\Big] < \mathbb{B}. \qquad \text{(A.43)}$$

Since $C^H > K_0^* > C^L$, for any feasible offer K_1 such that $C^H \geq K_0^* + K_1$, investor i's expected payoff from accepting the offer in a LME is

$$\underline{R}^{\text{LME}}_{\text{accept}}(\theta_i) = (1-p_2)\frac{K_0^*}{2} + \left(1 - \frac{\widetilde{P}^{\text{LME}}}{2}\right)\left[(1-p_2)K_1 - I_1(1+\theta_i)\right] + p_2\left[\widetilde{P}^{\text{LME}} \min\left\{\frac{C^L}{2}, \frac{K_0^* + K_1}{2}\right\} + (1-\widetilde{P}^{\text{LME}}) \min\left\{C^L, \frac{K_0^*}{2} + K_1\right\}\right], \tag{A.44}$$

and the expected payoff from rejecting the offer is still $R^{\text{LME}}_{\text{reject}}$ given by (7). Compared with $R^{\text{LME}}_{\text{accept}}(\theta_i)$ given in (6), we have $\underline{R}^{\text{LME}}_{\text{accept}}(\theta_i) \leq R^{\text{LME}}_{\text{accept}}(\theta_i)$. Thus, while achieving the same restructuring probability $A^{\text{LME}}(K_1) > 0$, K_1 solved from $\underline{R}^{\text{LME}}_{\text{accept}}(\theta_i) = R^{\text{LME}}_{\text{reject}}$ is weakly higher than K_1 solved from $R^{\text{LME}}_{\text{accept}}(\theta_i) = R^{\text{LME}}_{\text{reject}}$. Thus, it is sufficient to consider K_1 such that the subgame-perfect investor responses are described by (6) to (10): if K_1^{Alt} derived from (6) to (10) is not a profitable deviation, then K_1^{Alt} derived using $\underline{R}^{\text{LME}}_{\text{accept}}(\theta_i)$ is not a profitable deviation. Then we derive from (A.39) that the first part of Proposition 2 holds.

To prove the second part, assume equity can choose between an LME or traditional renegotiation at date 1. There are four possibilities for the equilibrium outcome: (A) an LME with $\widetilde{P} = 1$, (B) an LME with $\widetilde{P} = 1 - q$, (C) traditional renegotiation with $\widetilde{P} = 1$, or (D) traditional renegotiation with $\widetilde{P} = 1 - q$. Note that, ex post, equity will always prefer renegotiation with some positive probability to no renegotiation since the latter delivers an ex post objective of zero. First, observe that (C) cannot be the equilibrium outcome: equity can achieve $\widetilde{P} = 1$ for cheaper ex post with an LME, since (A.33) is lower than (A.16). Likewise, (D) cannot be an equilibrium outcome: equity can achieve $\widetilde{P} = 1 - q$ for cheaper ex post with an LME since (A.37) is lower than (A.19) for the same K_0^*. Thus, the equilibrium outcome must be an LME. That leaves (A) and (B), both of which entail LMEs, proving the second part of the Proposition 2.

A.7 Proof of Proposition 3

We begin by introducing the additional conditions required to prove Proposition 3.

Condition 5. $\mathbb{B}^{\text{LME}} < (1-p_2)(C^H - C^L) < \mathbb{B}^{\text{Trad}}$ *and*

$$(1-p_2)(C^H - C^L) > I_1 + \frac{1-p_2}{1+q}\frac{I_0 - C^L}{1 - p_1\big[1-(1-q)(1-p_2)\big]}, \tag{A.45}$$

$$(1-p_2)(C^H - C^L) > (1-p_2)\frac{I_0 - C^L}{1 - p_1\big[1-(1-q)(1-p_2)\big]}, \tag{A.46}$$

$$(1-p_2)(C^H - C^L) > I_1\left(1 + \frac{\overline{\theta}}{q^2}\right) + \frac{1}{q}\frac{(1-p_2)I_0 - (1-p_1p_2)C^L}{1 - p_1\big[1-(1-q)(1-p_2)\big]}. \tag{A.47}$$

Following the proofs of Propositions 1 and 2, we only consider an offer K_1 such that the subgame-perfect investor responses are described by (1) to (5) in a traditional renegotiation, and we only consider an offer K_1 such that the subgame-perfect investor responses are described by (6) to (10) in an LME.

Using Lemma 1, we can denote ex ante firm value as $E_0(A(K_1))$ and there are three possible cases. When $A(K_1) = 0$, the ex-ante firm value is

$$E_0(0) = (1-p_1)C^H + p_1 C^L - I_0. \tag{A.48}$$

When $A(K_1) = 1 - q^2$, the ex-ante firm value is

$$E_0(1-q^2) = (1-p_1)C^H + p_1 C^L + p_1(1-q^2)\Big[(1-p_2)(C^H - C^L) - I_1\Big] - I_0. \tag{A.49}$$

When $A(K_1) = 1$, the ex-ante firm value is

$$E_0(1) = (1-p_1)C^H + p_1 C^L + p_1\Big[(1-p_2)(C^H - C^L) - I_1(1 + q\overline{\theta})\Big] - I_0. \tag{A.50}$$

Since $(1-p_2)(C^H - C^L) > I_1$, we have $E_0(1-q^2) > E_0(0)$. The proof of Lemma 2 shows that $(1-p_2)(C^H - C^L) < I_1(1 + \overline{\theta}/q)$ implies that $E_0(1-q^2) > E_0(1)$. Thus, in order to prove the first part of Proposition 3, we only need to show that there exists K_0^*, K_1^* such that $C^H > K_0^* + K_1^* > C^L$, $C^H > K_0^* > C^L$, $A(K_1^*) = 1 - q^2$.

We derive from (A.19) that

$$K_1^* = \frac{I_1}{1-p_2} - \frac{q}{1+q}(K_0^* - C^L). \tag{A.51}$$

It follows that

$$K_1^* + K_0^* - C^L = \frac{I_1}{1-p_2} + \frac{K_0^* - C^L}{1+q}. \tag{A.52}$$

Substituting $A(K_1^*) = 1 - q^2$ and $\mathbb{E}[\Theta] = 0$ into (12), we obtain

$$\begin{aligned}
I_0 =&(1-p_1)K_0^* + p_1 C^L + p_1(1-q^2)[(1-p_2)(K_0^* + K_1^* - C^L) - I_1] \\
=&(1-p_1)K_0^* + p_1 C^L + p_1(1-q)(1-p_2)(K_0^* - C^L) \\
=&\Big[1 - p_1\big[1-(1-q)(1-p_2)\big]\Big]K_0^* + p_1\big[1-(1-q)(1-p_2)\big]C^L,
\end{aligned} \tag{A.53}$$

where the second equality uses (A.52). Rearranging, we obtain

$$K_0^* = \frac{I_0 - p_1\big[1-(1-q)(1-p_2)\big]C^L}{1 - p_1\big[1-(1-q)(1-p_2)\big]}. \tag{A.54}$$

Substituting $A(K_1^*) = 1 - q^2$ and $\mathbb{E}[\Theta] = 0$ into (12), we can see $C^H > K_0^* + K_1^*$ if and only if

$$I_0 <(1-p_1)K_0^* + p_1 C^L + p_1(1-q^2)[(1-p_2)(C^H - C^L) - I_1], \tag{A.55}$$

which is equivalent to

$$\begin{aligned}
&(1-p_2)(C^H - C^L) > I_1 + \frac{I_0 - (1-p_1)K_0^* - p_1 C^L}{p_1(1-q^2)} \\
=&I_1 + \frac{1-p_2}{1+q}(K_0^* - C^L) \\
=&I_1 + \frac{1-p_2}{1+q}\frac{I_0 - C^L}{1 - p_1\big[1-(1-q)(1-p_2)\big]},
\end{aligned} \tag{A.56}$$

where the first equality uses the second equality in (A.53), and the second equality uses (A.54). We can see from (A.45) that (A.56) holds. Hence, $C^H > K_0^* + K_1^*$ holds. Additionally, we derive from Internet Appendix A.2 that $K_0^* > C^L$. Then we derive from (A.52) that $K_0^* + K_1^* > C^L$.

Note that $C^H > K_0^*$ is equivalent to $(1-p_2)(C^H - C^L) > (1-p_2)(K_0^* - C^L)$. Using (A.54), we have

$$(1-p_2)(K_0^* - C^L) = (1-p_2)\frac{I_0 - C^L}{1 - p_1\big[1-(1-q)(1-p_2)\big]}. \tag{A.57}$$

Then we derive from (A.46) and (A.57) that $C^H > K_0^*$. This proves the first part of Proposition 3.

We now prove the nonexistence of an equilibrium $(\widetilde{K}_0, \widetilde{K}_1)$ with LMEs in which $A^{\text{LME}}(\widetilde{K}_1) = 1 - q^2$. If $C^H < \widetilde{K}_0 + \widetilde{K}_1$ or $C^H < \widetilde{K}_0$, then it is clear that $(\widetilde{K}_0, \widetilde{K}_1)$ is not an equilibrium. Thus, we only need to consider the case $C^H \geq \widetilde{K}_0 + \widetilde{K}_1$ and $C^H \geq \widetilde{K}_0$. According to the result proved in Internet Appendix A.2, we have $\widetilde{K}_0 > C^L$.

From equation (A.37) in the proof of Proposition 2, if $A^{\text{LME}}(\widetilde{K}_1) = 1 - q^2$, equity optimality implies that

$$\widetilde{K}_1 = \frac{I_1}{1-p_2} - \frac{q}{1+q}(\widetilde{K}_0 - C^L) - \frac{p_2 C^L}{(1+q)(1-p_2)}. \tag{A.58}$$

It follows that

$$\widetilde{K}_0 + \widetilde{K}_1 - C^L = \frac{I_1}{1-p_2} + \frac{1}{1+q}(\widetilde{K}_0 - C^L) - \frac{p_2 C^L}{(1+q)(1-p_2)}. \tag{A.59}$$

Using Condition 3, $\widetilde{K}_0 > C^L$, and (A.59), we conclude that $\widetilde{K}_0 + \widetilde{K}_1 > C^L$.

To calculate the $\widetilde{K}_0$ implied by this, we start with the break-even condition, noting that the expected investor funding cost is zero since high-cost investors reject the offer in this equilibrium, and calculate:

$$\begin{aligned}
I_0 &= (1-p_1)\widetilde{K}_0 + p_1\Big[C^L + (1-q^2)\,\big(\,(1-p_2)(\widetilde{K}_0 + \widetilde{K}_1 - C^L) - I_1\,\big)\Big] \\
&= (1-p_1)\widetilde{K}_0 + p_1\Big[C^L + (1-q^2)\,\Big(\,\frac{1-p_2}{1+q}(\widetilde{K}_0 - C^L) - \frac{p_2 C^L}{(1+q)}\,\Big)\Big] \\
&= (1-p_1)\widetilde{K}_0 + p_1\Big[C^L + (1-q)\,\Big(\,(1-p_2)\widetilde{K}_0 - C^L\,\Big)\Big] \\
&= (1-p_1)\widetilde{K}_0 + p_1\Big[qC^L + (1-q)(1-p_2)\widetilde{K}_0\Big],
\end{aligned} \tag{A.60}$$

where the second equality uses (A.59). Rearranging, we obtain

$$\widetilde{K}_0 = \frac{I_0 - p_1 q C^L}{1 - p_1\big[1 - (1-q)(1-p_2)\big]}. \tag{A.61}$$

Recall from equation (A.33) in the proof of Proposition 2 that the optimal K_1^* that ensures $A^{\text{LME}}(K_1^*) = 1$ is

$$K_1^* = K_1^{\text{LME}} = \frac{I_1(1+\overline{\theta}) - p_2 C^L}{1 - p_2}. \tag{A.62}$$

Using Condition 3, we have $K_1^{\text{LME}} > 0$. Since we have shown that $\widetilde{K}_0 > C^L$, we have $\widetilde{K}_0 + K_1^{\text{LME}} > C^L$.

If $C^H - \widetilde{K}_0 - K_1^{\text{LME}} > (1-q^2)\Big[C^H - \widetilde{K}_0 - \widetilde{K}_1\Big]$, using $C^H \geq \widetilde{K}_0 + \widetilde{K}_1$, we conclude that $C^H - \widetilde{K}_0 - K_1^{\text{LME}} > 0$, and deviating to K_1^{LME} is feasible and profitable. Thus, it is profitable to deviate from the strategy $\widetilde{K}_1$ to K_1^{LME} if

$$\begin{aligned}
& C^H - \widetilde{K}_0 - K_1^{\text{LME}} > (1-q^2)\Big[C^H - \widetilde{K}_0 - \widetilde{K}_1\Big] \\
\iff\ & C^H - \widetilde{K}_0 - K_1^{\text{LME}} > (1-q^2)\Bigg[C^H - \widetilde{K}_0 - \left(\frac{I_1}{1-p_2} - \frac{q}{1+q}(\widetilde{K}_0 - C^L) - \frac{p_2 C^L}{(1+q)(1-p_2)}\right)\Bigg] \\
\iff\ & C^H - \widetilde{K}_0 - K_1^{\text{LME}} > (1-q^2)\Bigg[C^H - \frac{I_1}{1-p_2}\Bigg] - (1-q)\widetilde{K}_0 - q(1-q)C^L + \frac{p_2 C^L (1-q)}{(1-p_2)} \\
\iff\ & q^2(C^H - C^L) > K_1^{\text{LME}} - (1-q^2)\frac{I_1}{1-p_2} + q(\widetilde{K}_0 - C^L) + \frac{p_2 C^L(1-q)}{(1-p_2)}.
\end{aligned} \tag{A.63}$$

Multiplying both sides of (A.63) by $(1-p_2)/q^2$, we have

$$(1-p_2)(C^H - C^L) > \frac{1-p_2}{q^2}\Bigg[K_1^{\text{LME}} - (1-q^2)\frac{I_1}{1-p_2} + q(\widetilde{K}_0 - C^L) + \frac{p_2 C^L(1-q)}{(1-p_2)}\Bigg]. \tag{A.64}$$

The right-hand side of (A.64) is identical to that of (A.41), except that K_0^* is replaced by $\widetilde{K}_0$ in

(A.61). Thus, the right side of (A.64) is

$$
\begin{aligned}
& I_1 \left(1+\frac{\overline{\theta}}{q^2}\right)+\frac{1-p_2}{q}\widetilde{K}_0-\frac{C^L}{q} \\
=& I_1 \left(1+\frac{\overline{\theta}}{q^2}\right)+\frac{1-p_2}{q}\frac{I_0-p_1 q C^L}{1-p_1\left[1-(1-q)(1-p_2)\right]}-\frac{C^L}{q} \\
=& I_1 \left(1+\frac{\overline{\theta}}{q^2}\right)+\frac{1}{q}\frac{(1-p_2)I_0-(1-p_1p_2)C^L}{1-p_1\left[1-(1-q)(1-p_2)\right]},
\end{aligned}
\tag{A.65}
$$

where the first equality uses (A.61).

Thus, when inequality (A.47) holds, we conclude that (A.63) holds, and there is no equilibrium with LMEs in which $A^{\mathrm{LME}}(\widetilde{K}_1)=1-q^2$. Further, from Proposition 2, there exists an equilibrium $(K_0^{\mathrm{LME}}, K_1^{\mathrm{LME}})$ with $A^{\mathrm{LME}}(K_1^{\mathrm{LME}})=1$ if

$$(1-p_2)(C^H-C^L)\geq \mathbb{B}^{\mathrm{LME}}.$$

Proposition 2 also implies that, under the above condition, shareholders will opt for an LME when given the choice between an LME and traditional renegotiation at date 1. According to the proof of Proposition 2, $A^{\mathrm{LME}}(K_1)=1$ if and only if $K_1\geq K_1^{\mathrm{LME}}$. For $K_1>K_1^{\mathrm{LME}}$, it is profitable to deviate from K_1 to K_1^{LME}. Combining these facts, and noting that equity would never choose $A^{\mathrm{LME}}(K_1)=0$ ex post since it guarantees them a zero payoff, we can see that the unique equilibrium is $(K_0^{\mathrm{LME}}, K_1^{\mathrm{LME}})$ with LMEs, which features $A^{\mathrm{LME}}(K_1^{\mathrm{LME}})=1$. Combining this and the first part of the Proposition, the third part of Proposition 3 follows.

All that remains is to prove the second part of Proposition 3. From the proof of Proposition 1, if $A^{\mathrm{Trad}}(K_1^*)=1-q^2$, equity optimality implies that K_1^* is given by (A.51). Using the break-even condition, we obtain K_0^* as shown in (A.54). Recall that we have shown that $C^H>K_0^*+K_1^*>C^L$ and $C^H>K_0^*>C^L$.

To verify that (K_0^*, K_1^*) comprises an equilibrium, we only need to consider the deviation K_1 at date 1 such that $A^{\mathrm{Trad}}(K_1)=1$. From equation (A.16) in the proof of Proposition 1, the optimal offer K_1 that achieves $A^{\mathrm{Trad}}(K_1)=1$ is $K_1=K_1^{\mathrm{Trad}}=I_1(1+\overline{\theta})/(1-p_2)$. Comparing it with K_1^* in (A.51) and using $K_0^*>C^L$, we have $K_1^{\mathrm{Trad}}>K_1^*$. Using $K_0^*+K_1^*>C^L$, we have $K_0^*+K_1^{\mathrm{Trad}}>C^L$.

Similar to (A.63), (K_0^*, K_1^*) comprises an equilibrium if

$$\begin{aligned} & C^H - K_0^* - K_1^{\text{Trad}} < (1-q^2)(C^H - K_0^* - K_1^*) \\ \iff & C^H - K_0^* - K_1^{\text{Trad}} < (1-q^2)\left[C^H - \frac{K_0^*}{1+q} - \left(\frac{I_1}{1-p_2} + \frac{qC^L}{1+q}\right)\right]. \end{aligned} \tag{A.66}$$

Following (A.20), we can see the above inequality is equivalent to

$$(1-p_2)(C^H - C^L) < \frac{1}{q^2}\left[I_1\left(q^2 + \overline{\theta}\right)\right] + \frac{1-p_2}{q}(K_0^* - C^L). \tag{A.67}$$

The right side of (A.67) is the same as $\mathbb{B}^{\text{Trad}}$ given in (A.21), except that K_0^{Trad}, as defined in (A.17), is replaced by K_0^* as given in (A.54). In the current proof, we assume $(1-p_2)(C^H - C^L) < \mathbb{B}^{\text{Trad}}$, so we only need to show that $K_0^{\text{Trad}} < K_0^*$.

Note that for any $Z < 1$,

$$\frac{\partial}{\partial Z}\frac{I_0 - ZC^L}{1-Z} = \frac{-C^L(1-Z) + I_0 - ZC^L}{(1-Z)^2} = \frac{I_0 - C^L}{(1-Z)^2} > 0. \tag{A.68}$$

It follows that

$$\begin{aligned} K_0^{\text{Trad}} &= \frac{I_0 - p_1\left[p_2C^L + I_1\overline{\theta}(1-q)\right]}{1-p_1p_2} \\ &< \frac{I_0 - p_1p_2C^L}{1-p_1p_2} < \frac{I_0 - p_1C^L\big[1-(1-q)(1-p_2)\big]}{1-p_1\big[1-(1-q)(1-p_2)\big]} = K_0^*, \end{aligned} \tag{A.69}$$

where the second inequality uses (A.68) and $1-(1-q)(1-p_2) = p_2 + q(1-p_2) > p_2$.

Thus, $(1-p_2)(C^H - C^L) < \mathbb{B}^{\text{Trad}}$ implies that (A.67) holds and (K_0^*, K_1^*) comprises an equilibrium. This proves the second part of Proposition 3, concluding the proof.

B Exclusive LMEs

In this Appendix, we formalize the extension summarized in Subsection 5.1. As described in that section, we make two changes to the baseline model. First, we introduce convex funding costs. Second, we introduce exclusive LMEs.

Convex funding costs. For simplicity, we continue to assume that low-cost investors ($\theta_i = 0$) have zero funding cost in all situations. However, we now assume that high-cost investors have convex funding costs. If two high-cost investors split the deal, each injecting $I_1/2$, then they each pay a funding cost $\overline{\theta}_2 I_1/2$ for a total funding cost $\overline{\theta}_2$. However, if one high-cost investor funds the deal alone, then they pay a higher funding cost $\overline{\theta}_1 I_1$, where $\overline{\theta}_1 > \overline{\theta}_2$. Finally, if a high-cost investor splits a deal with a low-cost investor, then the high-cost investor pays $\overline{\theta}_2 I_1/2$. In summary:

$$\text{One high-cost investor} \Rightarrow \text{Total funding cost} = \overline{\theta}_1 I_1 \tag{B.1}$$

$$\text{One or two low-cost investors} \Rightarrow \text{Total funding cost} = 0 \tag{B.2}$$

$$\text{One low-cost investor and one high-cost investor} \Rightarrow \text{Total funding cost} = \overline{\theta}_2 I_1/2 \tag{B.3}$$

$$\text{Two high-cost investors} \Rightarrow \text{Total funding cost} = \overline{\theta}_2 I_1. \tag{B.4}$$

Exclusive LMEs. Consider a subgame in which the date-0 contract specified a payment K_0^*, the firm entered distress at date 1, and equity holders make an exclusive LME offer K_1 to debt holder i. The debt holder knows that if they reject, the deal fails and the firm enters bankruptcy, so they receive $C^L/2$. If the debt holder accepts, they receive their original debt value and the entire LME payoff:

$$R^{\text{Exc}}_{\text{accept}}(\theta_i) = \underbrace{\frac{(1-p_2)K_0^*}{2}}_{\text{Original debt contract}} + \underbrace{\left[p_2 C^L - I_1(1+\overline{\theta}_1 \mathbf{1}(\theta_i > 0)) + (1-p_2)K_1\right]}_{\text{Entire renegotiation payoff}}. \tag{B.5}$$

Note that since investor i funds the whole deal, their funding cost is either 0 or $\overline{\theta}_1$, depending on their type. Also, note that $R^{\text{Exc}}_{\text{accept}}(\theta_i)$ does not depend on $\widetilde{P}_j$, the probability that the other investor j accepts the offer, since investor j is not given the opportunity to accept in an exclusive LME. Other than this change, if equity holders contractually commit to an exclusive LME, the equilibrium is defined in the same manner as before.

At $t = 0$, equity holders choose whether to (1) allow any form of renegotiation, (2) ban any LME, (3) only ban nonexclusive LMEs, or (4) only ban exclusive LMEs. The following proposition shows that banning exclusive LMEs can create value:

Proposition 4. *Assume the Conditions 1-4 and (19) hold for $\overline{\theta} = \overline{\theta}_2$. Then in the full commitment equilibrium, $A(K_1^*) = 1$, and banning nonexclusive LMEs at $t = 0$ is strictly suboptimal.*

1. *As long as $\overline{\theta}_1 > \overline{\theta}_2$, equity holders weakly prefer a contract that (1) allows nonexclusive LMEs and (2) bans exclusive LMEs.*

2. *Equity holders strictly prefer a contract that (1) allows nonexclusive LMEs and (2) bans exclusive LMEs if*

$$\frac{(1-p_2)}{2}\underbrace{\left(\frac{I_0 - p_1 I_1 \overline{\theta}_2(1-q)}{1-p_1p_2}\right)}_{K_0^{\text{LME}} \text{ from Proposition 2}} > I_1(\overline{\theta}_1 - \overline{\theta}_2) + \frac{C^L}{2}. \tag{B.6}$$

Intuitively, the assumption that $\overline{\theta}_1 > \overline{\theta}_2$ introduces convex funding costs that make exclusive LMEs ex-ante inefficient, relative to the constrained-efficient ex-ante firm value that arises from guaranteeing two-investor renegotiation through traditional renegotiation or nonexclusive LMEs. Under the conditions of Proposition 2, this constrained-efficient ex-ante firm value can be achieved through nonexclusive LMEs. Thus, when nonexclusive LMEs already achieve the constrained-efficient outcome, exclusive LMEs can only do worse. This leads to the first result of Proposition 4.

Moreover, exclusive LMEs can be attractive ex-post to equity holders. Comparing equation (B.5) to the rejection payoff of $C^L/2$, we observe that a debt holder is likely to accept even a low-K_1 exclusive LME because it preserves their ability to realize the original debt value K_0^*. In this sense, exclusive LMEs can be more coercive, allowing equity holders to achieve renegotiation with low K_1 payments. While this could potentially improve efficiency if renegotiations otherwise fail in equilibrium, the attractive nature of exclusive LMEs becomes a problem if nonexclusive LMEs already guarantee renegotiation. In this case, which is considered in the above proposition, guaranteeing renegotiation with a nonexclusive LME is preferable ex-ante to guaranteeing renegotiation with an exclusive LME, which entails higher funding costs due to the convexity assumption. Thus, equity holders want to commit ex-ante to a nonexclusive LME to reassure lenders that renegotiation will be less costly. This leads to the second result of Proposition 4: equity holders can strictly benefit ex-ante from banning exclusive LMEs. However, if $\overline{\theta}_1$ become very large relative to $\overline{\theta}_2$, then exclusive LMEs become less attractive ex post: equity holders must compensate the investor for their high convex funding cost. In this case, there is no need to ban exclusive LMEs because equity holders do not choose them ex-post even when they are available. Likewise, equity holders will not choose exclusive LMEs if K_0^{LME} is low, since then they are less coercive (and thus more expensive) than nonexclusive LMEs.

Proposition 4 provides an explanation for the empirical evidence of Buccola and Nini (2024): while new debt contracts intentionally allow for nonexclusive LMEs, they increasingly feature clauses that block exclusive LMEs, suggesting that exclusive LMEs are harmful ex ante. In our model, this can be explained by convex funding costs. However, we acknowledge that there are other reasons outside of our model that a firm may prefer to ban exclusive LMEs. For example, firms might worry that exclusive LMEs deter small investors in the secondary market, who worry they would not be among the chosen few to participate in an exclusive LME. Similarly, investors in the secondary market might have some slight aversion to the uncertainty regarding whether they will be in the exclusive group. Either of these effects could cause banks to charge more ex-ante, anticipating that it will be harder to sell the loans in the secondary market later.

B.1 Proof of Proposition 4

In this new setup, we continue to assume the same date-0 debt pricing condition: exclusive LMEs simply shift the potential acceptance probability $A(K_1)$ and the expected funding costs of participating lenders $\mathbb{E}[\Theta]$. Thus, Lemma 1 and Lemma 2 continue to hold. We begin by using these lemmas to characterize the potential ex-ante firm values that could arise from different ex-post outcomes.

First, if the ex-post outcome is that restructuring always fails, then $A(K_1) = 0$, $\mathbb{E}[\Theta] = 0$, and the ex-ante firm value is

$$(1 - p_1)C^H + p_1 C^L - I_0. \tag{B.7}$$

Second, if the ex-post outcome is that (1) equity holders forgo an exclusive offer and (2) only low-cost investors accept offers, then $A(K_1) = 1 - q^2$, $\mathbb{E}[\Theta] = 0$, and the ex-ante firm value is

$$(1 - p_1)C^H + p_1 C^L - I_0 + p_1(1 - q^2)[(1 - p_2)(C^H - C^L) - I_1]. \tag{B.8}$$

Third, if the ex-post outcome is that (1) equity holders forgo an exclusive offer and (2) both low-cost and high-cost investors accept offers, then $A(K_1) = 1$, $\mathbb{E}[\Theta] = q\overline{\theta}_2$, the ex-ante firm value is

$$(1 - p_1)C^H + p_1 C^L - I_0 + p_1[(1 - p_2)(C^H - C^L) - I_1(1 + q\overline{\theta}_2)], \tag{B.9}$$

where $\overline{\theta}_2$ appears in this expression since there are always two investors funding the deal.

Fourth, if the ex-post outcome is that (1) equity holders make an exclusive offer and (2) only a low-cost investor accepts, then $A(K_1) = 1 - q$, $\mathbb{E}[\Theta] = 0$, ex-ante firm value is:

$$(1 - p_1)C^H + p_1 C^L - I_0 + p_1(1 - q)[(1 - p_2)(C^H - C^L) - I_1]. \tag{B.10}$$

Finally, if the ex-post outcome is that (1) equity holders make an exclusive offer and (2) any type of investor accepts, then $A(K_1) = 1$, $\mathbb{E}[\Theta] = q\overline{\theta}_1$, ex-ante firm value is:

$$(1 - p_1)C^H + p_1 C^L - I_0 + p_1[(1 - p_2)(C^H - C^L) - I_1(1 + q\overline{\theta}_1)]. \tag{B.11}$$

Given the assumptions of the proposition, Lemma 2 implies that the third ex-post outcome delivers the highest ex-ante firm value of the first three options. The fourth option is strictly worse than the second because $q > q^2$ and Condition 4 implies that $(1 - p_2)(C^H - C^L) > I_1$. The fifth option is strictly worse than the third since $\overline{\theta}_1 > \overline{\theta}_2$. Thus, any contract that delivers the third ex-post outcome is optimal ex-ante.

According to Proposition 2, the following contract delivers the third outcome ex-post: (1) allow for nonexclusive LMEs and (2) ban exclusive LMEs. Thus, this contract is ex-ante weakly optimal for any $\overline{\theta}_1 > \overline{\theta}_2$, which proves the first part of Proposition 4.

It remains to show that the contract is strictly optimal for low values of $\overline{\theta}_1 - \overline{\theta}_2$. To show this, we prove that a contract enabling exclusive LMEs cannot support an equilibrium featuring the third ex-post outcome described above. Since the third outcome is strictly the best outcome ex-ante under Condition 4, equity holders thus strictly benefit from banning exclusive LMEs so that the third outcome can be achieved (Proposition 2).

To show that a contract enabling exclusive LMEs cannot support the third ex-post outcome, we proceed in a proof by contradiction. Suppose that enabling exclusive LMEs still supports the third ex-post outcome. By definition of the third outcome, lenders at $t = 0$ expect equity holders to forgo an exclusive LME and they expect any investor type will accept the offer. Since we have assumed $\mathbb{B}^{\text{LME}} < (1-p_2)(C^H - C^L) < \mathbb{B}^{\text{Trad}}$, it cannot be that lenders at $t = 0$ expect a traditional restructuring with $A^{\text{Trad}}(K_1^{\text{Trad}}) = 1$: if debt were priced accordingly, Proposition 1 implies that equity holders would deviate to a strategy that implements the second outcome described above. Thus, lenders must expect a nonexclusive LME with $A^{\text{LME}}(K_1^{\text{LME}}) = 1$. By Proposition 2, they expect

$$K_1^{\text{LME}} = \frac{I_1(1+\overline{\theta}_2) - p_2 C^L}{1-p_2}, \tag{B.12}$$

and they set a $t = 0$ price of

$$K_0^{\text{LME}} = \frac{I_0 - p_1 I_1 \overline{\theta}_2 (1-q)}{1 - p_1 p_2}. \tag{B.13}$$

Moreover, in the proof of Proposition 2, we have shown that $C^L < K_0^{\text{LME}} + K_1^{\text{LME}} < C^H$ and $C^L < K_0^{\text{LME}} < C^H$.

However, we have assumed that exclusive LMEs are now an available deviation ex-post for equity holders. If equity holders make an exclusive offer K_1, it is accepted by a high cost investor if

$$\frac{(1-p_2)K_0^{\text{LME}}}{2} + \left[p_2 C^L - I_1(1+\overline{\theta}_1) + (1-p_2)K_1\right] \geq \frac{C^L}{2}. \tag{B.14}$$

Rearranging, we obtain

$$K_1 \geq \frac{I_1(1+\overline{\theta}_1)}{1-p_2} + \frac{C^L(1-2p_2)}{2(1-p_2)} - \frac{K_0^{\text{LME}}}{2}. \tag{B.15}$$

Thus, equity holders can deviate to offer the exclusive LME and ensure a successful restructuring by offering a price K_1 determined by replacing the above inequality with equality:

$$K_1^{\text{Exc}} = \frac{I_1(1+\overline{\theta}_1)}{1-p_2} + \frac{C^L(1-2p_2)}{2(1-p_2)} - \frac{K_0^{\text{LME}}}{2}. \tag{B.16}$$

Recall that $C^H - K_0^{\text{LME}} - K_1^{\text{LME}} > 0$. If $K_1^{\text{LME}} > K_1^{\text{Exc}}$, then $C^H - K_0^{\text{LME}} - K_1^{\text{Exc}} > 0$ and K_1^{Exc} is a feasible deviation. Thus, K_1^{Exc} is a profitable deviation ex post if

$$\begin{aligned} & K_1^{\text{LME}} > K_1^{\text{Exc}} \\ \iff & \frac{I_1(1+\overline{\theta}_2) - p_2 C^L}{1-p_2} > \frac{I_1(1+\overline{\theta}_1)}{1-p_2} + \frac{C^L(1-2p_2)}{2(1-p_2)} - \frac{K_0^{\text{LME}}}{2}. \end{aligned} \tag{B.17}$$

Multiplying by $1-p_2$ and adding $p_2 C^L - I_1$ to both sides, this holds if and only if

$$\begin{aligned} & I_1\overline{\theta}_2 > I_1\overline{\theta}_1 + \frac{C^L}{2} - \frac{(1-p_2)K_0^{\text{LME}}}{2} \\ \iff & \frac{(1-p_2)K_0^{\text{LME}}}{2} > I_1(\overline{\theta}_1 - \overline{\theta}_2) + \frac{C^L}{2} \\ \iff & \frac{(1-p_2)}{2}\left(\frac{I_0 - p_1 I_1 \overline{\theta}_2(1-q)}{1-p_1p_2}\right) > I_1(\overline{\theta}_1 - \overline{\theta}_2) + \frac{C^L}{2}. \end{aligned} \tag{B.18}$$

Then we prove the second part of Proposition 4.

C Exit Consents

In our baseline model, we focus on the role of LMEs in enabling liquidity injections because raising liquidity is the stated objective of many LMEs. However, this focus makes it difficult to compare LMEs to distressed bond exchanges, in which the primary goal is to reduce the face value of debt outstanding and extend maturities.[47] In this extension, we modify our baseline model to study how LMEs could be used to facilitate the reduction of term-loan debt in a distressed exchange. Indeed, some LMEs feature both a liquidity injection and a haircut paid by participating lenders to reduce the face value of existing debt (Subsection 2.3). This extension enables a direct comparison between LMEs, which are used to restructure term-loan debt, and exit consents, which are used to reduce bond debt. Such a comparison is useful because exit consents share a key component with LMEs: bond holders who refuse to participate in an exchange lose value that they were previously promised in default. Nonetheless, we prove that under certain conditions LMEs can achieve efficient debt reduction better than exit consents.

We now describe the modified assumptions in this extension. Our assumptions regarding firm fundamentals are the same as before. A firm raises I_0 at date zero through debt financing. It promises to pay lenders $K_0 \in [I_0, C^H]$. At $t = 1$, the firm enters distress with probability p_1. It avoids distress with probability $1 - p_1$. If it avoids distress, it pays C^H and lenders collect K_0.

However, in this extension, we make different assumptions regarding distress. If a firm enters distress, its highest feasible payoff falls from C^H to C^M. We assume that $C^M \in (C^L, I_0)$. Further, we assume that the firm must cut its debt obligation to avoid certain failure. Specifically, the firm must convince debt holders to forgive debt such that the outstanding obligation is no greater than C^M. If debt holders refuse, we assume that the firm fails immediately and pays out C^L. This assumption is a reduced-form manner of capturing, for example, a firm's trade suppliers and employees fleeing in anticipation of a certain bankruptcy. We assume the firm must cut debt such that there is a chance that it will avoid bankruptcy (debt oustanding is less than C^M), or else trade suppliers flee preemptively and the firm fails.

If the firm successfully negotiates with lenders to reduce debt obligations to a lower amount that

[47] Naturally, extending maturities preserves liquidity in the short term, but a typical LME explicitly raises new liquidity for operations.

is less than C^M, then the firm avoids bankruptcy in the short term. After a successful negotiation, the firm ultimately pays C^M with probability $1 - p_2$ or C^L with probability p_2.

In distress, we assume that equity holders make a take-it-or-leave-it offer to cut debt. In a successful restructuring, the firm cuts debt from K_0 to $K_0 + K_1$, where $K_1 \leq C^M - K_0$. Note that K_1 is an endogenous choice of equity holders in a restructuring offer, but the restructuring automatically fails (leaving equity with nothing) if $K_0 + K_1 > C^M$. A successful restructuring is always beneficial in that it increases total firm cashflow from C^L to $p_2 C^L + (1 - p_2)C^M > C^L$.

Within this extension, we consider two settings. In one setting, the firm issues bonds at date 0 to a continuum of atomistic bond holders. In this first setting, the friction impeding debt renegotiation is the holdout problem: each bond holder's action has no impact on the probability of a restructuring succeeding, creating an incentive to reject a debt haircut and free ride on the sacrifices of other bond holders that prevent bankruptcy. The second setting more closely resembles our baseline model: the firm issues a syndicated loan that is purchased by two distressed investors. As before, the friction is asymmetric information. Each investor pays a reputation cost θ_i if they participate in restructuring, where $\theta_i = \overline{\theta}$ with probability q or 0 with probability $1 - q$. For example, a hedge fund might be perceived as weak for participating in a restructuring, risking client relationships, or a CLO might risk encouraging other firms in its portfolio to seek debt haircuts. For simplicity, we assume that bond holders have no such reputation cost. Comparing these two settings, we illustrate the differences between LMEs and exit consents.

C.1 Bond restructuring and exit consents

First, we assume the firm issued bond debt at date 0. There is a continuum of bond holders of measure 1. At date 1, if the firm enters distress, equity holders make a take-it-or-leave-it offer to cut debt from K_0 to $K_0 + K_1$. Each individual bond holder decides whether to accept or reject. Because each bondholder is atomistic, her individual decision has no impact on the likelihood of the restructuring succeeding.

As in Subsection 4.2, an equilibrium has three components: (1) in any subgame in which equity holders offer K_1, the equilibrium response strategies of bond holders comprise a Nash equilibrium of the subgame, (2) equity holders choose a restructuring strategy at date 1 to maximize their ex-post value, and (3) at date 0, bond holders require a promised payoff K_0^* that makes them break even.

First, we show that the holdout problem prevents restructuring unless exit consents (an analog of LMEs for bond restructurings) can be used to steal value from holdouts in default. To see this, assume first that a bond holder who rejects an exchange offer cannot have their payoff altered. Fix an equilibrium. Suppose that equity holders offer K_1 and bond holders expect a restructuring to succeed with probability $A^{\text{Bond}}(K_1)$. Each individual bond holder's decision to accept or reject has no impact on the probability of the restructuring succeeding. Thus, if an individual bondholder rejects, she gets:[48]

$$\left(\,1 - A^{\text{Bond}}(K_1)\,\right) C^L + A^{\text{Bond}}(K_1)\Big[p_2 C^L + (1-p_2)K_0^*\Big]. \tag{C.1}$$

If the individual bond holder accepts, she gets:

$$\left(\,1 - A^{\text{Bond}}(K_1)\,\right) C^L + A^{\text{Bond}}(K_1)\Big[p_2 \min\{C^L, K_0^* + K_1\} + (1-p_2)(K_0^* + K_1)\Big]. \tag{C.2}$$

This implies that it is never optimal for an individual bond holder to accept a restructuring offer for any $A^{\text{Bond}}(K_1) > 0$ as $K_1 < 0$.

In order for a bond restructuring to succeed, equity holders must have the ability to allocate value away from a bondholder who rejects an offer. For example, the restructuring must feature an exit consent in which consenting bond holders agree to vote to alter their original debt contract and void covenants or collateral rights. Equivalently, the firm must obtain the right to make the new bonds created in the exchange senior to the existing bonds. This finding is known from Gertner and Scharfstein (1991); Kahan and Tuckman (1993); Hege and Mella-Barral (2005); Donaldson et al. (2020).

In a distressed exchange with an exit consent, the payoff for accepting is still given by (C.2).

[48] Using the break even condition, $C^L < C^M < I_0$, and $K_0 + K_1 \leq C^M$, we can obtain $K_0 > I_0 > C^M$. Then we derive from $K_0 + K_1 \leq C^M$ that $K_1 < 0$.

However, the payoff for rejecting becomes

$$\big(\, 1 - A^{\text{Bond}}(K_1) \,\big)\, C^L + A^{\text{Bond}}(K_1)\Big[(1-p_2)K_0^*\Big]. \tag{C.3}$$

This is the same as (C.1), except that the rejecting bond holder loses their right to receive C^L if the restructuring succeeds and the firm ultimately goes bankrupt. In Lemma 5, we show that exit consents can lead to successful bond restructurings.

For any fixed (K_0, K_1), as bond holders need to break even, they require

$$\begin{aligned} I_0 = \int_0^1 & (1-p_1)K_0 \\ & + p_1\Big[(1 - A^{\text{Bond}}(K_1))C^L + A^{\text{Bond}}(K_1)\,\Big(\, p_2 \min\{C^L, K_0 + K_1\} + (1-p_2)\Big(K_0 + K_1\Big)\,\Big)\Big]\,dh, \end{aligned} \tag{C.4}$$

where h indexes atomistic bond holders.

The date-0 equity value of the bond-issuing firm is given by

$$E_0^{\text{Bond}} = \int_0^1 (1-p_1)(C^H - K_0) + p_1 A^{\text{Bond}}(K_1)\Big[p_2(C^L - K_0 - K_1)^+ + (1-p_2)\Big(C^M - K_0 - K_1\Big)\Big]\,dh\,. \tag{C.5}$$

Combining (C.4) and (C.5), we obtain

$$E_0^{\text{Bond}} = (1-p_1)C^H + p_1 C^L - I_0 + p_1 A^{\text{Bond}}(K_1)(1-p_2)\Big(C^M - C^L\Big)\,. \tag{C.6}$$

According to (C.6), we can see that achieving $A^{\text{Bond}}(K_1) = 1$ is always efficient ex ante because $C^M > C^L$. Thus, it is immediate that $A^{\text{Bond}}(K_1) = 1$ maximizes ex ante firm value E_0^{Bond}.

We impose a condition implying that limited liability is satisfied: $C^H > K_0^*$ for the bond-issuing firm.

Condition 6. $C^H > \max\{\frac{I_0}{1-p_1p_2}, \frac{I_0 - p_1 C^L}{1-p_1}\}$.

The following lemma checks when $A^{\text{Bond}}(K_1) = 1$ can be supported in equilibrium.

Lemma 5. *Suppose Condition 6 holds and that the firm issues bonds and attempts an exit consent restructuring.*

1. *If* $I_0 \leq \frac{1-p_1 p_2}{1-p_2}[(1-p_2)C^M + p_2 C^L]$*, then there exists an equilibrium* $(K_0^{\text{Bond}}, K_1^{\text{Bond}})$ *in which* $A^{\text{Bond}}(K_1^{\text{Bond}}) = 1$*. In this equilibrium, ex-ante firm value is*

$$E_0^{\text{Bond}} = (1-p_1)C^H + p_1 C^L - I_0 + p_1(1-p_2)(C^M - C^L). \tag{C.7}$$

2. *If* $I_0 > \frac{1-p_1 p_2}{1-p_2}[(1-p_2)C^M + p_2 C^L]$*, then the exit consent restructuring always fails in equilibrium:* $A^{\text{Bond}}(K_1^*) = 0$*. In this equilibrium, ex-ante firm value is*

$$E_0^{\text{Bond}} = (1-p_1)C^H + p_1 C^L - I_0. \tag{C.8}$$

In summary, exit consents can achieve the ex-ante efficient outcome ($A^{\text{Bond}}(K_1) = 1$) if and only if the original amount of debt is small.

C.2 Comparing exit consents to LMEs

We now use this extension to compare exit consents in bond restructurings to LMEs in loan restructurings. For this comparison, we again consider a setting in which the firm issues a term loan to banks at date zero. As in our baseline model, the banks sell the loans on a secondary market to two distressed debt investors if the firm experiences distress at date one. If the firm experiences distress, equity holders make a take-it-or-leave-it offer to cut debt from K_0 to $K_0 + K_1$. We assume an exchange-offer structure such that if one investor accepts and the other rejects, the debt falls from K_0 to $K_0 + K_1/2$. Each investor privately observes the reputation cost θ_i that they pay if they accept a restructuring offer. As motivated earlier, this reflects the potential for a hedge fund to appear weak if they accept a debt haircut, losing clients, or for a CLO to appear overly generous, encouraging future debt-restructuring attempts. As in our baseline model, $\theta_i = \overline{\theta}$ with probability q and it equals 0 with probability $1-q$. Let $N(K_1) \in \{0, 1, 2\}$ denote the number of investors who accept. Then, the total reputation cost is given by

$$\Theta = \mathbf{1}_{K_0+N(K_1)K_1/2\leq C^M} \sum_i \theta_i \mathbf{1}_{\text{accept}}(K_1, \theta_i), \tag{C.9}$$

where $\mathbf{1}_{K_0+N(K_1)K_1/2\leq C^M}$ indicates that the reputation cost is paid only when restructuring is successful. Then $A^{\text{LME}}(K_1)$, the probability that the LME successfully reduces debt obligations to a lower amount that is less than C^M, is given by

$$A^{\text{LME}}(K_1) = \mathbb{P}\Big(K_0 + \frac{N(K_1)}{2} K_1 \leq C^M\Big). \tag{C.10}$$

We apply an analogous equilibrium definition to the one used in Subsection 4.3: (1) lender responses to offers comprise a Nash equilibrium in any subgame, (2) equity holders choose offers to maximize ex-post value, and (3) lenders break even at date zero.

As before, we derive conditions under which LMEs can support the ex-ante efficient outcome (restructurings always succeed) in equilibrium.

Proposition 5. *Assume Condition 6 and the following hold:*

$$I_0 > \max\Bigg\{ 2p_1\overline{\theta}(1-q) + (1-p_1)\Big(2C^M + \frac{[q+(1-q)p_2]C^L}{(1-p_2)(1-q)}\Big), \\ \frac{1-p_1p_2}{1-p_2}\big[(1-p_2)C^M + p_2C^L\big] \Bigg\}, \tag{C.11}$$

$$(1-p_2)(C^M - C^L) \geq \frac{2\overline{\theta}}{q}, \tag{C.12}$$

$$C^H > \frac{I_0 - 2p_1\overline{\theta}(1-q)}{1-p_1}, \tag{C.13}$$

$$2\overline{\theta} > C^L. \tag{C.14}$$

1. *If the firm issues term-loan debt and enables LMEs, there exists an equilibrium in which LME renegotiations always succeed: $A^{\text{LME}}(K_1^{\text{LME}}) = 1$. In this equilibrium, ex-ante firm value is*

$$E_0^{\text{LME}} = (1-p_1)C^H + p_1C^L - I_0 + p_1\Big[(1-p_2)(C^M - C^L) - 2\overline{\theta}q\Big]. \tag{C.15}$$

2. *Combining the above results and Lemma 5, LMEs improve ex-ante firm value, $E_0^{\text{LME}} > E_0^{\text{Bond}}$,*

even if the firm could issue bonds and use exit consents.

In summary, LMEs can achieve efficient debt restructuring better than exit consents for heavily levered firms. For a heavily levered firm, the holders of a term loan internalize that all of them must forgive debt in order for the firm to avoid bankruptcy. Each debt holder does not want to incur the cost θ_i, but the LME helps incentivize them to do so through the threat of losing collateral if they reject. In contrast, for a heavily levered firm, the holdout problem for bonds becomes insurmountable. The threat of losing any payoff in bankruptcy can only achieve a limited haircut, which is insufficient to reduce debt below the necessary threshold. Piecing this together, when a firm is heavily levered and the reputation cost $\overline{\theta}$ for term loan holders is small, LMEs improve efficiency.

C.3 Proof of Lemma 5

Comparing (C.3) to (C.2), it is an equilibrium for a bond holder to accept if

$$p_2 \min\{C^L, K_0^* + K_1\} + (1 - p_2)K_1 \geq 0. \tag{C.16}$$

If (C.16) holds with strict inequality, then $A^{\text{Bond}}(K_1) = 1$. In this case, it is profitable to slightly reduce K_1 while still maintaining $A^{\text{Bond}}(K_1) = 1$. Therefore, this situation cannot constitute an equilibrium. If (C.16) holds with equality, then bond holders are indifferent between accepting or rejecting, so any probability $A^{\text{Bond}}(K_1) \in [0, 1]$ can be supported in a mixed-strategy Nash equilibrium of the subgame.

Let K_1^* satisfy (C.16) with equality and let K_0^* satisfies the break even condition (C.4) given $K_1 = K_1^*$. It is suboptimal to deviate from K_1^* to any $K_1 > K_1^*$ as we can set $A^{\text{Bond}}(K_1^*) = 1$. In addition, we have $A^{\text{Bond}}(K_1) = 0$ for any $K_1 < K_1^*$. In order to prove that (K_0^*, K_1^*) is an equilibrium, we only need to verify that $K_0^* + K_1^* \leq C^M$, $C^M < K_0^* < C^H$.

We first consider the case $C^L > K_0^* + K_1^*$. Imposing equality in (C.16) yields

$$K_1^* = -p_2 K_0^*. \tag{C.17}$$

Plugging $K_1^* = -p_2 K_0^*$ and $A^{\text{Bond}}(K_1^*) = 1$ into (C.4) and rearranging,

$$K_0^* = \frac{I_0}{1 - p_1 p_2}, \text{ and } K_1^* = -\frac{p_2 I_0}{1 - p_1 p_2}. \tag{C.18}$$

We have $K_0^* > I_0 > C^M$. Using Condition 6, we have $C^H > K_0^*$. We derive from (C.18) that $K_0^* + K_1^* < C^L$ if and only if $I_0 < \frac{1-p_1p_2}{1-p_2} C^L$. Substituting $A^{\text{Bond}}(K_1^*) = 1$ into (C.6), we obtain (C.7).

Next, we consider the case $C^L \le K_0^* + K_1^*$. Imposing equality in (C.16) yields

$$K_1^{\text{Bond}} = -\frac{p_2 C^L}{1 - p_2}. \tag{C.19}$$

Plugging $K_1 = K_1^{\text{Bond}}$ given in (C.19) and $A^{\text{Bond}}(K_1) = 1$ into (C.4) and rearranging,

$$K_0^{\text{Bond}} = \frac{I_0}{1 - p_1 p_2}. \tag{C.20}$$

We have $K_0^{\text{Bond}} > I_0 > C^M$. Using Condition 6, we have $C^H > K_0^{\text{Bond}}$. Combining (C.19) and (C.20), we obtain $K_0^{\text{Bond}} + K_1^{\text{Bond}} \le C^M$ if and only if

$$\begin{aligned} & \frac{I_0}{1 - p_1 p_2} - \frac{p_2 C^L}{1 - p_2} \le C^M, \\ \iff \quad & I_0 \le \frac{1 - p_1 p_2}{1 - p_2}[(1 - p_2)C^M + p_2 C^L]. \end{aligned} \tag{C.21}$$

In addition, we have $K_0^{\text{Bond}} + K_1^{\text{Bond}} - C^L = \frac{I_0}{1-p_1p_2} - \frac{C^L}{1-p_2}$. Thus, $C^L \le K_0^{\text{Bond}} + K_1^{\text{Bond}}$ if and only if $I_0 \ge \frac{1-p_1p_2}{1-p_2} C^L$. Substituting $A^{\text{Bond}}(K_1^{\text{Bond}}) = 1$ into (C.6), we obtain (C.7). This proves the first part of Lemma 5.

Next, we prove the second part of Lemma 5. To obtain a contradiction, assume that there exists an equilibrium $(\widehat{K}_0, \widehat{K}_1)$ such $A^{\text{Bond}}(\widehat{K}_1) > 0$. Thus, some bond holders are willing to accept the restructuring offer. Using $A^{\text{Bond}}(\widehat{K}_1) > 0$ and (C.16), we conclude that $\widehat{K}_1 = -p_2 \widehat{K}_0$ if $C^L > \widehat{K}_0 + \widehat{K}_1$ and $\widehat{K}_1 = K_1^{\text{Bond}}$ if $C^L \le \widehat{K}_0 + \widehat{K}_1$.

For the case $C^L > \widehat{K}_0 + \widehat{K}_1$, plugging $\widehat{K}_1 = -p_2\widehat{K}_0$ into (C.4) and rearranging, we obtain

$$\widehat{K}_0 = \widehat{K}_0(A^{\text{Bond}}) := \frac{I_0 - p_1 C^L(1 - A^{\text{Bond}})}{1 - p_1 + p_1(1 - p_2)A^{\text{Bond}}}. \tag{C.22}$$

We have

$$\frac{\partial \widehat{K}_0(A^{\text{Bond}})}{\partial A^{\text{Bond}}} = p_1 \frac{C^L(1 - p_1 p_2) - I_0(1 - p_2)}{\left[1 - p_1 + p_1(1 - p_2)A^{\text{Bond}}\right]^2} < 0, \tag{C.23}$$

where the inequality uses $I_0 > \frac{1-p_1p_2}{1-p_2}[(1-p_2)C^M + p_2C^L] > \frac{1-p_1p_2}{1-p_2}C^L$. Then we have

$$\widehat{K}_0 + \widehat{K}_1 = (1 - p_2)\widehat{K}_0 \geq (1 - p_2)K_0^{\text{Bond}} = \frac{1 - p_2}{1 - p_1 p_2} I_0 > (1 - p_2)C^M + p_2 C^L, \tag{C.24}$$

where the first inequality uses (C.23) and $\widehat{K}_0(A^{\text{Bond}}) = \frac{I_0}{1-p_1p_2} = K_0^{\text{Bond}}$ when $A^{\text{Bond}} = 1$. Then we arrive at a contradiction as we are considering the case $C^L > \widehat{K}_0 + \widehat{K}_1$.

For the case $C^L \leq \widehat{K}_0 + \widehat{K}_1$, plugging $K_1 = K_1^{\text{Bond}}$ given in (C.19) into (C.4) and rearranging, we also have (C.22) and $\widehat{K}_0 \geq K_0^{\text{Bond}}$. As we have shown $\widehat{K}_1 = K_1^{\text{Bond}}$ in this case, we have

$$\widehat{K}_0 + \widehat{K}_1 \geq K_0^{\text{Bond}} + K_1^{\text{Bond}} = \frac{I_0}{1 - p_1 p_2} - \frac{p_2 C^L}{1 - p_2} > C^M, \tag{C.25}$$

where the second inequality follows from the condition $I_0 > \frac{1-p_1p_2}{1-p_2}[(1-p_2)C^M + p_2C^L]$.

Thus, we arrive at a contradiction, thereby proving the nonexistence of an equilibrium $(\widehat{K}_0, \widehat{K}_1)$ such that $A^{\text{Bond}}(\widehat{K}_1) > 0$.

Finally, we show there exists an equilibrium (K_0^*, K_1^*) such that $A^{\text{Bond}}(K_1^*) = 0$. Substituting $A^{\text{Bond}}(K_1^*) = 0$ into (C.4), we obtain $K_0^* = \frac{I_0 - p_1C^L}{1-p_1}$. Since $C^L < I_0$, we have

$$K_0^* = \frac{I_0 - p_1 C^L}{1 - p_1} > \frac{I_0 - p_1 I_0}{1 - p_1} = I_0 > C^M. \tag{C.26}$$

Using Condition 6, we have $C^H > K_0^*$. To show that $(K_0^*, K_1^*) = \left(\frac{I_0-p_1C^L}{1-p_1}, 0\right)$ is an equilibrium, it remains to prove that deviating from $K_1^* = 0$ to other K_1^{Alt} with $A^{\text{Bond}}(K_1^{\text{Alt}}) > 0$ is suboptimal.

Using $A^{\text{Bond}}(K_1^{\text{Alt}}) > 0$ and (C.16), we conclude that $K_1^{\text{Alt}} \geq -p_2 K_0^*$ if $C^L > K_0^* + K_1^{\text{Alt}}$, and that $K_1^{\text{Alt}} \geq K_1^{\text{Bond}}$ if $C^L \leq K_0^* + K_1^{\text{Alt}}$. Using (C.22) and (C.23) , we have

$$K_0^* = \frac{I_0 - p_1 C^L}{1 - p_1} = \widehat{K}_0(0) > \widehat{K}_0(1) = K_0^{\text{Bond}}. \tag{C.27}$$

When $C^L > K_0^* + K_1^{\text{Alt}}$, we have

$$K_0^* + K_1^{\text{Alt}} \geq (1 - p_2)K_0^* > (1 - p_2)K_0^{\text{Bond}} = \frac{1 - p_2}{1 - p_1 p_2} I_0 > (1 - p_2)C^M + p_2 C^L \tag{C.28}$$

for any K_1^{Alt} with $A^{\text{Bond}}(K_1^{\text{Alt}}) > 0$. Then we arrive at a contradiction as $C^L > K_0^* + K_1^{\text{Alt}}$. When $C^L \leq K_0^* + K_1^{\text{Alt}}$, we have

$$K_0^* + K_1^{\text{Alt}} > K_0^{\text{Bond}} + K_1^{\text{Bond}} = \frac{I_0}{1 - p_1 p_2} - \frac{p_2 C^L}{1 - p_2} > C^M \tag{C.29}$$

for any K_1^{Alt} with $A^{\text{Bond}}(K_1^{\text{Alt}}) > 0$. This shows that $(K_0^*, K_1^*) = \left(\frac{I_0 - p_1 C^L}{1 - p_1}, 0\right)$ is an equilibrium.

Substituting $A^{\text{Bond}}(K_1^*) = 0$ into (C.6), we obtain (C.8). Then we prove the second part of Lemma 5.

C.4 Proof of Proposition 5

Recall that we only consider (K_0, K_1) such that $K_0 + K_1 \leq C^M$ and $K_0 \leq C^H$. For any fixed K_1, total reputation cost Θ, the number of investors who accept $N(K_1)$, and the probability of a successful renegotiation $A(K_1)$, the banks' date-0 break-even condition that determines K_0 is:

$$\begin{aligned} I_0 =& (1 - p_1)K_0 + p_1[1 - A(K_1)] \min\{C^L, K_0\} - p_1 \mathbb{E}[\Theta] \\ &+ p_1 A(K_1)\left[(1 - p_2)\left(K_0 + \frac{\mathbb{E}[N(K_1)]}{2} K_1\right) + p_2 \min\left\{K_0 + \frac{\mathbb{E}[N(K_1)]}{2} K_1, C^L\right\}\right]. \end{aligned} \tag{C.30}$$

The ex-ante firm value is:

$$E_0 = (1-p_1)(C^H - K_0) + p_1[1 - A(K_1)](C^L - K_0)^+ \\ + p_1 A(K_1)\left[(1-p_2)\Big(C^M - K_0 - \frac{\mathbb{E}[N(K_1)]}{2}K_1\Big) + p_2\Big(C^L - K_0 - \frac{\mathbb{E}[N(K_1)]}{2}K_1\Big)^+\right]. \tag{C.31}$$

Combining (C.30) and (C.31), we obtain

$$E_0 = (1-p_1)C^H + p_1 C^L - I_0 + p_1\left[A(K_1)(1-p_2)\Big(C^M - C^L\Big) - \mathbb{E}[\Theta]\right]. \tag{C.32}$$

First, if $A(K_1) = 0$, then $\mathbf{1}_{\text{accept}}(K_1, \theta_i) = 0$, $\mathbb{E}[\Theta] = 0$, and ex-ante firm value (C.32) is

$$E_0 = (1-p_1)C^H + p_1 C^L - I_0. \tag{C.33}$$

Second, suppose K_1 is such that $\mathbf{1}_{\text{accept}}(K_1, \theta_i) = 1$ for $\theta_i = 0$ but $\mathbf{1}_{\text{accept}}(K_1, \theta_i) = 0$ for $\theta_i = \overline{\theta}$. Assume also that $K_0 + K_1/2 \leq C^M$, so only one investor is necessary to avoid bankruptcy. Then $A(K_1) = 1 - q^2$, $\mathbb{E}[\Theta] = 0$, and ex-ante firm value (C.32) is

$$E_0 = (1-p_1)C^H + p_1 C^L - I_0 + p_1(1-q^2)(1-p_2)(C^M - C^L). \tag{C.34}$$

If $K_0 + K_1/2 > C^M$, so that both investors must accept to avoid bankruptcy, then the same expression holds replacing $(1-q^2)$ with $(1-q)^2$.

Finally, suppose K_1 is such that $\mathbf{1}_{\text{accept}}(K_1, \theta_i) = 1$ for both high and low θ_i. Since we only consider (K_0, K_1) such that $K_0 + K_1 \leq C^M$, then $A(K_1) = 1$, $\mathbb{E}[\Theta] = 2\overline{\theta}q$, and ex-ante firm value (C.32) is

$$E_0 = (1-p_1)C^H + p_1 C^L - I_0 + p_1\left[(1-p_2)(C^M - C^L) - 2\overline{\theta}q\right]. \tag{C.35}$$

Comparing (C.33), (C.34), and (C.35), we can see that (C.35) is the highest if and only if

$$\begin{aligned} &(1-p_2)(C^M - C^L) - 2\overline{\theta}q > (1-q^2)(1-p_2)(C^M - C^L) \\ \iff &(1-p_2)(C^M - C^L) > \frac{2\overline{\theta}}{q}. \end{aligned} \tag{C.36}$$

This inequality is supported by (C.12). Further, if this holds, then it holds replacing $1-q^2$ with the smaller quantity $(1-q)^2$, so it holds whether $K_0 + K_1/2$ is larger or smaller than C^M.

Suppose that LMEs are enabled and fix an equilibrium in which $A^{\text{LME}}(K_1^*) = 1$, $C^L \leq K_0^* + K_1^* \leq C^M$, $C^L \leq K_0^* \leq C^H$. We will verify these conditions later. A debt holders' payoff from accepting an offer K_1 is then

$$R_{\text{accept}}(\theta_i) = \frac{1}{2}\Big[p_2 C^L + (1-p_2)(K_0^* + K_1)\Big] - \theta_i. \tag{C.37}$$

There are two potential cases for the payoff from rejecting. First, we consider K_1 such that $K_0^* + \frac{K_1}{2} > C^M$. Then rejection causes bankruptcy because there is insufficient debt reduction, so the payoff from rejecting is zero. In this case, the offer is accepted by a high cost investor if and only if $R_{\text{accept}}(\overline{\theta}) \geq 0$, which is equivalent to

$$p_2 C^L + (1-p_2)(K_0^* + K_1) - 2\overline{\theta} \geq 0. \tag{C.38}$$

Second, we consider K_1 such that $K_0^* + \frac{K_1}{2} \leq C^M$. Then the renegotiation still succeeds when one debt holder rejects, so the payoff from rejection is

$$R_{\text{reject}} = \frac{1}{2}(1-p_2)K_0^*.$$

In this case, the offer is accepted by a high cost investor if and only if $R_{\text{accept}}(\overline{\theta}) \geq R_{\text{reject}}$, which is equivalent to

$$p_2 C^L + (1-p_2)K_1 - 2\overline{\theta} \geq 0. \tag{C.39}$$

Since equity holders optimally choose the lowest K_1 that ensures acceptance, we derive from

(C.38) and (C.39) that two potential equilibrium offers:

$$K_1^* = -K_0^* + \frac{2\overline{\theta} - p_2 C^L}{1-p_2} \qquad \text{if} \qquad \frac{K_0^*}{2} + \frac{2\overline{\theta} - p_2 C^L}{2(1-p_2)} > C^M, \tag{C.40}$$

$$K_1^* = \frac{2\overline{\theta} - p_2 C^L}{1-p_2} \qquad \text{if} \qquad K_0^* + \frac{2\overline{\theta} - p_2 C^L}{2(1-p_2)} \leq C^M. \tag{C.41}$$

Next, we solve for (K_0^*, K_1^*) for the case (C.40). Substituting $A^{\mathrm{LME}}(K_1^*) = 1$, $\mathbb{E}[\Theta] = 2q\overline{\theta}$, and $N(K_1^*) = 2$ into the break even condition (C.30), and using $C^L \leq K_0^* + K_1^*$, we obtain

$$I_0 = (1-p_1)K_0^* + p_1\Big[p_2 C^L + (1-p_2)(K_0^* + K_1^*) - 2q\overline{\theta}\Big]. \tag{C.42}$$

Conjecturing the case (C.40) above and plugging in $K_1^* = -K_0^* + \frac{2\overline{\theta} - p_2 C^L}{1-p_2}$,

$$\begin{aligned} I_0 &= (1-p_1)K_0^* + p_1\Big[p_2 C^L + 2\overline{\theta} - p_2 C^L - 2q\overline{\theta}\Big] \\ &= (1-p_1)K_0^* + 2p_1\overline{\theta}(1-q). \end{aligned} \tag{C.43}$$

Rearranging, we obtain

$$K_0^* = K_0^{\mathrm{LME}} = \frac{I_0 - 2p_1\overline{\theta}(1-q)}{1-p_1}. \tag{C.44}$$

Putting this together, the inequality in (C.40) holds if

$$\frac{I_0 - 2p_1\overline{\theta}(1-q)}{1-p_1} + \frac{2\overline{\theta} - p_2 C^L}{1-p_2} > 2C^M. \tag{C.45}$$

Note that the above inequality is implied by the following inequality:

$$\begin{aligned} &\frac{I_0 - 2p_1\overline{\theta}(1-q)}{1-p_1} - \frac{[q+(1-q)p_2]C^L}{(1-p_2)(1-q)} > 2C^M \\ \Longleftrightarrow\ & I_0 - 2p_1\overline{\theta}(1-q) > (1-p_1)\Big(2C^M + \frac{[q+(1-q)p_2]C^L}{(1-p_2)(1-q)}\Big) \\ \Longleftrightarrow\ & I_0 > 2p_1\overline{\theta}(1-q) + (1-p_1)\Big(2C^M + \frac{[q+(1-q)p_2]C^L}{(1-p_2)(1-q)}\Big). \end{aligned} \tag{C.46}$$

It follows from (C.11) that the above inequality holds. Additionally, using (C.13) and (C.44), we can see that $C^H > K_0^{\text{LME}}$. Using (C.14) and the equality in (C.40), we have $K_0^{\text{LME}} + K_1^{\text{LME}} - C^L = \frac{2\overline{\theta} - C^L}{1-p_2} > 0$.

In addition, we derive from (C.44) and the equality in (C.40) that

$$K_1^{\text{LME}} = -K_0^{\text{LME}} + \frac{2\overline{\theta} - p_2 C^L}{1 - p_2} = \frac{2\overline{\theta} - p_2 C^L}{1 - p_2} - \frac{I_0 - 2p_1\overline{\theta}(1-q)}{1 - p_1}. \tag{C.47}$$

According to (C.11) and (C.44), we have $K_0^{\text{LME}} > C^M$. In addition, note that $K_0^{\text{LME}} + K_1^{\text{LME}} = \frac{2\overline{\theta} - p_2 C^L}{1-p_2} < C^M$ if and only if

$$\begin{aligned} &(1-p_2)C^M > 2\overline{\theta} - p_2 C^L \\ \iff &(1-p_2)(C^M - C^L) > 2\overline{\theta} - C^L. \end{aligned} \tag{C.48}$$

According to (C.12), we can see the above inequality holds. Hence, $K_0^{\text{LME}} + K_1^{\text{LME}} < C^M$.

Next, we verify that $(K_0^{\text{LME}}, K_1^{\text{LME}})$ is an equilibrium. By the definition of K_1^{LME}, it is suboptimal to deviate to any K_1 such that $K_0^* + \frac{K_1}{2} > C^M$ and $A^{\text{LME}}(K_1) = 1$. Next, we consider K_1 such that $K_0^{\text{LME}} + \frac{K_1}{2} \le C^M$ and $A^{\text{LME}}(K_1) = 1$. Then we derive from (C.41) that $K_1 \ge \frac{2\overline{\theta} - p_2 C^L}{1-p_2}$, which implies that

$$K_0^{\text{LME}} + \frac{K_1}{2} > \frac{K_0^{\text{LME}}}{2} + \frac{2\overline{\theta} - p_2 C^L}{2(1-p_2)} > C^M, \tag{C.49}$$

where the first inequality uses $K_0^{\text{LME}} > C^M > 0$, and the second inequality holds because we have shown that the inequality in (C.40) is implied by (C.11). Thus, there is no relevant deviation for which $K_0^{\text{LME}} + \frac{K_1}{2} \le C^M$ and $A^{\text{LME}}(K_1) = 1$.

All that is left to check is that equity cannot profitably deviate to an offer with $\widetilde{P} = 1 - q$. If $\widetilde{P} = 1 - q$, there are again two cases. First, we consider a deviation K_1 such that $K_0^{\text{LME}} + \frac{K_1}{2} > C^M$. If $K_0^{\text{LME}} + K_1 \ge C^L$, then the payoff from accepting is:[49]

$$R_{\text{accept}}(\theta_i) = qC^L + (1-q)\left[\frac{1}{2}\Big[p_2 C^L + (1-p_2)(K_0^{\text{LME}} + K_1)\Big] - \theta_i\right]. \tag{C.50}$$

[49]Note that an investor obtains the entire bankruptcy payoff in an LME if they accept and the other investor rejects, even if there is insufficient debt reduction to avoid bankruptcy. Moreover, note that we have assumed that the reputation cost θ_i is only paid if the investor accepts and there is sufficient debt reduction.

The payoff from rejecting is

$$R_{\text{reject}} = \frac{1}{2} q C^L.$$

Thus, the offer is accepted by a low cost investor if $R_{\text{accept}}(0) \geq R_{\text{reject}}$, i.e.

$$[q + (1-q)p_2]C^L + (1-q)(1-p_2)(K_0^{\text{LME}} + K_1) \geq 0. \tag{C.51}$$

Since $K_0^{\text{LME}} + K_1 \geq C^L$ implies the above holds with strict inequality, it optimal to choose

$$K_1 = -K_0^{\text{LME}} + C^L. \tag{C.52}$$

If $K_0^{\text{LME}} + K_1 < C^L$, then the payoff from accepting is

$$R_{\text{accept}}(\theta_i) = qC^L + (1-q)\left[\frac{1}{2}(K_0^{\text{LME}} + K_1) - \theta_i\right]. \tag{C.53}$$

The payoff from rejecting is still given by $R_{\text{reject}} = \frac{1}{2}qC^L$. Thus, the offer is accepted by a low cost investor if $R_{\text{accept}}(0) \geq R_{\text{reject}}$, i.e.

$$qC^L + (1-q)\frac{1}{2}(K_0^{\text{LME}} + K_1) \geq \frac{1}{2}qC^L. \tag{C.54}$$

This implies

$$K_1^{\text{Alt}} = -K_0^{\text{LME}} - \frac{qC^L}{1-q}. \tag{C.55}$$

Since (C.55) is lower than (C.52), it is better to choose (C.55) if (C.55) is feasible. We have

$$\begin{aligned} K_0^{\text{LME}} + \frac{K_1^{\text{Alt}}}{2} &= \frac{1}{2}\Big[K_0^{\text{LME}} - \frac{qC^L}{1-q}\Big] \\ &> \frac{1}{2}\Big[K_0^{\text{LME}} - \frac{[q+(1-q)p_2]C^L}{(1-p_2)(1-q)}\Big] \\ &= \frac{1}{2}\Big[\frac{I_0 - 2p_1\overline{\theta}(1-q)}{1-p_1} - \frac{[q+(1-q)p_2]C^L}{(1-p_2)(1-q)}\Big] > C^M, \end{aligned} \tag{C.56}$$

where the last inequality uses (C.46) and (C.11). Moreover, $K_0^{\text{LME}} + K_1^{\text{Alt}} < 0 < C^L$. Thus, K_1^{Alt} given in (C.55) is a feasible deviation.

Second, we consider a deviation K_1 such that $K_0^{\text{LME}}+\frac{K_1}{2} \leq C^M$. Then the payoff from accepting is

$$R_{\text{accept}}(\theta_i) = -\theta_i + (1-p_2)\frac{K_0^{\text{LME}}+K_1}{2} + p_2\left[q\min\left\{\frac{K_0^{\text{LME}}+K_1}{2}, C^L\right\} + (1-q)\min\left\{\frac{K_0^{\text{LME}}+K_1}{2}, \frac{C^L}{2}\right\}\right]. \tag{C.57}$$

The payoff from rejecting is

$$R_{\text{reject}} = \frac{1}{2}[qC^L + (1-q)(1-p_2)K_0^{\text{LME}}]. \tag{C.58}$$

Thus, the offer is accepted by a low cost investor if and only if $R_{\text{accept}}(0) \geq R_{\text{reject}}$. Since $R_{\text{accept}}(0) \leq (1-p_2)\frac{K_0^{\text{LME}}+K_1}{2} + p_2[q+\frac{1-q}{2}]C^L$, we have

$$\begin{aligned}
&(1-p_2)\frac{K_0^{\text{LME}}+K_1}{2} + p_2[q+\frac{1-q}{2}]C^L \geq \frac{1}{2}[qC^L + (1-q)(1-p_2)K_0^{\text{LME}}] \\
\Longleftrightarrow\ &\frac{q(1-p_2)(K_0^{\text{LME}}-C^L)}{2} + \frac{p_2C^L}{2} + \frac{(1-p_2)K_1}{2} \geq 0 \\
\Longleftrightarrow\ &K_1 \geq -q(K_0^{\text{LME}}-C^L) - \frac{p_2C^L}{1-p_2}.
\end{aligned} \tag{C.59}$$

This implies

$$K_1 \geq \widetilde{K_1}^{\text{Alt}} = -q(K_0^{\text{LME}}-C^L) - \frac{p_2C^L}{1-p_2}. \tag{C.60}$$

Since we have shown that $K_0^{\text{LME}} > C^M > 0$ and $C^L > 0$, we have

$$\widetilde{K_1}^{\text{Alt}} > -K_0^{\text{LME}} - \frac{p_2C^L}{1-p_2} > -K_0^{\text{LME}} - \frac{[q+(1-q)p_2]C^L}{(1-p_2)(1-q)}. \tag{C.61}$$

Thus, for any K_1 satisfying (C.59), we have

$$K_0^{\text{LME}} + \frac{K_1}{2} \geq K_0^{\text{LME}} + \frac{\widetilde{K_1}^{\text{Alt}}}{2} > \frac{1}{2}\left[K_0^{\text{LME}} - \frac{[q+(1-q)p_2]C^L}{(1-p_2)(1-q)}\right] > C^M, \tag{C.62}$$

where the second inequality follows from (C.61), the last inequality uses (C.56). Then we arrive at a contradiction as we are considering deviation K_1 such that $K_0^{\text{LME}} + \frac{K_1}{2} \leq C^M$.

Therefore, we only need to verify whether the deviation K_1^{Alt} given in (C.55) is profitable. Since $C^M > K_0^{\text{LME}} + K_1^{\text{LME}} > C^L$, $K_0^{\text{LME}} > C^M$, $K_0^{\text{LME}} + K_1^{\text{Alt}} < 0$, $A^{\text{LME}}(K_1^{\text{LME}}) = 1$, $A^{\text{LME}}(K_1^{\text{Alt}}) = (1-q)^2$, K_1^{Alt} is not a profitable deviation relative to K_1^{LME} if

$$\begin{aligned}
&(1-p_2)(C^M - K_0^{\text{LME}} - K_1^{\text{LME}}) \\
\geq&(1-q)^2\Big[(1-p_2)(C^M - K_0^{\text{LME}} - K_1^{\text{Alt}}) + p_2(C^L - K_0^{\text{LME}} - K_1^{\text{Alt}})\Big] \\
\Longleftrightarrow\ &(1-p_2)\Big(C^M - \frac{2\overline{\theta} - p_2 C^L}{1-p_2}\Big) \geq (1-q)^2\Big((1-p_2)C^M + p_2 C^L + \frac{qC^L}{1-q}\Big) \\
\Longleftrightarrow\ &[1-(1-q)^2](1-p_2)C^M + \Big[[1-(1-q)^2]p_2 - (1-q)q\Big]C^L \geq 2\overline{\theta} \\
\Longleftrightarrow\ &[1-(1-q)^2](1-p_2)(C^M - C^L) + \Big[1-(1-q)^2 - (1-q)q\Big]C^L \geq 2\overline{\theta} \\
\Longleftrightarrow\ &(1-p_2)(C^M - C^L) \geq \frac{2\overline{\theta} - qC^L}{1-(1-q)^2}
\end{aligned} \tag{C.63}$$

using (C.40) and (C.55). Since $1-(1-q)^2 > q$, then it follows from condition (C.12) that the above inequality holds.

Therefore, we have shown that $(K_0^{\text{LME}}, K_1^{\text{LME}})$ is an equilibrium. Finally, according to (C.35) and $A^{\text{LME}}(K_1^*) = 1$, we obtain (C.15).

D Outside Investors

What if the firm could raise financing from distressed investors who do not hold the existing debt? As discussed in Section 2, a core feature of an LME is that a majority of existing debtholders must typically agree to transfer collateral. However, an insolvent firm contemplating raising liquidity without an LME does not have the same need to raise financing from its existing debtholders.

We now consider an extension in which a firm chooses between three options: (a) raise liquidity from two specialized distressed debt investors who do not hold the firm's debt, (b) raise liquidity, without an LME, from two specialized distressed debt investors who currently hold the firm's debt, or (c) raise liquidity from two specialized distressed debt investors who currently hold the firm's debt through an LME.

As in our main model, we assume that a firm raises I_0 at $t = 0$ by promising a future payoff of K_0. Lenders have rational expectations and sell the debt to two specialist investors if the firm enters distress. As before, at $t = 1$, the firm must raise I_1 to avoid a certain bankruptcy (paying C^L). A successful liquidity raise leads to a high payoff C^H with probability $1 - p_2$ or a bankruptcy with probability p_2.

Suppose that the initial debt contract banned LMEs. At $t = 1$, we now introduce a choice for equity holders. They can approach a group of two investors who do not hold debt and make a take-it-or-leave-it offer. Alternatively, they can approach the two existing debtholders and make a take-it-or-leave-it offer. We assume that equity holders can only pursue one of these approaches. This assumption is a simple reduced-form manner of capturing how one group of investors might infer from deal failure that the other group of investors received a negative signal in the process of negotiating and conducting due diligence.

If lenders expect that, at date 1, the firm will make an offer K_1 to external investors, the date-0 lenders' break-even condition that determines K_0 is

$$I_0 = (1-p_1+p_1 A^{\text{Outside}}(K_1)(1-p_2))K_0+p_1[1-A^{\text{Outside}}(K_1)+A^{\text{Outside}}(K_1)p_2]\min\{C^L, K_0\}. \quad \text{(D.1)}$$

where $A^{\text{Outside}}(K_1)$ is the probability of a successful liquidity injection and is determined later. It follows from (D.1) that $I_0 \leq K_0$. Since $I_0 > C^L$, we have $K_0 > C^L$. Combining this result with the result in Internet Appendix A.2, we conclude that $K_0 > C^L$ whether the equity holders make an offer to external investors or existing debtholders.

If equity holders make an offer K_1 to the non-debt-holder group, each of the two potential investors weighs a payoff of 0 from rejecting against the following payoff from accepting:

$$R^{\text{Outside}}_{\text{accept}}(\theta_i) \equiv \underbrace{\left(1 - \frac{\widetilde{P}}{2}\right)}_{\text{Share}} \times \underbrace{\big[(1-p_2)K_1 - I_1(1+\theta_i)\big]}_{\text{Renegotiation payoff}}, \tag{D.2}$$

where θ_i is the investor's privately observed funding cost and $\widetilde{P}$ is their perceived probability that the other investor accepts. Since the payoff from rejecting is zero, the potential lender accepts if and only if

$$K_1 \geq \frac{I_1(1+\theta_i)}{1-p_2}. \tag{D.3}$$

Then $A^{\text{Outside}}(K_1)$, the probability of a successful liquidity injection, is given by

$$A^{\text{Outside}}(K_1) = 1, \qquad \text{if} \quad K_1 \geq \frac{I_1(1+\overline{\theta})}{1-p_2}, \tag{D.4}$$

$$A^{\text{Outside}}(K_1) = 1 - q^2, \qquad \text{if} \quad \frac{I_1}{1-p_2} \leq K_1 < \frac{I_1(1+\overline{\theta})}{1-p_2}, \tag{D.5}$$

$$A^{\text{Outside}}(K_1) = 0, \qquad \text{if} \quad K_1 < \frac{I_1}{1-p_2}. \tag{D.6}$$

Thus, equity holders choose between four options at $t = 1$: (1) forgo financing and file for bankruptcy, (2) offer $K_1 = I_1/(1-p_2)$ to external investors and risk failure when $\theta_1 = \theta_2 = \overline{\theta}$, (3) offer $K_1 = I_1(1+\overline{\theta})/(1-p_2)$ to external lenders and ensure financing success, (4) ignore external investors and follow the equilibrium offer to existing debtholders described by Proposition 1. Equity holders choose between these options to maximize their date-1 objective. Other than this change, we define an equilibrium in the same manner as in Subsection 4.2. We now show that it is without loss of generality to ignore the external investors:

Proposition 6. *Suppose that equity holders ban LMEs at $t = 0$ and Conditions 1-4 hold. Let*

$$\mathbb{B}^{\text{Outside}} \equiv I_1\left(1 + \frac{\overline{\theta}}{q^2}\right) + \frac{1-p_2}{q}\frac{I_0 - C^L}{1-p_1p_2}. \tag{D.7}$$

We have $\mathbb{B}^{\text{Outside}} > \mathbb{B}^{\text{Trad}}$*, where* $\mathbb{B}^{\text{Trad}}$ *is given in Proposition 1.*

1. *Proposition 1 continues to hold even if equity holders have the option to instead make an offer to external investors at* $t = 1$*.*

2. *If* $(1 - p_2)(C^H - C^L) \geq \mathbb{B}^{\text{Outside}}$*, there exists an equilibrium* $(K_0^{\text{Outside}}, K_1^{\text{Outside}})$ *in which* $A^{\text{Outside}}(K_1^{\text{Outside}}) = 1$*. There is only one equilibrium in which equity holders make an offer to external investors. Moreover,* $K_0^{\text{Outside}} > K_0^{\text{Trad}}$*,* $K_1^{\text{Outside}} = K_1^{\text{Trad}}$*, and this equilibrium achieves a lower ex-ante firm value than that in Proposition 1, where* K_0^{Trad} *and* K_1^{Trad} *are given in Proposition 1.*

3. *If* $(1 - p_2)(C^H - C^L) < \mathbb{B}^{\text{Outside}}$*, there does not exist an equilibrium in which equity holders make an offer to external investors.*

This proposition shows that the availability of external investors has no impact on equilibrium efficiency. Intuitively, equity holders always weakly prefer to make offers to existing debtholders because these debtholders enjoy some of the surplus from a successful renegotiation, which leads them to accept lower interest rates. Thus, if equity holders have a choice, they will offer a deal to existing debtholders, even when they do not need their approval to move collateral as in an LME.

Finally, we show that LMEs continue to improve efficiency even when outside investors are available. This follows naturally from the above proposition and the fact that LMEs can only be achieved with some consent from existing debtholders.

Corollary 2. *Proposition 2 continues to hold even if equity holders have the option to instead make an offer to external investors at* $t = 1$*.*

D.1 Proof of Proposition 6

We first show that $(K_0^{\text{Trad}}, K_1^{\text{Trad}})$ is an equilibrium when $(1 - p_2)(C^H - C^L) \geq \mathbb{B}^{\text{Trad}}$ and equity holders have the option to make an offer to external investors at $t = 1$, where $\mathbb{B}^{\text{Trad}}$ is defined in (16), K_0^{Trad} is given in (A.17) and K_1^{Trad} is given in (A.16).

By the proof of Proposition 1, it is suboptimal to offer any other K_1 to existing debtholders.

In particular, it is suboptimal to offer the following to existing debtholders:

$$K_1 = \frac{I_1}{1-p_2} - \frac{q}{1+q}(K_0^{\text{Trad}} - C^L). \tag{D.8}$$

As discussed in (D.3), if equity holders approach outside investors, they can ensure success by offering

$$K_1 = \frac{I_1(1+\overline{\theta})}{1-p_2}, \tag{D.9}$$

and they can ensure approval only from low-cost investors by offering

$$K_1 = \frac{I_1}{1-p_2}. \tag{D.10}$$

Since (D.9) is the same as K_1^{Trad}, offering outside investors (D.9) yields the same equity value at time 1. Thus, equity holders are indifferent between these. Further, equity holders achieve the same restructuring probability $1-q^2$ by offering existing debtholders (D.8) or offering outside investors (D.10). It is clear that the expression in (D.10) is higher than (D.8) as $K_0^{\text{Trad}} > C^L$, so equity holder always prefer offering existing debtholders (D.8). However, by Proposition 1, this is not a profitable deviation, so (D.10) is also not a profitable deviation. This proves that $(K_0^{\text{Trad}}, K_1^{\text{Trad}})$ is an equilibrium when $(1-p_2)(C^H - C^L) \geq \mathbb{B}^{\text{Trad}}$.

By Proposition 1, there does not exist an equilibrium in which equity holders make an offer K_1^* to existing debtholders such that $A^{\text{Trad}}(K_1^*) = 1$ when $(1-p_2)(C^H - C^L) < \mathbb{B}^{\text{Trad}}$. To prove the first part of Proposition 6, we only need to show that when $(1-p_2)(C^H - C^L) < \mathbb{B}^{\text{Outside}}$, there does not exist an equilibrium (K_0^*, K_1^*) in which date-0 lenders expect equity holders to make the offer to external investors, guaranteeing acceptance $A^{\text{Outside}}(K_1^*) = 1$. We proceed in a proof by contradiction. Suppose that there exists an equilibrium $(K_0^{\text{Outside}}, K_1^{\text{Outside}})$ in which equity holders make the offer to external investors and $A^{\text{Outside}}(K_1^{\text{Outside}}) = 1$. If $C^H < K_0^{\text{Outside}} + K_1^{\text{Outside}}$ or $C^H < K_0^{\text{Outside}}$, then it is clear that $(K_0^{\text{Outside}}, K_1^{\text{Outside}})$ is not an equilibrium. Then we only need to consider the case $C^H \geq K_0^{\text{Outside}} + K_1^{\text{Outside}}$ and $C^H \geq K_0^{\text{Outside}}$. We derive from (D.4) that

$$K_1^{\text{Outside}} = \frac{I_1(1+\overline{\theta})}{1-p_2} = K_1^{\text{Trad}}. \tag{D.11}$$

Substituting $A^{\text{Outside}} = 1$ into (D.1) and using $K_0^* > C^L$, we obtain

$$K_0^{\text{Outside}} = \frac{I_0 - p_1 p_2 C^L}{1 - p_1 p_2} > K_0^{\text{Trad}}. \tag{D.12}$$

Since we have shown that $K_0^{\text{Trad}} + K_1^{\text{Trad}} > C^L$ and $K_0^{\text{Trad}} > C^L$, then we derive from (D.11) and (D.12) that $K_0^{\text{Outside}} + K_1^{\text{Outside}} > C^L$ and $K_0^{\text{Outside}} > C^L$.

Following the proof of Proposition 1, the firm can ensure approval only from low-cost investors by offering the following to existing debtholders:

$$K_1^{\text{Alt}} = \frac{I_1}{1 - p_2} - \frac{q}{1+q}(K_0^{\text{Outside}} - C^L). \tag{D.13}$$

We have

$$K_0^{\text{Outside}} + K_1^{\text{Alt}} - C^L = \frac{I_1}{1 - p_2} + \frac{1}{1+q}(K_0^{\text{Outside}} - C^L) > 0, \tag{D.14}$$

where the inequality uses $K_0^{\text{Outside}} > C^L$. Recall that we are considering the case $C^H \geq K_0^{\text{Outside}} + K_1^{\text{Outside}}$. If $C^H - K_0^{\text{Outside}} - K_1^{\text{Outside}} < (1-q^2)\big(C^H - K_0^{\text{Outside}} - K_1^{\text{Alt}}\big)$, then $C^H - K_0^{\text{Outside}} - K_1^{\text{Alt}} > 0$ and K_1^{Alt} is a feasible deviation. Thus, K_1^{Alt} is a profitable deviation ex post if and only if

$$C^H - K_0^{\text{Outside}} - K_1^{\text{Outside}} < (1 - q^2)\big(C^H - K_0^{\text{Outside}} - K_1^{\text{Alt}}\big). \tag{D.15}$$

Using (D.13), (D.11), and following (A.20), we can show (D.15) is equivalent to

$$(1 - p_2)(C^H - C^L) < \frac{1}{q^2}\left[I_1\left(q^2 + \overline{\theta}\right)\right] + \frac{1 - p_2}{q}(K_0^{\text{Outside}} - C^L) = \mathbb{B}^{\text{Outside}}. \tag{D.16}$$

Therefore, when $(1 - p_2)(C^H - C^L) < \mathbb{B}^{\text{Outside}}$, K_1^{Alt} is a feasible and profitable deviation, implying $(K_0^{\text{Outside}}, K_1^{\text{Outside}})$ is not an equilibrium. This proves the first part of Proposition 6.

Next, we prove the second part of Proposition 6. We first prove that $K_0^{\text{Outside}} = \frac{I_0 - p_1 p_2 C^L}{1 - p_1 p_2}$, and offering external investors $K_1^{\text{Outside}} = \frac{I_1(1+\overline{\theta})}{1-p_2}$, is an equilibrium when $(1 - p_2)(C^H - C^L) \geq \mathbb{B}^{\text{Outside}}$. It is clear that $K_0^{\text{Outside}} > C^L$. Moreover, $K_0^{\text{Outside}} + K_1^{\text{Outside}} < C^H$ is equivalent to

$$(1 - p_2)(C^H - C^L) > I_1(1 + \overline{\theta}) + \left(I_0 - C^L\right)\frac{1 - p_2}{1 - p_1 p_2}, \tag{D.17}$$

which is implied by $(1-p_2)(C^H - C^L) \geq \mathbb{B}^{\text{Outside}}$. Since $K_1^{\text{Outside}} > 0$, we have $C^L < K_0^{\text{Outside}} < K_0^{\text{Outside}} + K_1^{\text{Outside}} < C^H$.

Since $K_1^{\text{Outside}} = K_1^{\text{Trad}}$, equity holders are indifferent between offering existing debtholders K_1^{Trad} and offering external investors K_1^{Outside}. Equity holders achieve the same restructuring probability $1-q^2$ by offering existing debtholders (D.13) or offering outside investors (D.10). Because (D.10) is higher than (D.13), we only need to verify whether it is profitable to offer existing debtholders (D.13). According to (D.15) and (D.16), when $(1-p_2)(C^H - C^L) \geq \mathbb{B}^{\text{Outside}}$, it is suboptimal to make offer (D.13) to existing debtholders. This shows that $(K_0^{\text{Outside}}, K_1^{\text{Outside}})$ is an equilibrium. Using (D.4), we have $A^{\text{Outside}}(K_1^{\text{Outside}}) = 1$.

For the sake of contradiction, suppose there exists an equilibrium in which equity holders offer external investors K_1^* satisfying $A^{\text{Outside}}(K_1^*) = 1-q^2$. Using (D.5), K_1^* is given by (D.10). Recall that we have shown $K_0^* > C^L$ using (D.1) and $I_0 > C^L$.

Firm can ensure approval only from low-cost investors by offering the following to existing debtholders:

$$K_1^{\text{Alt}} = \frac{I_1}{1-p_2} - \frac{q}{1+q}(K_0^* - C^L), \tag{D.18}$$

which is lower than (D.10). It could never be an equilibrium to offer (D.10) to external investors, since equity holders would deviate and offer (D.18) to debtholders to get the same restructuring probability success. Therefore, we prove that there does not exist an equilibrium in which the equity holders offer external investors K_1^* that satisfies $A^{\text{Outside}}(K_1^*) = 1-q^2$. It follows that $(K_0^{\text{Outside}}, K_1^{\text{Outside}})$ is the unique equilibrium, provided that equity holders make an offer to external investors.

Note that

$$\begin{aligned} E_0^{\text{Outside}} &= (1-p_1)(C^H - K_0^{\text{Outside}}) + p_1 A^{\text{Outside}}(K_1^{\text{Outside}})(1-p_2)(C^H - K_0^{\text{Outside}} - K_1^{\text{Outside}}), \\ E_0^{\text{Trad}} &= (1-p_1)(C^H - K_0^{\text{Trad}}) + p_1 A^{\text{Trad}}(K_1^{\text{Trad}})(1-p_2)(C^H - K_0^{\text{Trad}} - K_1^{\text{Trad}}). \end{aligned} \tag{D.19}$$

Using $A^{\text{Outside}}(K_1^{\text{Outside}}) = A^{\text{Trad}}(K_1^{\text{Trad}}) = 1$, $K_0^{\text{Outside}} > K_0^{\text{Trad}}$, $K_1^{\text{Outside}} = K_1^{\text{Trad}}$, we conclude $E_0^{\text{Outside}} < E_0^{\text{Trad}}$. This proves the second part of Proposition 6.

Finally, recall that we have proved that 1.) there does not exist an equilibrium in which equity holders offer external investors K_1^* satisfying $A^{\text{Outside}}(K_1^*) = 1 - q^2$, and 2.) when $(1 - p_2)(C^H - C^L) < \mathbb{B}^{\text{Outside}}$, there does not exist an equilibrium in which equity holders offer external investors K_1^* satisfying $A^{\text{Outside}}(K_1^*) = 1$. Then we prove the third part of Proposition 6.

D.2 Proof of Corollary 2

For Proposition 2 to fail, there must exist a profitable deviation from an equilibrium with LMEs to make an offer to outside investors. By the arguments used in the proof of Proposition 6, any deviation to make an offer to outside investors is worse than an analogous deviation to make a traditional restructuring offer to existing debtholders. By Proposition 2, there are no profitable deviations to make traditional restructuring offers to existing debtholders, so there must be no profitable deviations to make offers to external investors.

E Secured Debt

In this extension, at $t = 0$, the firm chooses between (1) unsecured debt, (2) secured debt banning LMEs (which is equivalent to traditional renegotiation in Subsection 4.2), or (3) secured debt enabling LMEs (which is equivalent to the LME equilibrium in Subsection 4.3). We focus on option (1), which is novel in this extension. We characterize when unsecured debt can reach the constrained efficient ex-ante firm value. We show that in some circumstances, equity holders strictly prefer to issue secured debt with an LME option.

We now characterize the equilibrium when a firm issues unsecured debt at $t = 0$. At $t = 1$, if the firm enters distress, we assume that it has two options: it can issue secured debt to existing debt holders (similar to traditional renegotiation in Subsection 4.2) or it can issue secured debt to outside investors (as in the extension of Subsection 5.3). Either way, as in Subsection 5.3, we assume the firm makes a take-it-or-leave-it offer to a pool of two distressed debt investors. Each investor has a funding cost θ_i and decides whether to accept or reject. Once the firm makes an offer to one investor group (debt holders or outside investors), they cannot make an offer to the other group due to a perception that the first group must have observed a negative signal.

Consider a subgame in which the firm issued unsecured debt at $t = 0$ with promised payment K_0^*, the firm entered distress, and equity holders chose to make an offer to issue secured debt with promised payment K_1 to outside investors. Crucially, we assume that the new secured debt is senior to the previously issued unsecured debt. Thus, for any $K_1 < C^L$, the debt is oversecured and risk free. For any $K_1 < C^L$, we accordingly assume that outside investors receive $K_1 - I_1(1 + \theta_i)$ for funding the entire deal, regardless of what subsequently happens to the firm. An outside investor receives half of this for funding half of the deal. We assume an outside investor receives nothing if they reject (they do not hold the firm's debt so they are unaffected by its failure). Given this, outside investors have a simple decision rule if $K_1 < C^L$: accept if and only if $K_1 \geq I_1(1 + \theta_i)$.

Then $A^{\text{Outside}}(K_1)$, the probability of a successful liquidity injection, is given by

$$A^{\text{Outside}}(K_1) = 1, \qquad \text{if} \quad K_1 \geq I_1(1+\overline{\theta}), \tag{E.1}$$

$$A^{\text{Outside}}(K_1) = 1 - q^2, \qquad \text{if} \quad I_1 \leq K_1 < I_1(1+\overline{\theta}), \tag{E.2}$$

$$A^{\text{Outside}}(K_1) = 0, \qquad \text{if} \quad K_1 < I_1. \tag{E.3}$$

We assume that $I_1(1+\overline{\theta}) < C^L$, so equity holders can always guarantee renegotiation with risk free secured debt if they are willing to pay $K_1 = I_1(1+\overline{\theta})$. Thus, we only need to consider $K_1 \leq I_1(1+\overline{\theta}) < C^L$. In addition, as in our baseline model, we assume $K_0 \leq C^H$ and $K_0 + K_1 \leq C^H$.

Alternatively, equity holders can make a financing offer to the two distressed debt investors who currently own the unsecured debt. If a debt holder i accepts an offer, they receive the following payoff:

$$R^{\text{Sec}}_{\text{accept}}(\theta_i) = \underbrace{\frac{(1-p_2)K_0^* + p_2 \min\{C^L - K_1, K_0^*\}}{2}}_{\text{Original debt contract}} + \underbrace{\left(1 - \frac{\widetilde{P}}{2}\right)}_{\text{Share}} \times \underbrace{\left[K_1 - I_1(1+\theta_i)\right]}_{\text{renegotiation payoff}}. \tag{E.4}$$

The first term captures the fact that accepting prevents bankruptcy with probability $1-p_2$, yielding the original unsecured debt payoff K_0^*, and leads to bankruptcy with probability p_2, which yields an impaired unsecured payoff equal to half of the residual value $C^L - K_1$ after paying new secured debt. The second term captures the payoff from funding the new secured debt, which is potentially split if the other investor accepts.

If debt holder i rejects an offer, they get the following payoff:

$$R^{\text{Sec}}_{\text{reject}} = (1-\widetilde{P})\frac{C^L}{2} + \widetilde{P}\left[\frac{(1-p_2)K_0^* + p_2 \min\{C^L - K_1, K_0^*\}}{2}\right]. \tag{E.5}$$

The first term captures the bankruptcy payoff if no secured debt is issued. The second term captures the payoff from holding the previously issued unsecured debt given that the new secured debt avoids certain bankruptcy and takes priority.

As before, equity holders choose the offer K_1 to maximize their ex-post payoff, where the probability of renegotiation acceptance depends on which investor pool receives the offer and which types of investors accept the offer. At $t = 0$, all lenders have rational expectations. If lenders anticipate that equity holders make an offer to the distressed debt investors holding the debt, they can price this in when they sell the debt:

$$I_0 = (1-p_1)K_0^* + p_1\Big[(1-A^{\text{Inside}}(K_1^*))\min\{C^L, K_0^*\} + A^{\text{Inside}}(K_1^*)(1-p_2)K_0^*\Big]$$
$$+ p_1\Big[A^{\text{Inside}}(K_1^*)p_2\min\{C^L - K_1^*, K_0^*\} + \underbrace{A^{\text{Inside}}(K_1^*)(K_1^* - I_1) - I_1\mathbb{E}[\Theta]}_{\text{New secured debt}}\Big], \tag{E.6}$$

where $\Theta = \frac{\sum_i \theta_i \mathbf{1}_{\text{accept}}(K_1^*,\theta_i)}{1+\prod_i \mathbf{1}_{\text{accept}}(K_1^*,\theta_i)}$.

Alternatively, if lenders anticipate that equity holders will offer the secured debt to outside investors (not the ones who buy the unsecured debt), then debt is priced as

$$I_0 = (1-p_1)K_0^* + p_1\Big[(1-A^{\text{Outside}}(K_1^*))\min\{C^L, K_0^*\} + A^{\text{Outside}}(K_1^*)(1-p_2)K_0^*\Big]$$
$$+ p_1 A^{\text{Outside}}(K_1^*)p_2\min\{C^L - K_1^*, K_0^*\}. \tag{E.7}$$

In equilibrium, we will show that the constrained efficient ex-ante firm value can only be achieved if equity holders find it optimal ex-post to issue secured debt to existing debt holders. Otherwise, the information rent that is usually transferred from equity holders to lenders is instead transferred to third party investors, reducing ex-ante firm value.

Proposition 7. *Suppose that equity holders issue unsecured debt at $t = 0$, Conditions 1-4 hold, and*

$$I_0 - C^L + \Big(1 - \overline{\theta}p_1(1-p_2-q)\Big)I_1 > 0, \tag{E.8}$$
$$\mathbb{B}^{\text{LME}} < (1-p_2)(C^H - C^L) < \max\{\mathbb{B}^{\text{Sec}}_{\text{Inside}}\ ,\ \mathbb{B}^{\text{Sec}}_{\text{Outside}}\}\ , \tag{E.9}$$

where

$$\mathbb{B}^{\text{Sec}}_{\text{Inside}} = \frac{q+1-p_2}{q+1-qp_2}\mathbb{B}^{\text{Trad}} + \frac{I_1 p_2}{q(q+1-qp_2)}\Big[(1+\overline{\theta})(p_1 q + p_1 - 1)\frac{1-p_2}{1-p_1p_2} - (q^2+\overline{\theta})\Big], \tag{E.10}$$
$$\mathbb{B}^{\text{Sec}}_{\text{Outside}} = q\mathbb{B}^{\text{Trad}} + I_1\Bigg[\bigg(1+\overline{\theta}\Big(\frac{1-p_1p_2}{q^2} + p_1p_2\Big)\bigg)\frac{1-p_2}{1-p_1p_2} - \Big(q + \frac{\overline{\theta}}{q}\Big)\Bigg], \tag{E.11}$$

$\mathbb{B}^{\text{Trad}}$ *is given in Proposition 1 and* $\mathbb{B}^{\text{LME}}$ *is given in Proposition 2. Then ex-ante firm value is strictly lower with unsecured debt than the value that is achieved by issuing secured debt with an LME option.*

Intuitively, LME-exposed secured debt can do better than unsecured debt ex ante because (1) the coercive nature of an LME makes it even cheaper than issuing secured debt with unencumbered collateral, incentivizing equity holders to ensure renegotiation success, and (2) offering the information rent to existing debt holders maximizes ex ante firm value. Note that (E.8) holds for any $\overline{\theta} \leq 1/p_1$ as $C^L < I_0$.

E.1 Proof of Proposition 7

First, we calculate ex-ante firm value as follows:

$$\begin{aligned} E_0 =&(1-p_1)(C^H - K_0) + p_1[1 - A(K_1)](C^L - K_0)^+ \\ &+ p_1 A(K_1)\Big[p_2(C^L - K_0 - K_1)^+ + (1-p_2)(C^H - K_0 - K_1)\Big]. \end{aligned} \tag{E.12}$$

When there is an inside offer to debtholders, using $E_0 = E_0 + I_0 - I_0$, (E.12) and (E.6), we get

$$\begin{aligned} E_0^{\text{Inside}} =&(1-p_1)C^H - I_0 + p_1\Big[(1 - A^{\text{Inside}}(K_1^*))C^L + A^{\text{Inside}}(K_1^*)\left(p_2 C^L + (1-p_2)C^H - K_1^*\right)\Big] \\ &+ p_1\left[A^{\text{Inside}}(K_1^*)(K_1^* - I_1) - I_1\mathbb{E}[\Theta]\right] \\ =&(1-p_1)C^H + p_1 C^L - I_0 + p_1\Big[A^{\text{Inside}}(K_1^*)(1-p_2)(C^H - C^L) - I_1\big(A^{\text{Inside}}(K_1^*) + \mathbb{E}[\Theta]\big)\Big]. \end{aligned} \tag{E.13}$$

This is the expression from Lemma 1. Using Lemma 2, E_0^{Inside} is maximized when $A^{\text{Inside}}(K_1^*) = 1$.

When there is an outside offer to outside investors (non debt holders), using $E_0 = E_0 + I_0 - I_0$, (E.12) and (E.7), we get

$$
\begin{aligned}
&E_0^{\text{Outside}}\\
=&(1-p_1)C^H - I_0 + p_1\Big[(1-A^{\text{Outside}}(K_1^*))C^L + A^{\text{Outside}}(K_1^*)\left(\ p_2C^L + (1-p_2)C^H - K_1^*\ \right)\Big]\\
=&(1-p_1)C^H + p_1C^L - I_0 + p_1A^{\text{Outside}}(K_1^*)\left(\ (1-p_2)(C^H-C^L) - K_1^*\ \right). \qquad \text{(E.14)}
\end{aligned}
$$

According to (E.1), $K_1^* = I_1(1+\overline{\theta})$ is the lowest one satisfying $A^{\text{Outside}}(K_1^*) = 1$, and the corresponding ex-ante firm value for the outside offer is

$$
E_0^{\text{Outside}} = (1-p_1)C^H + p_1C^L - I_0 + p_1\left(\ (1-p_2)(C^H-C^L) - I_1(1+\overline{\theta})\ \right). \qquad \text{(E.15)}
$$

This is strictly lower than E_0^{Inside} in (E.13) when $A^{\text{Inside}}(K_1^*) = 1$ as $\mathbb{E}[\Theta] = q\overline{\theta}$. According to (E.2), $K_1^* = I_1$ is the lowest one satisfying $A^{\text{Outside}}(K_1^*) = 1-q^2$, and the corresponding ex-ante firm value for the outside offer is

$$
E_0^{\text{Outside}} = (1-p_1)C^H + p_1C^L - I_0 + p_1(1-q^2)\left(\ (1-p_2)(C^H-C^L) - I_1\ \right). \qquad \text{(E.16)}
$$

This is the same as E_0^{Inside} in (E.13) when $A^{\text{Inside}}(K_1^*) = 1-q^2$ as $\mathbb{E}[\Theta] = 0$.

Because we have shown that $E_0^{\text{Outside}} = E_0^{\text{Inside}}$ when $A^{\text{Outside}}(K_1^*) = A^{\text{Inside}}(K_1^*) = 1-q^2$, and $E_0^{\text{Outside}} < E_0^{\text{Inside}}$ when $A^{\text{Outside}}(K_1^*) = A^{\text{Inside}}(K_1^*) = 1$, and because E_0^{Inside} is maximized when $A^{\text{Inside}}(K_1^*) = 1$, ex-ante firm value is maximized by an inside offer that ensures $A^{\text{Inside}}(K_1^*) = 1$. We now show that this can not be supported in equilibrium when $(1-p_2)(C^H-C^L) < \max\{\mathbb{B}^{\text{Sec}}_{\text{Inside}}, \mathbb{B}^{\text{Sec}}_{\text{Outside}}\}$. To obtain a contradiction, assume that there exists an equilibrium (K_0^*, K_1^*) with $A^{\text{Inside}}(K_1^*) = 1$.

Investor i accepts the offer if and only if $R^{\text{Sec}}_{\text{accept}}(\theta_i) \geq R^{\text{Sec}}_{\text{reject}}$, i.e.

$$
\begin{aligned}
&\frac{(1-p_2)K_0^* + p_2\min\{C^L-K_1, K_0^*\}}{2} + \left(1-\frac{\widetilde{P}}{2}\right)\times\big[K_1 - I_1(1+\theta_i)\big]\\
\geq&(1-\widetilde{P})\frac{C^L}{2} + \widetilde{P}\Big[\frac{(1-p_2)K_0^* + p_2\min\{C^L-K_1, K_0^*\}}{2}\Big]. \qquad \text{(E.17)}
\end{aligned}
$$

Substituting $\widetilde{P}=1$ into the above inequality, we can see investor i accepts the offer if and only if

$$\frac{1}{2}\Big[K_1 - I_1(1+\theta_i)\Big] \geq 0. \tag{E.18}$$

Thus, if equity holders wish to achieve $\widetilde{P}=1$ with an offer to debt holders, they optimally offer

$$K_1^* = I_1(1+\overline{\theta})$$

and all investors accept.

From the above, the equilibrium inside offer that guarantees acceptance is $K_1^* = I_1(1+\overline{\theta})$. Plugging this, $A^{\text{Inside}}(K_1^*) = 1$ and $\mathbb{E}[\Theta] = q\overline{\theta}$ into the ex-ante debt pricing equation for inside offers (E.6),

$$\begin{aligned} I_0 &= (1-p_1)K_0^* + p_1\Big[p_2 \min\{C^L - K_1^*, K_0^*\} + (1-p_2)K_0^* + K_1^* - I_1(1+q\overline{\theta})\Big] \\ &= (1-p_1p_2)K_0^* + p_1\Big[p_2 \min\{C^L - K_1^*, K_0^*\} + K_1^* - I_1(1+q\overline{\theta})\Big]. \end{aligned} \tag{E.19}$$

It follows that

$$\begin{aligned} I_0 &\leq (1-p_1p_2)K_0^* + p_1\Big[p_2(C^L - K_1^*) + K_1^* - I_1(1+q\overline{\theta})\Big] \\ &= (1-p_1p_2)K_0^* + p_1\Big[p_2C^L + (1-p_2)I_1(1+\overline{\theta}) - I_1(1+q\overline{\theta})\Big]. \end{aligned} \tag{E.20}$$

Thus,

$$K_0^* \geq \frac{I_0 - p_1p_2C^L - p_1I_1\Big[-p_2 + \overline{\theta}(1-p_2-q)\Big]}{1-p_1p_2}. \tag{E.21}$$

Combing the above with (E.18), we have

$$\begin{aligned} K_0^* + K_1^* - C^L &\geq \frac{I_0 - C^L - p_1I_1\Big[-p_2 + \overline{\theta}(1-p_2-q)\Big]}{1-p_1p_2} + I_1(1+\overline{\theta}) \\ &= \frac{I_0 - C^L + I_1\Big[1 + \overline{\theta}(1-p_1+p_1q)\Big]}{1-p_1p_2} > 0. \end{aligned} \tag{E.22}$$

Thus, (E.20) and (E.21) become equalities. Then we have

$$K_0^* = \frac{I_0 - p_1 p_2 C^L - p_1 I_1 \Big[- p_2 + \overline{\theta}(1 - p_2 - q) \Big]}{1 - p_1 p_2}. \tag{E.23}$$

If $C^H < K_0^* + K_1^*$, then it is clear that (K_0^*, K_1^*) is not an equilibrium. Then we only need to consider the case $C^H \geq K_0^* + K_1^*$. Using $K_1^* > 0$, we have $C^H \geq K_0^* + K_1^* > K_0^*$.

Next, we consider off the equilibrium path in which $\widetilde{P} = 1 - q$. Substituting $\widetilde{P} = 1 - q$ into (E.17), an existing debtholder with $\theta_i = 0$ accepts an offer K_1 if and only if

$$q\frac{(1-p_2)K_0^* + p_2 \min\{C^L - K_1, K_0^*\}}{2} + \frac{1+q}{2}\Big[K_1 - I_1\Big] \geq q\frac{C^L}{2}. \tag{E.24}$$

Since $\min\{C^L - K_1, K_0^*\} \leq C^L - K_1$, we derive from (E.24) that

$$\begin{aligned} & K_1[1 + q(1-p_2)] \geq qC^L + (1+q)I_1 - q((1-p_2)K_0^* + p_2 C^L) \\ \iff \quad & K_1 \geq \frac{1}{1 + q(1-p_2)}\big[q(1-p_2)(C^L - K_0^*) + (1+q)I_1\big]. \end{aligned} \tag{E.25}$$

Thus,

$$\begin{aligned} & K_0^* + K_1 - C^L \geq \frac{K_0^* - C^L + (1+q)I_1}{1 + q(1-p_2)} \\ = & \frac{1}{1+q(1-p_2)}\left[\frac{I_0 - C^L - p_1 I_1 \Big[- p_2 + \overline{\theta}(1 - p_2 - q) \Big]}{1 - p_1 p_2} + (1+q)I_1\right] \\ = & \frac{1}{[1+q(1-p_2)](1-p_1p_2)}\left[I_0 - C^L + \Big(1 + (1-p_1p_2)q - \overline{\theta}p_1(1-p_2-q)\Big)I_1\right] > 0, \end{aligned} \tag{E.26}$$

where the inequality uses (E.8).

Thus, (E.24) is equivalent to (E.25), and a type 0 debtholder is indifferent between accepting and rejecting if

$$K_1^{\text{Alt}} = \frac{1}{1 + q(1-p_2)}\big[q(1-p_2)(C^L - K_0^*) + (1+q)I_1\big]. \tag{E.27}$$

Similar to (E.26), we have $K_0^* + K_1^{\text{Alt}} - C^L > 0$.

Using $C^H \geq K_0^* + K_1^* > C^L$ and $K_0^* + K_1^{\text{Alt}} > C^L$, we can see an inside offer K_1^{Alt} that achieves

$A^{\text{Inside}}(K_1^{\text{Alt}}) = 1 - q^2$ yields a higher payoff, relative to the strategy $K_1^* = I_1(1+\overline{\theta})$, if

$$C^H - K_0^* - K_1^* < (1-q^2)(C^H - K_0^* - K_1^{\text{Alt}}). \tag{E.28}$$

Note that (E.28) is equivalent to

$$\begin{aligned}
& C^H - K_0^* - K_1^* < (1-q^2)(C^H - K_0^* - K_1^{\text{Alt}}) \\
\iff\ & q^2(C^H - K_0^*) < K_1^* - (1-q^2)K_1^{\text{Alt}} \\
\iff\ & C^H < K_0^* + \frac{K_1^* - (1-q^2)K_1^{\text{Alt}}}{q^2} \\
\iff\ & C^H - C^L < K_0^* - C^L + \frac{K_1^* - (1-q^2)K_1^{\text{Alt}}}{q^2}.
\end{aligned} \tag{E.29}$$

Plugging in the K_1^{Alt} given by (E.27) and plugging in $K_1^* = I_1(1+\overline{\theta})$, the right side is

$$\begin{aligned}
& K_0^* - C^L + \frac{K_1^* - (1-q^2)\left(\ I_1 + \frac{1}{1+q(1-p_2)}\left[q(1-p_2)(C^L - K_0^*) + p_2 q I_1\right]\ \right)}{q^2} \\
=& K_0^* - C^L + I_1 + \frac{1}{q^2}\left(\ I_1\overline{\theta} - \frac{1-q^2}{1+q(1-p_2)}\left[q(1-p_2)(C^L - K_0^*) + p_2 q I_1\right]\ \right) \\
=& (K_0^* - C^L)\left(1 + \frac{(1-q^2)(1-p_2)}{q(1+q(1-p_2))}\right) + I_1\left(1 + \frac{1}{q^2}\left(\overline{\theta} - \frac{(1-q^2)p_2 q}{1+q(1-p_2)}\right)\right) \\
=& (K_0^* - C^L)\frac{q+1-p_2}{q(1+q-qp_2)} + I_1\left(1 + \frac{\overline{\theta}}{q^2} - \frac{(1-q^2)p_2}{q(1+q(1-p_2))}\right).
\end{aligned} \tag{E.30}$$

Plugging in $K_0^* - C^L = \frac{I_0 - C^L - p_1 I_1\left[\overline{\theta}(1-p_2-q) - p_2\right]}{1-p_1p_2}$ and rearranging, this is

$$\begin{aligned}
& \frac{I_0 - C^L - p_1 I_1\left[\overline{\theta}(1-p_2-q) - p_2\right]}{1-p_1p_2}\frac{q+1-p_2}{q(1+q-qp_2)} + I_1\left(1 + \frac{\overline{\theta}}{q^2} - \frac{(1-q^2)p_2}{q(1+q(1-p_2))}\right) \\
=& \frac{I_0 - C^L - p_1 I_1\overline{\theta}(1-q)}{1-p_1p_2}\frac{q+1-p_2}{q(1+q-qp_2)} + I_1(1+\overline{\theta})\frac{p_1p_2}{1-p_1p_2}\frac{q+1-p_2}{q(1+q-qp_2)} \\
& + I_1\left(1 + \frac{\overline{\theta}}{q^2} - \frac{(1-q^2)p_2}{q(1+q(1-p_2))}\right) \\
=& I_1\left(1 + \frac{\overline{\theta}}{q^2}\right) + \frac{I_0 - C^L - p_1 I_1\overline{\theta}(1-q)}{1-p_1p_2}\frac{q+1-p_2}{q(q+1-qp_2)} \\
& + \frac{I_1 p_2}{q(q+1-qp_2)}\left((1+\overline{\theta})\frac{p_1(q+1-p_2)}{1-p_1p_2} - (1-q^2)\right).
\end{aligned} \tag{E.31}$$

Multiplying the above equation by $(1-p_2)$, we get

$$\mathbb{B}^{\text{Sec}}_{\text{Inside}} = I_1\left(1+\frac{\overline{\theta}}{q^2}\right)(1-p_2) + \frac{I_0 - C^L - p_1 I_1 \overline{\theta}(1-q)}{1-p_1p_2}\frac{1-p_2}{q}\frac{q+1-p_2}{q+1-qp_2}$$
$$+ \frac{I_1 p_2(1-p_2)}{q(q+1-qp_2)}\left((1+\overline{\theta})\frac{p_1(q+1-p_2)}{1-p_1p_2} - (1-q^2)\right). \tag{E.32}$$

Combining the above with (16), we have

$$\begin{aligned}
&\mathbb{B}^{\text{Sec}}_{\text{Inside}} - \frac{q+1-p_2}{q+1-qp_2}\mathbb{B}^{\text{Trad}} = I_1\left(1+\frac{\overline{\theta}}{q^2}\right)\left(1-p_2-\frac{q+1-p_2}{q+1-qp_2}\right)\\
&\quad + \frac{I_1 p_2(1-p_2)}{q(q+1-qp_2)}\left((1+\overline{\theta})\frac{p_1(q+1-p_2)}{1-p_1p_2} - (1-q^2)\right)\\
=&I_1\left(1+\frac{\overline{\theta}}{q^2}\right)(-p_2)q\left(\frac{2-p_2}{q+1-qp_2}\right) + \frac{I_1 p_2(1-p_2)}{q(q+1-qp_2)}\left((1+\overline{\theta})\frac{p_1(q+1-p_2)}{1-p_1p_2} - (1-q^2)\right)\\
=&\frac{I_1p_2}{q(q+1-qp_2)}\left[(1-p_2)\left((1+\overline{\theta})\frac{p_1(q+1-p_2)}{1-p_1p_2} - (1-q^2)\right) - (q^2+\overline{\theta})(2-p_2)\right]\\
=&\frac{I_1p_2}{q(q+1-qp_2)}\left[(1-p_2)\left((1+\overline{\theta})\frac{p_1(q+1-p_2)}{1-p_1p_2} - (1+\overline{\theta}))\right) - (q^2+\overline{\theta})\right]\\
=&\frac{I_1p_2}{q(q+1-qp_2)}\left[(1-p_2)(1+\overline{\theta})\frac{p_1q+p_1-1}{1-p_1p_2} - (q^2+\overline{\theta})\right].
\end{aligned} \tag{E.33}$$

Then we have

$$\mathbb{B}^{\text{Sec}}_{\text{Inside}} = \frac{q+1-p_2}{q+1-qp_2}\mathbb{B}^{\text{Trad}} + \frac{I_1p_2}{q(q+1-qp_2)}\left[(1+\overline{\theta})(p_1q+p_1-1)\frac{1-p_2}{1-p_1p_2} - (q^2+\overline{\theta})\right]. \tag{E.34}$$

Thus, if $(1-p_2)(C^H - C^L)$ is smaller than $\mathbb{B}^{\text{Sec}}_{\text{Inside}}$, then there is a strictly profitable deviation to the equilibrium strategy of $K_1^* = I_1(1+\overline{\theta})$: make an inside offer K_1^{Alt}.

Next, we consider an outside offer $K_1 = I_1$, which achieves $A^{\text{Outside}}(K_1) = 1-q^2$. We have

$$\begin{aligned}
&K_0^* + I_1 - C^L\\
=&\frac{I_0 - C^L - p_1I_1\Big[-p_2+\overline{\theta}(1-p_2-q)\Big]}{1-p_1p_2} + I_1\\
=&\frac{1}{1-p_1p_2}\left[I_0 - C^L + \Big(1-\overline{\theta}p_1(1-p_2-q)\Big)I_1\right] > 0,
\end{aligned} \tag{E.35}$$

where the inequality uses (E.8).

Using $C^H \geq K_0^* + K_1^* > C^L$ and $K_0^* + I_1 > C^L$, we can see that an outside offer $K_1 = I_1$ yields a higher payoff, relative to the strategy $K_1^* = I_1(1+\overline{\theta})$, if

$$C^H - K_0^* - K_1^* < (1-q^2)(C^H - K_0^* - I_1). \tag{E.36}$$

Next, we simplify (E.36). Similar to (E.29), we have

$$\begin{aligned} & C^H - K_0^* - K_1^* < (1-q^2)(C^H - K_0^* - I_1) \\ \iff \quad & C^H - C^L < K_0^* - C^L + \frac{K_1^* - (1-q^2)I_1}{q^2}. \end{aligned} \tag{E.37}$$

Plugging in $K_1^* = I_1(1+\overline{\theta})$ and $K_0^* - C^L = \frac{I_0 - C^L - p_1 I_1 \left[\overline{\theta}(1-p_2-q) - p_2\right]}{1-p_1p_2}$, and rearranging, the right side is

$$\begin{aligned} & \frac{I_0 - C^L - p_1 I_1\left[\overline{\theta}(1-p_2-q) - p_2\right]}{1-p_1p_2} + \frac{I_1(1+\overline{\theta}) - (1-q^2)I_1}{q^2} \\ =& \frac{I_0 - C^L}{1-p_1p_2} + I_1\left[\frac{\overline{\theta}+q^2}{q^2} - \frac{p_1\left[\overline{\theta}(1-p_2-q) - p_2\right]}{1-p_1p_2}\right] \\ =& \frac{I_0 - C^L}{1-p_1p_2} + I_1\left[\frac{\overline{\theta}}{q^2} - \frac{p_1\overline{\theta}(1-p_2-q) - 1}{1-p_1p_2}\right] \\ =& \frac{I_0 - C^L - p_1 I_1(1-q)\overline{\theta}}{1-p_1p_2} + I_1\left[\frac{\overline{\theta}}{q^2} - \frac{p_1\overline{\theta}(-p_2) - 1}{1-p_1p_2}\right] \\ =& \frac{I_0 - C^L - p_1 I_1(1-q)\overline{\theta}}{1-p_1p_2} + I_1\left[1 + \frac{\overline{\theta}}{q^2} + \frac{p_1p_2(1+\overline{\theta})}{1-p_1p_2}\right]. \end{aligned} \tag{E.38}$$

Multiplying the above equation by $(1-p_2)$, we get

$$\begin{aligned} \mathbb{B}^{\text{Sec}}_{\text{Outside}} =& I_1\left[1 + \frac{\overline{\theta}}{q^2} + \frac{p_1p_2(1+\overline{\theta})}{1-p_1p_2}\right](1-p_2) + \frac{I_0 - C^L - p_1 I_1\overline{\theta}(1-q)}{1-p_1p_2}(1-p_2) \\ =& I_1\left(1 + \overline{\theta}\Big(\frac{1-p_1p_2}{q^2} + p_1p_2\Big)\right)\frac{1-p_2}{1-p_1p_2} + \frac{I_0 - C^L - p_1 I_1\overline{\theta}(1-q)}{1-p_1p_2}(1-p_2). \end{aligned} \tag{E.39}$$

Combining the above with (16), we have

$$\mathbb{B}^{\text{Sec}}_{\text{Outside}} - q\mathbb{B}^{\text{Trad}} = I_1\left[\left(1 + \overline{\theta}\Big(\frac{1-p_1p_2}{q^2} + p_1p_2\Big)\right)\frac{1-p_2}{1-p_1p_2} - \Big(q + \frac{\overline{\theta}}{q}\Big)\right]. \tag{E.40}$$

Then we have

$$\mathbb{B}^{\text{Sec}}_{\text{Outside}} = q\mathbb{B}^{\text{Trad}} + I_1 \left[\left(1 + \overline{\theta} \Big(\frac{1 - p_1 p_2}{q^2} + p_1 p_2 \Big) \right) \frac{1 - p_2}{1 - p_1 p_2} - \Big(q + \frac{\overline{\theta}}{q} \Big) \right]. \tag{E.41}$$

Thus if $(1 - p_2)(C^H - C^L)$ is smaller than $\mathbb{B}^{\text{Sec}}_{\text{Outside}}$, then there is at least one strictly profitable deviation to the strategy $K_1^* = I_1(1 + \overline{\theta})$: make an outside offer I_1.

Therefore, if $(1 - p_2)(C^H - C^L) < \max\{\mathbb{B}^{\text{Sec}}_{\text{Inside}} \ , \ \mathbb{B}^{\text{Sec}}_{\text{Outside}}\}$, there does not exist an equilibrium with an inside offer that satisfies $A^{\text{Inside}}(K_1^*) = 1$. Since we have shown that ex-ante firm value is maximized by an inside offer that ensures $A^{\text{Inside}}(K_1^*) = 1$, and E_0^{Inside} is identical to (14), equity can do better ex-ante with an LME when

$$\mathbb{B}^{\text{LME}} < (1 - p_2)(C^H - C^L) < \max\{\mathbb{B}^{\text{Sec}}_{\text{Inside}} \ , \ \mathbb{B}^{\text{Sec}}_{\text{Outside}}\}. \tag{E.42}$$

F Optimal Mechanism Design Proofs and Details

Our baseline model is intended to provide intuition in the simplest setting possible. Accordingly, it relies on many simplifying assumptions to streamline the exposition: there are only two distressed debt holders, the private type θ_i takes one of two values, both investors are allowed to inject liquidity even if one has a higher funding cost, and equity holders are restricted to make take-it-or-leave-it offers.

In this section, we show that our main result is robust to relaxing all of these assumptions. We consider a general setting with N distressed investors. Each investor has a type $\theta_i \in [0, \overline{\theta}]$ drawn independently from a cumulative distribution function F with probability density function f. The only restrictions we make on this distribution are that it has full support and satisfies a monotone likelihood ratio property: $F(x)/f(x)$ increases monotonically in x. This property is satisfied by many common distributions (e.g., uniform). Further, we allow equity holders to design an arbitrary optimal mechanism to raise liquidity. By the revelation principle, it is without loss of generality to focus on mechanisms that elicit the truthful revelation of each investor's type. Once equity holders incentivize investors to truthfully reveal their types, equity holders are free to raise liquidity from any arbitrary combination of investors, including from only the investor with the lowest funding cost.

We assume that renegotiation is efficient in the following sense:

$$(1 - p_2)(C^H - C^L) > I_1(1 + \overline{\theta}). \tag{F.1}$$

Given this assumption, once equity holders incentivize investors to truthfully report their private cost θ_i, raising liquidity from the lowest cost investor always increases ex-post joint surplus. In spite of this, in equilibrium, equity holders optimally choose a mechanism that sometimes fails to raise liquidity. Intuitively, a mechanism that always raises liquidity is too "expensive" for privately-optimizing equity holders, because they must pay investors a great deal to truthfully reveal their private cost. Equity holders do not internalize that this cost of incentivizing the revelation of information is simply a transfer. Equity holders instead find it privately optimal to reduce costs by committing to fail if investor funding costs are too high, which reduces joint surplus. This is a core insight from the mechanism design literature, which was applied in the context of debt renegotiations in Detragiache and Garella (1996). Differing from earlier work, we now study how LMEs interact with this inefficiency.

As in our baseline model, we compare two renegotiation approaches. In a traditional renegotiation, no debt holder can be made worse off by a mechanism. In particular, while all investors optimally choose to participate in the mechanism on the equilibrium path, we assume that an off-equilibrium path deviation to not participate in the mechanism would not harm an investor: the investor retains their right to collateral in bankruptcy and their right to the initially promised payment K_0^* if the firm avoids bankruptcy. In contrast, in an LME, the firm can take away collateral rights from any investor who does not participate in the mechanism.[50]

We replicate the main insight of our baseline model.

Proposition 8. *Assume that (F.1) holds. Fix a time-0 debt payment $K_0^* \in (C^L, C^H)$. At date 1, equity holders maximize their ex-post payoff by optimally designing a mechanism.*

1. *If equity holders design a "traditional renegotiation" mechanism that cannot make an investor worse off for refusing to participate, the ex-post optimal mechanism raises liquidity if and only if*

$$(1-p_2)\underbrace{(C^H - K_0^*)}_{\text{debt overhang}} > I_1 \min_i \left(1 + \theta_i + \underbrace{\frac{F(\theta_i)}{f(\theta_i)}}_{\text{Incentive to reveal information}}\right). \tag{F.2}$$

2. *If equity holders design a "LME" mechanism that can take collateral from an investor who does not participate, the ex-post optimal mechanism raises liquidity if and only if*

$$(1-p_2)\underbrace{(C^H - K_0^*)}_{\text{debt overhang}} > I_1 \min_i \left(1 + \theta_i + \frac{F(\theta_i)}{f(\theta_i)}\right) - \underbrace{p_2 C^L}_{\text{LME efficiency}}. \tag{F.3}$$

Proposition 8 formalizes the above intuition. Given equation (F.1), surplus is always maximized by renegotiation. However, equation (F.2) shows that equity holders optimally choose a mechanism

[50] For tractability, we must assume that at most $N-1$ investors participate in the mechanism. Thus, if all investors choose to participate as they do in equilibrium, one is randomly excluded. If, off the equilibrium path, an investor refuses to participate, the other $N-1$ investors proceed. This assumption allows for a clean calculation of how a mechanism determines the off-equilibrium path payoff that results from refusal to participate. This assumption is analogous to assuming that one investor misses the announcement that an LME or renegotiation is occurring.

that can fail to restructure in equilibrium. The failure has two sources. First, the quantity on the left side is the private improvement $(1-p_2)(C^H - K_0^*)$ for equity holders from a successful renegotiation, rather than the total improvement $(1-p_2)(C^H - C^L)$ in firm value. This is the debt overhang effect. Second, the quantity on the right side is the cost of compensating a lender for both their funding cost and an incentive to reveal their private information (an "information rent"). Even when $(1-p_2)(C^H - C^L) > I_1 \min_i(1+\theta_i)$, so the social benefit is higher than the social cost, it could be that $(1-p_2)(C^H - C^L) < I_1 \min_i(1+\theta_i + F(\theta_i)/f(\theta_i))$, where the right side includes the transfer that equity holders pay to investors to reveal their private information. In this sense, equity holders optimally choose not to restructure when it would be too privately costly to induce information revelation.

We provide further details about this mechanism design setup and the proof of Proposition 8 below.

F.1 Details

We now provide further details about our mechanism design setup. We focus on date 1 and assume that the firm has already issued debt K_0^*.

The firm is in distress. Differing from our baseline model, we assume the debt is sold to N investors. Each investor i independently draws a funding cost θ_i from a distribution F_i with pdf f_i. We assume that each f_i has full support on the interval $[0, \overline{\theta}]$ and satisfies a monotone likelihood ratio property: $F_i(\theta_i)/f_i(\theta_i)$ increases monotonically in θ_i.

Equity holders design a mechanism. By the revelation principle, it is without loss of generality to focus on direct mechanisms in which investors send a report $\widehat{\theta}_i$ of their type θ_i.

For tractability, we assume that n investors participate in the mechanism, where $n \leq N-1$ is fixed. If all investors choose to participate as they do in equilibrium, $N-n$ are randomly excluded. This assumption allows for a clean calculation of how a mechanism determines the off-equilibrium path payoff that results from refusal to participate. In the above exposition, we have $n = N-1$, but the result holds more generally for $n \leq N-1$, as we show below.

We assume each investor has the same distribution F_i and we let $\boldsymbol{F}(\boldsymbol{\theta})$, $\boldsymbol{f}(\boldsymbol{\theta})$ denote the CDF and pdf, respectively, of the $n \times 1$ vector $\boldsymbol{\theta} = (\theta_1, ..., \theta_n)$.

A mechanism is a correspondence that maps the n reports $\widehat{\boldsymbol{\theta}} = (\widehat{\theta}_1, ..., \widehat{\theta}_n)$ to three objects: (1) a probability $A(\widehat{\boldsymbol{\theta}})$ that the firm raises liquidity, (2) a payment $K_1^i(\widehat{\boldsymbol{\theta}})$ for each lender $i = 1, ..., n$ who participates if the restructuring succeeds and the firm avoids bankruptcy, and (3) an amount $I_1^i(\widehat{\boldsymbol{\theta}})$ that investor i injects, where $\sum_{i=1}^n I_1^i(\widehat{\boldsymbol{\theta}}) = I_1$.

By the revelation principle, it is without loss of generality to focus on truthtelling equilibria: equity holders optimally choose a mechanism subject to the constraint that there is an equilibrium in which each investor finds it optimal to report their true type θ_i.

We also focus on mechanisms and equilibria in which each investor finds it optimal to participate. If, off the equilibrium path, an investor refuses to participate, they receive a payoff X that does not depend on their type or their report because they inject no liquidity, making their funding cost θ_i irrelevant. Likewise, if all investors participate, $N - n$ investors are randomly excluded, with each excluded investor receiving X.

In a traditional restructuring,

$$X = \frac{C^L}{N} + \int_{J_n} A(\boldsymbol{\theta})(1 - p_2)\frac{K_0^* - C^L}{N}\boldsymbol{f}(\boldsymbol{\theta})d\boldsymbol{\theta}, \tag{F.4}$$

where $J_n = \underbrace{[0, \overline{\theta}] \times [0, \overline{\theta}] \cdots \times [0, \overline{\theta}]}_{n}$ and $d\boldsymbol{\theta} = d\theta_1 d\theta_2 \cdots d\theta_n$. The investor gets their bankruptcy payoff C^L/N if the firm files for bankruptcy or the originally promised payment K_0^*/N if the firm avoids bankruptcy. Note that the term $A(\boldsymbol{\theta})$ in the integral reflects the fact that the participating n investors follow their equilibrium strategies and report truthfully.

In an LME,

$$X = \frac{C^L}{N} + \int_{J_n} A(\boldsymbol{\theta})\left[(1 - p_2)\frac{K_0^* - C^L}{N} - p_2\frac{C^L}{N}\right]\boldsymbol{f}(\boldsymbol{\theta})d\boldsymbol{\theta}. \tag{F.5}$$

The term $-p_2C^L/N$ reflects the investor's loss of collateral if the restructuring succeeds, which occurs with probability $A(\boldsymbol{\theta})$.

In either type of restructuring mechanism, if an investor i with type θ_i chooses to participate in the mechanism and reports $\widehat{\theta}_i$, and if all other investors follow their equilibrium strategies of participation and truthtelling, then investor i receives the following payoff in expectation:

$$U(\theta_i,\widehat{\theta}_i) \equiv \frac{X(N-n)}{N} + \frac{n}{N}\left[\frac{C^L}{N} + \int_{J_{n-1}} A\left(\{\widehat{\theta}_i,\boldsymbol{\theta}_{-i}\}\right)\left[(1-p_2)\left(K_1^i\left(\{\widehat{\theta}_i,\boldsymbol{\theta}_{-i}\}\right) + \frac{K_0^*-C^L}{N}\right)\right.\right.$$
$$\left.\left. + Y - I_1^i\left(\{\widehat{\theta}_i,\boldsymbol{\theta}_{-i}\}\right)(1+\theta_i)\right]\boldsymbol{f_{-i}}(\boldsymbol{\theta}_{-i})d\boldsymbol{\theta}_{-i}\right], \tag{F.6}$$

where $Y=0$ in a traditional restructuring and $Y = p_2 C^L\left(\frac{1}{n}-\frac{1}{N}\right)$ in an LME, $\boldsymbol{\theta}_{-i}$ is the vector obtained by removing θ_i from the vector $\boldsymbol{\theta} = (\theta_1,...,\theta_n)$, and $\boldsymbol{f_{-i}}(\boldsymbol{\theta}_{-i})$ is the probability density of $\boldsymbol{\theta}_{-i}$. In an LME, Y represents the n participating investors sharing the benefit of priming $N-n$ non-participating investors. The first term in (F.6) reflects the probability $(N-n)/N$ that the investor is excluded and receives X. The rest of the terms in (F.6) reflect the probability n/N that the investor is included. The integral takes an expectation over the vector $\boldsymbol{\theta}_{-i}$ of the $n-1$ investors who participate in the mechanism (they are not excluded) and are not investor i. Investor i receives $K_1^i + K_0^*/N$ if the firm avoids bankruptcy, receives C^L/N in a traditional restructuring that is followed by bankruptcy, and C^L/n in an LME that is followed by bankruptcy. In addition, investor i pays $I_1^i(1+\theta_i)$ if the restructuring succeeds.

We can write the truthtelling constraint as follows:

$$\theta_i \in \operatorname{argmax}_{\widehat{\theta}_i} U(\theta_i,\widehat{\theta}_i). \tag{F.7}$$

We can write the participation constraint as

$$U(\theta_i,\theta_i) \geq X \quad \text{for all } \theta_i. \tag{F.8}$$

If both hold, then it is an equilibrium for each investor to participate and truthfully report. In such an equilibrium, the ex-post payoff for equity holders is:

$$V \equiv \int_{J_n} A(\boldsymbol{\theta})(1-p_2)\left(C^H - K_0^* - \sum_{i=1}^{n} K_1^i(\boldsymbol{\theta})\right)\boldsymbol{f}(\boldsymbol{\theta})d\boldsymbol{\theta}. \tag{F.9}$$

With this setup, we prove Proposition 8.

F.2 Proof of Proposition 8

Fix any mechanism with a truthtelling equilibrium. Let U_1, U_2 denote the partial derivatives of U with respect to the first and second arguments. Equation (F.7) holds if and only if

$$U_2(\theta_i, \theta_i) = 0. \tag{F.10}$$

Define $\widehat{U}(\theta_i) \equiv U(\theta_i, \theta_i)$. Then the above equation holds if and only if

$$\frac{d}{d\theta_i}\widehat{U}(\theta_i) = \frac{d}{d\theta_i}U(\theta_i, \theta_i) = U_1(\theta_i, \theta_i) + U_2(\theta_i, \theta_i) = U_1(\theta_i, \theta_i). \tag{F.11}$$

This is the Envelope theorem. From (F.6),

$$\begin{aligned}
U_1(\theta_i, \widehat{\theta}_i) &= \frac{\partial}{\partial\theta_i}U(\theta_i, \widehat{\theta}_i) \\
&= \frac{\partial}{\partial\theta_i}\frac{n}{N}\int_{J_{n-1}} A\left(\{\widehat{\theta}_i, \boldsymbol{\theta}_{-i}\}\right)\Bigg[(1-p_2)\left(K_1^i\left(\{\widehat{\theta}_i, \boldsymbol{\theta}_{-i}\}\right) + \frac{K_0^* - C^L}{N}\right) \\
&\quad + Y - I_1^i\left(\{\widehat{\theta}_i, \boldsymbol{\theta}_{-i}\}\right)(1+\theta_i)\Bigg]\boldsymbol{f_{-i}}(\boldsymbol{\theta}_{-i})d\boldsymbol{\theta}_{-i} \\
&= -\frac{n}{N}\int_{J_{n-1}} A\left(\{\widehat{\theta}_i, \boldsymbol{\theta}_{-i}\}\right) I_1^i\left(\{\widehat{\theta}_i, \boldsymbol{\theta}_{-i}\}\right)\boldsymbol{f_{-i}}(\boldsymbol{\theta}_{-i})d\boldsymbol{\theta}_{-i}.
\end{aligned} \tag{F.12}$$

Thus,

$$\frac{d}{d\theta_i}\widehat{U}(\theta_i) = -\frac{n}{N}\int_{J_{n-1}} A\left(\{\theta_i, \boldsymbol{\theta}_{-i}\}\right) I_1^i\left(\{\theta_i, \boldsymbol{\theta}_{-i}\}\right)\boldsymbol{f_{-i}}(\boldsymbol{\theta}_{-i})d\boldsymbol{\theta}_{-i}. \tag{F.13}$$

This shows that $\frac{d}{d\theta_i}U(\theta_i, \theta_i) \leq 0$.

Applying the fundamental theorem of calculus:

$$\begin{aligned}
\widehat{U}(\theta_i) &= \widehat{U}(\overline{\theta}) - \int_{\theta_i}^{\overline{\theta}} \frac{d}{du}\widehat{U}(u)du \\
&= \widehat{U}(\overline{\theta}) - \frac{n}{N}\int_{\theta_i}^{\overline{\theta}}\left[-\int_{J_{n-1}} A\left(\{u, \boldsymbol{\theta}_{-i}\}\right) I_1^i\left(\{u, \boldsymbol{\theta}_{-i}\}\right)\boldsymbol{f_{-i}}(\boldsymbol{\theta}_{-i})d\boldsymbol{\theta}_{-i}\right]du \\
&= \widehat{U}(\overline{\theta}) + \frac{n}{N}\int_{J_{n-1}}\left[\int_{\theta_i}^{\overline{\theta}} A\left(\{u, \boldsymbol{\theta}_{-i}\}\right) I_1^i\left(\{u, \boldsymbol{\theta}_{-i}\}\right)du\right]\boldsymbol{f_{-i}}(\boldsymbol{\theta}_{-i})d\boldsymbol{\theta}_{-i}.
\end{aligned}$$

Equating this with the definition implied by (F.6) with $\widehat{\theta}_i = \theta_i$,

$$
\begin{aligned}
&\widehat{U}(\overline{\theta}) + \frac{n}{N} \int_{J_{n-1}} \left[\int_{\theta_i}^{\overline{\theta}} A\left(\{u, \boldsymbol{\theta}_{-i}\} \right) I_1^i\left(\{u, \boldsymbol{\theta}_{-i}\} \right) du \right] \boldsymbol{f_{-i}}(\boldsymbol{\theta}_{-i}) d\boldsymbol{\theta}_{-i} \\
=& \frac{X(N-n)}{N} + \frac{n}{N} \left[\frac{C^L}{N} + \int_{J_{n-1}} A\left(\{\theta_i, \boldsymbol{\theta}_{-i}\} \right) \left[(1-p_2) \left(K_1^i\left(\{\theta_i, \boldsymbol{\theta}_{-i}\} \right) + \frac{K_0^* - C^L}{N} \right) \right.\right. \\
&\left.\left. + Y - I_1^i\left(\{\theta_i, \boldsymbol{\theta}_{-i}\} \right) (1+\theta_i) \right] \boldsymbol{f_{-i}}(\boldsymbol{\theta}_{-i}) d\boldsymbol{\theta}_{-i} \right]. \qquad \text{(F.14)}
\end{aligned}
$$

We now take an expectation of both sides of (F.14) with respect to θ_i: we integrate both sides of this equation over all possible values of θ_i, weighting by the pdf $f_i(\theta_i)$. The right side of the equation simply becomes

$$
\frac{X(N-n)}{N} + \frac{n}{N} \left[\frac{C^L}{N} + \int_{J_n} A(\boldsymbol{\theta}) \left[(1-p_2) \left(K_1^i(\boldsymbol{\theta}) + \frac{K_0^* - C^L}{N} \right) + Y - I_1^i(\boldsymbol{\theta})(1+\theta_i) \right] \boldsymbol{f}(\boldsymbol{\theta}) d\boldsymbol{\theta} \right]. \qquad \text{(F.15)}
$$

For the left side of (F.14), define

$$
\widetilde{A}(\{u, \boldsymbol{\theta}_{-i}\}) = \frac{n}{N} A\left(\{u, \boldsymbol{\theta}_{-i}\} \right) I_1^i\left(\{u, \boldsymbol{\theta}_{-i}\} \right). \qquad \text{(F.16)}
$$

Note that $f_i(\theta_i) = F_i'(\theta_i)$ and apply integration by parts:

$$
\begin{aligned}
&\int_0^{\overline{\theta}} \left[\int_{\theta_i}^{\overline{\theta}} \widetilde{A}\left(\{u, \boldsymbol{\theta}_{-i}\} \right) du \right] f_i(\theta_i) d\theta_i \\
=& \left(\int_{\theta_i}^{\overline{\theta}} \widetilde{A}\left(\{u, \boldsymbol{\theta}_{-i}\} \right) du \right) F_i(\theta_i) \Big|_{\theta_i=0}^{\theta_i=\overline{\theta}} - \int_0^{\overline{\theta}} \left(-\widetilde{A}\left(\{\theta_i, \boldsymbol{\theta}_{-i}\} \right) \right) F_i(\theta_i) d\theta_i. \qquad \text{(F.17)}
\end{aligned}
$$

The second term in the second line follows from Leibniz rule. The first term on the second line is zero because $F_i(0) = 0$ and $\int_{\overline{\theta}}^{\overline{\theta}} \widetilde{A}\left(\{u, \boldsymbol{\theta}_{-i}\} \right) du = 0$. Thus,

$$
\begin{aligned}
&\int_0^{\overline{\theta}} \left[\int_{\theta_i}^{\overline{\theta}} \widetilde{A}(\{u, \boldsymbol{\theta}_{-i}\}) du \right] f_i(\theta_i) d\theta_i = \int_0^{\overline{\theta}} \widetilde{A}(\{\theta_i, \boldsymbol{\theta}_{-i}\}) F_i(\theta_i) d\theta_i \\
=& \int_0^{\overline{\theta}} \widetilde{A}(\{\theta_i, \boldsymbol{\theta}_{-i}\}) \frac{F_i(\theta_i)}{f_i(\theta_i)} f_i(\theta_i) d\theta_i. \qquad \text{(F.18)}
\end{aligned}
$$

The expectation of the left side of (F.14) over θ_i is thus

$$
\begin{aligned}
&\widehat{U}(\overline{\theta}) + \int_{J_{n-1}} \int_0^{\overline{\theta}} \widetilde{A}(\{\theta_i, \boldsymbol{\theta}_{-i}\}) \frac{F_i(\theta_i)}{f_i(\theta_i)} f_i(\theta_i) d\theta_i \boldsymbol{f_{-i}}(\boldsymbol{\theta}_{-i}) d\boldsymbol{\theta}_{-i} \\
=&\widehat{U}(\overline{\theta}) + \frac{n}{N} \int_{J_n} A(\boldsymbol{\theta}) I_1^i(\boldsymbol{\theta}) \frac{F_i(\theta_i)}{f_i(\theta_i)} \boldsymbol{f}(\boldsymbol{\theta}) d\boldsymbol{\theta}.
\end{aligned}
\tag{F.19}
$$

Using (F.14), (F.15) and (F.19), we obtain

$$
\begin{aligned}
&\widehat{U}(\overline{\theta}) + \frac{n}{N} \int_{J_n} A(\boldsymbol{\theta}) I_1^i(\boldsymbol{\theta}) \frac{F_i(\theta_i)}{f_i(\theta_i)} \boldsymbol{f}(\boldsymbol{\theta}) d\boldsymbol{\theta} = \frac{(N-n)X}{N} \\
&+ \frac{n}{N} \left[\frac{C^L}{N} + \int_{J_n} A(\boldsymbol{\theta}) \Big[(1-p_2) \left(K_1^i(\boldsymbol{\theta}) + \frac{K_0^* - C^L}{N} \right) + Y - I_1^i(\boldsymbol{\theta})(1+\theta_i) \Big] \boldsymbol{f}(\boldsymbol{\theta}) d\boldsymbol{\theta} \right].
\end{aligned}
\tag{F.20}
$$

Rearranging, multiplying through by N/n, and summing over $i = 1$ to n,

$$
\begin{aligned}
&N\widehat{U}(\overline{\theta}) - (N-n)X - \frac{n}{N} C^L \\
&+ \int_{J_n} A(\boldsymbol{\theta}) \left((1-p_2) \frac{n}{N} C^L - nY + \sum_{i=1}^n I_1^i(\boldsymbol{\theta}) \Big(1 + \theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)} \Big) \right) \boldsymbol{f}(\boldsymbol{\theta}) d\boldsymbol{\theta} \\
=& \int_{J_n} A(\boldsymbol{\theta})(1-p_2) \left(\frac{n}{N} K_0^* + \sum_{i=1}^n K_1^i(\boldsymbol{\theta}) \right) \boldsymbol{f}(\boldsymbol{\theta}) d\boldsymbol{\theta}.
\end{aligned}
\tag{F.21}
$$

We begin with the traditional restructuring, where X is given by (F.4) and $Y = 0$. It must be that $\widehat{U}(\overline{\theta}) = X$. Thus,

$$
N\widehat{U}(\overline{\theta}) - (N-n)X = nX = \frac{n}{N} \left(C^L + \int_{J_n} A(\boldsymbol{\theta})(1-p_2)(K_0^* - C^L) \boldsymbol{f}(\boldsymbol{\theta}) d\boldsymbol{\theta} \right). \tag{F.22}
$$

Plugging this into (F.21) and rearranging terms,

$$
\begin{aligned}
&\int_{J_n} A\,(\,\theta\,) \left(\sum_{i=1}^n I_1^i(\boldsymbol{\theta}) \Big(1 + \theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)} \Big) \right) \boldsymbol{f}(\boldsymbol{\theta}) d\boldsymbol{\theta} \\
=& \int_{J_n} A(\boldsymbol{\theta})(1-p_2) \left(\sum_{i=1}^n K_1^i(\boldsymbol{\theta}) \right) \boldsymbol{f}(\boldsymbol{\theta}) d\boldsymbol{\theta}.
\end{aligned}
\tag{F.23}
$$

Plugging this into the definition of V given in (F.9),

$$\begin{aligned} V &= \int_{J_n} A(\boldsymbol{\theta})(1-p_2)\left(C^H - K_0^* - \sum_{i=1}^n K_1^i(\boldsymbol{\theta}) \right) \boldsymbol{f}(\boldsymbol{\theta})d\boldsymbol{\theta} \\ &= \int_{J_n} A(\boldsymbol{\theta})\Big[(1-p_2)\left(C^H - K_0^* \right) - \sum_{i=1}^n I_1^i(\boldsymbol{\theta})\Big(1+\theta_i+\frac{F_i(\theta_i)}{f_i(\theta_i)}\Big)\Big] \boldsymbol{f}(\boldsymbol{\theta})d\boldsymbol{\theta}. \end{aligned} \tag{F.24}$$

Thus, we have shown that a mechanism has a truthtelling equilibrium if and only if this is the equity objective in that equilibrium. Thus, equity can freely manipulate $A(\boldsymbol{\theta})$, $I_1^i(\boldsymbol{\theta})$, $i = 1, 2 \cdots, n$ to maximize this objective, and then set the transfers K_1^i that ensure truthtelling and participation. Since $F_i(\theta_i)/f_i(\theta_i)$ increases in θ_i, it is clear that V is maximized by setting $I_1^i = I_1$ for the lowest θ_i and 0 for other i and setting

$$A(\boldsymbol{\theta}) = \mathbf{1}\left((1-p_2)(C^H - K_0^*) - I_1 \min_{1\le i\le n}\left(1+\theta_i+\frac{F_i(\theta_i)}{f_i(\theta_i)} \right) > 0 \right). \tag{F.25}$$

Hence, we have

$$V = \int_{J_n} \left[(1-p_2)(C^H - K_0^*) - I_1 \min_{1\le i\le n}\left(1+\theta_i+\frac{F_i(\theta_i)}{f_i(\theta_i)} \right)\right]^+ \boldsymbol{f}(\boldsymbol{\theta})d\boldsymbol{\theta}. \tag{F.26}$$

Since $F_i(\theta_i) = F(\theta_i)$ and $f_i(\theta_i) = f(\theta_i)$, we prove part 1 of Proposition 8.

Next, we prove part 2 of Proposition 8. Recall that in an LME, X is given by (F.5) and $Y = p_2 C^L\big(\frac{1}{n} - \frac{1}{N}\big)$. As before, it must be that $\widehat{U}(\overline{\theta}) = X$. Thus,

$$N\widehat{U}(\overline{\theta}) - (N-n)X = nX = \frac{n}{N}\left(C^L + \int_{J_n} A(\boldsymbol{\theta})\Big[(1-p_2)(K_0^* - C^L) - p_2 C^L\Big]\boldsymbol{f}(\boldsymbol{\theta})d\boldsymbol{\theta} \right). \tag{F.27}$$

Plugging into (F.21),

$$\begin{aligned} &\int_{J_n} A(\boldsymbol{\theta})\left(-p_2\frac{n}{N}C^L - nY + \sum_{i=1}^n I_1^i(\boldsymbol{\theta})\Big(1+\theta_i+\frac{F_i(\theta_i)}{f_i(\theta_i)}\Big) \right) \boldsymbol{f}(\boldsymbol{\theta})d\boldsymbol{\theta} \\ &= \int_{J_n} A(\boldsymbol{\theta})(1-p_2)\left(\sum_{i=1}^n K_1^i(\boldsymbol{\theta}) \right) \boldsymbol{f}(\boldsymbol{\theta})d\boldsymbol{\theta}. \end{aligned} \tag{F.28}$$

Plugging this into the definition of V given in (F.9), and using $p_2 \frac{n}{N} C^L + nY = p_2 C^L$, we obtain

$$
\begin{aligned}
V &= \int_{J_n} A(\boldsymbol{\theta})(1-p_2)\left(C^H - K_0^* - \sum_{i=1}^{n} K_1^i(\boldsymbol{\theta}) \right) \boldsymbol{f}(\boldsymbol{\theta}) d\boldsymbol{\theta} \\
&= \int_{J_n} A(\boldsymbol{\theta}) \left[(1-p_2)\left(C^H - K_0^* \right) + p_2 C^L - \sum_{i=1}^{n} I_1^i(\boldsymbol{\theta}) \Big(1 + \theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)} \Big) \right] \boldsymbol{f}(\boldsymbol{\theta}) d\boldsymbol{\theta}. \qquad \text{(F.29)}
\end{aligned}
$$

It is clear that this is maximized by setting $I_1^i = I_1$ for the lowest θ_i and

$$
A(\boldsymbol{\theta}) = \mathbf{1}\left((1-p_2)(C^H - K_0^*) + p_2 C^L - I_1 \sum_{i=1}^{n} \left(1 + \theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)} \right) > 0 \right). \qquad \text{(F.30)}
$$

Then we have

$$
V = \int_{J_n} \left[(1-p_2)(C^H - K_0^*) + p_2 C^L - I_1 \min_{1 \le i \le n} \left(1 + \theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)} \right) \right]^+ \boldsymbol{f}(\boldsymbol{\theta}) d\boldsymbol{\theta}. \qquad \text{(F.31)}
$$

Since $F_i(\theta_i) = F(\theta_i)$ and $f_i(\theta_i) = f(\theta_i)$, we prove part 2 of Proposition 8.

Thus, if restructuring is efficient, LMEs increase value.

G Differences in Beliefs

In this extension, we show that our results hold when we assume all required returns are zero ($\theta_i = 0$) and instead assume a different friction: differences in beliefs and asymmetric information about the probability p_2 of bankruptcy after a liquidity injection.

At $t = 0$, the firm and the bank agree on the value of p_2. At $t = 1$, after buying the distressed debt, each hedge fund receives new information and forms an updated belief p_2^i about the probability of bankruptcy. We assume that each investor i is certain that their estimate p_2^i is correct: the investor does not update based on the actions of other agents. This is often called a "differences in beliefs" or "agree to disagree" framework. We assume $p_2^i \in \{p_2^L, p_2^H\}$, where $0 < p_2^L < p_2^H < 1$. We let q denote the probability that $p_2^i = p_2^H$. We assume that at $t = 0$, the firm and the bank share a belief that $p_2 = \overline{p}_2$, where $\overline{p}_2$ is the expected belief of hedge funds at $t = 1$:

$$\overline{p}_2 = q p_2^H + (1 - q) p_2^L. \tag{G.1}$$

Moreover, the firm believes $p_2 = \overline{p}_2$ at $t = 1$.

As in the baseline model, we always consider (K_0, K_1) such that $C^H \geq K_0$ and $C^H \geq K_0 + K_1$. Similar to Condition 1 in our baseline model, we impose the following condition, implying that the firm is insolvent if it uses debt financing to pay the initial expense I_0:

Condition 7. $\overline{p}_2 C^L + (1 - \overline{p}_2) C^H < I_0 + I_1$.

In distress, the bank expects to sell half the debt to an investor who will pay the expected value of their post-signal valuation

$$\begin{aligned}
&(1 - A(K_1))\frac{\min\{C^L, K_0\}}{2} + A(K_1)\frac{q\ \big(\ p_2^H \min\{C^L, K_0 + K_1\} + (1 - p_2^H)(K_0 + K_1)\ \big)}{2} \\
&+ A(K_1)\frac{(1 - q)\ \big(\ p_2^L \min\{C^L, K_0 + K_1\} + (1 - p_2^L)(K_0 + K_1)\ \big) - I_1}{2} \\
=&(1 - A(K_1))\frac{\min\{C^L, K_0\}}{2} + A(K_1)\frac{\overline{p}_2 \min\{C^L, K_0 + K_1\} + (1 - \overline{p}_2)(K_0 + K_1) - I_1}{2}.
\end{aligned}$$

Summing across investors, at $t = 0$ the debt pricing equation is

$$\begin{aligned} I_0 =& (1-p_1)K_0 \\ &+ p_1\Big[(1-A(K_1))\min\{C^L, K_0\} + A(K_1)\Big(\overline{p}_2 \min\{C^L, K_0+K_1\} + (1-\overline{p}_2)(K_0+K_1) - I_1\Big)\Big]. \end{aligned} \tag{G.2}$$

Ex-ante firm value is given as follows:

$$\begin{aligned} E_0 =& (1-p_1)(C^H - K_0) + p_1[1-A(K_1)](C^L - K_0)^+ \\ &+ p_1 A(K_1)\Big[\overline{p}_2(C^L - K_0 - K_1)^+ + (1-\overline{p}_2)(C^H - K_0 - K_1)\Big]. \end{aligned} \tag{G.3}$$

Combining (G.2) and (G.3), we obtain

$$E_0 = (1-p_1)C^H + p_1 C^L - I_0 + p_1 A(K_1)\left[\, (1-\overline{p}_2)(C^H - C^L) - I_1 \,\right]. \tag{G.4}$$

G.1 Efficiency and the limited liability condition

If the firm never raises liquidity at $t = 1$, the ex-ante firm value at $t = 0$ is

$$(1-p_1)C^H + p_1 C^L - I_0. \tag{G.5}$$

Next, suppose that the firm raises liquidity only if at least one hedge fund is optimistic, i.e. $\mathbf{1}_{\text{accept}}(K_1, p_2^i) = 1$ for $p_2^i = p_2^L$ but $\mathbf{1}_{\text{accept}}(K_1, p_2^i) = 0$ for $p_2^i = p_2^H$. Then at $t = 0$, the ex-ante firm value is

$$(1-p_1)C^H + p_1 C^L - I_0 + p_1(1-q^2)\left[\, (1-\overline{p}_2)(C^H - C^L) - I_1 \,\right]. \tag{G.6}$$

Firm value with certain liquidity injection is

$$(1-p_1)C^H + p_1 C^L - I_0 + p_1\left[\, (1-\overline{p}_2)(C^H - C^L) - I_1 \,\right]. \tag{G.7}$$

Thus, liquidity injection is efficient if

$$(1-\overline{p}_2)(C^H - C^L) > I_1. \tag{G.8}$$

Similar to Condition 2 in our baseline model, we assume a condition implying that liquidity injection satisfies limited liability.

Condition 8. $(1-\overline{p}_2)(C^H - C^L) > I_1\left(1 + \frac{1-p_1}{1-p_1\overline{p}_2}\frac{p_2^H-\overline{p}_2}{1-p_2^H}\right) + (I_0 - C^L)\frac{1-\overline{p}_2}{1-p_1\overline{p}_2}$.

We can see that Condition 8 implies that (G.8) is satisfied.

Lemma 6. *Suppose that Condition 7 and the break-even condition (G.2) holds. Then (1) $C^H \geq K_0 + K_1$ implies that $K_0 > C^L$, and (2) Condition 8 implies that $C^H > K_0 + K_1$ when $A(K_1) = 1$ and $K_0 \geq K_0^{\text{Trad}}$, where*

$$K_0^{\text{Trad}} = \frac{I_0 - p_1\left(\overline{p}_2 C^L + \frac{p_2^H-\overline{p}_2}{1-p_2^H} I_1\right)}{1-p_1\overline{p}_2}. \tag{G.9}$$

G.2 Equilibria in the absence and presence of LMEs

Proposition 9. *Assume that Conditions 7-8 hold. In the absence of LMEs, an efficient equilibrium exists if and only if $C^H - C^L \geq \mathbb{B}_{\text{belief}}^{\text{Trad}}$, where $\mathbb{B}_{\text{belief}}^{\text{Trad}}$ is given by*

$$\mathbb{B}_{\text{belief}}^{\text{Trad}} = \frac{I_1}{q^2}\left(\frac{1}{1-p_2^H} - \frac{1-q^2}{1-p_2^L}\right) + \frac{1}{q}\left(\frac{I_0 - C^L - p_1\frac{p_2^H-\overline{p}_2}{1-p_2^H}I_1}{1-p_1\overline{p}_2}\right). \tag{G.10}$$

Moreover, $(1-\overline{p}_2)\mathbb{B}_{\text{belief}}^{\text{Trad}}$ is strictly larger than the right-hand side of the inequality in Condition 8.

Thus, an efficient equilibrium exists if and only if I_0 is low.

Proposition 10. *Assume that Conditions 7-8 hold. Assume also that the pessimistic investor is more willing to accept offers:*

$$\frac{I_1 - p_2^H C^L}{1-p_2^H} > \frac{I_1 - p_2^L C^L}{1-p_2^L}. \tag{G.11}$$

Then an efficient equilibrium with LME renegotiations exists if and only if $C^H - C^L \geq \mathbb{B}_{\text{belief}}^{\text{LME}}$, *where* $\mathbb{B}^{\text{LME}}_{\text{belief}}$ *is given by*

$$\mathbb{B}^{\text{LME}}_{\text{belief}} = \mathbb{B}^{\text{Trad}}_{\text{belief}} + \underbrace{\frac{C^L}{q^2}\left[\frac{p_2^H}{1-p_2^H}\left(\frac{qp_1(1-\overline{p}_2)}{1-p_1\overline{p}_2}-1\right)+\frac{(1-q)p_2^L}{1-p_2^L}\right]}_{<0}. \tag{G.12}$$

Combining Proposition 9 and Proposition 10, we see that LMEs improve ex-ante firm value under Conditions 7-8 and the following condition:

$$\mathbb{B}^{\text{LME}}_{\text{belief}} < C^H - C^L < \mathbb{B}^{\text{Trad}}_{\text{belief}}. \tag{G.13}$$

G.3 Proofs

G.3.1 Proof of Lemma 6

It follows from (G.2) that

$$\begin{aligned}
I_0 &\leq (1-p_1)K_0 + p_1\Big[(1-A(K_1))C^L + A(K_1)\Big(\overline{p}_2 C^L + (1-\overline{p}_2)(K_0+K_1) - I_1\Big)\Big] \\
&\leq (1-p_1)K_0 + p_1\Big[C^L + A(K_1)\ \big(\ (1-\overline{p}_2)(C^H - C^L) - I_1\ \big)\Big] \\
&\leq (1-p_1)K_0 + p_1\Big[C^L + A(K_1)\ \big(\ I_0 - C^L\ \big)\Big] \\
&\leq (1-p_1)K_0 + p_1\Big[C^L + \big(\ I_0 - C^L\ \big)\Big] = (1-p_1)K_0 + p_1 I_0,
\end{aligned} \tag{G.14}$$

where the second inequality uses $K_0 + K_1 \leq C^H$ and the third inequality uses Condition 7. Thus, we have $K_0 \geq I_0 > C^L$ if $C^H \geq K_0 + K_1$.

Next, we prove that Condition 8 implies that $C^H > K_0 + K_1$ when $A(K_1) = 1$ and $K_0 \geq K_0^{\text{Trad}}$. If $K_0 + K_1 \leq C^L$, then we have $C^H > K_0 + K_1$. Thus, we only need to consider the case $K_0 + K_1 > C^L$.

In the proof of Proposition 9, we will show that $A(K_1) = 1$ for any $K_1 \geq K_1^{\text{Trad}} := \frac{I_1}{1-p_2^H}$, and $K_0 = K_0^{\text{Trad}}$, $K_1 = K_1^{\text{Trad}}$ satisfy the break even condition.

Using the break-even condition (G.2), $A(K_1) = 1$ and $K_0 + K_1 > C^L$, we obtain

$$I_0 =(1-p_1)K_0 + p_1\Big[\Big(\overline{p}_2 C^L + (1-\overline{p}_2)(K_0+K_1) - I_1\Big)\Big]. \tag{G.15}$$

Then we see that

$$\begin{aligned} & C^H > K_0 + K_1 \\ \iff \quad & I_0 < (1-p_1)K_0 + p_1 C^L + p_1[(1-\overline{p}_2)(C^H - C^L) - I_1], \end{aligned} \tag{G.16}$$

which is equivalent to

$$(1-\overline{p}_2)(C^H - C^L) > I_1 + \frac{I_0 - [(1-p_1)K_0 + p_1 C^L]}{p_1}. \tag{G.17}$$

Thus, we have shown that (G.17) is equivalent to $C^H > K_1 + K_0$. We see that the right side of (G.17) is decreasing in K_0, so if (G.17) holds for K_0^{Trad} then it holds for $K_0 \geq K_0^{\text{Trad}}$.

Plugging K_0^{Trad} into (G.17), a sufficient condition for $C^H > K_0^{\text{Trad}} + K_1^{\text{Trad}}$ is

$$\begin{aligned} (1-\overline{p}_2)(C^H - C^L) &> I_1 + \frac{I_0 - p_1 C^L}{p_1} - \frac{1-p_1}{p_1}\frac{I_0 - p_1\left(\overline{p}_2 C^L + \frac{p_2^H - \overline{p}_2}{1-p_2^H} I_1\right)}{1-p_1\overline{p}_2} \\ =& I_1 + \frac{1-p_1}{1-p_1\overline{p}_2}\frac{p_2^H - \overline{p}_2}{1-p_2^H} I_1 + \frac{I_0}{p_1}\Big[1 - \frac{1-p_1}{1-p_1\overline{p}_2}\Big] - C^L\Big[1 - \frac{1-p_1}{p_1}\frac{p_1\overline{p}_2}{1-p_1\overline{p}_2}\Big] \\ =& I_1\left(1 + \frac{1-p_1}{1-p_1\overline{p}_2}\frac{p_2^H - \overline{p}_2}{1-p_2^H}\right) + (I_0 - C^L)\frac{1-\overline{p}_2}{1-p_1\overline{p}_2}. \end{aligned} \tag{G.18}$$

The final line is the right side of Condition 8. Thus, under the conditions of the lemma, Condition 8 ensures that $C^H > K_0 + K_1$.

G.3.2 Proof of Proposition 9

Assume LMEs are banned and consider a subgame in which equity offers K_1. Each investor i forms a rational expectation of the probability $\widetilde{P}(K_1)$ that the competing investor accepts. Following the proof of Proposition 1, we only need to consider K_1 which yields the accepting investor's payoff:

$$R^{\text{Trad}}_{\text{accept}}(p_2^i) = \frac{(1-p_2^i)K_0^* + p_2^i C^L}{2} + \Big(1 - \frac{\widetilde{P}(K_1)}{2}\Big)\big[(1-p_2^i)K_1 - I_1\big], \tag{G.19}$$

and rejecting investor's payoff:

$$R^{\text{Trad}}_{\text{reject}}(p_2^i) = \big[1 - \widetilde{P}(K_1) + \widetilde{P}(K_1)p_2^i\big]\frac{C^L}{2} + \widetilde{P}(K_1)\frac{(1-p_2^i)K_0^*}{2}. \tag{G.20}$$

First, we consider K_1 such that $\widetilde{P}(K_1) = 1$. Investor i accepts the offer if and only if $R^{\text{Trad}}_{\text{accept}}(p_2^i) \geq R^{\text{Trad}}_{\text{reject}}(p_2^i)$. Using $\widetilde{P}(K_1) = 1$, (G.19) and (G.20), we can see that the offer is accepted if and only if

$$\begin{aligned}
&\frac{(1-p_2^i)K_0^* + p_2^i C^L}{2} + \Big(1 - \frac{1}{2}\Big)\big[(1-p_2^i)K_1 - I_1\big] \geq p_2^i\frac{C^L}{2} + \frac{(1-p_2^i)K_0^*}{2} \\
\iff \quad & (1-p_2^i)K_1 - I_1 \geq 0 \\
\iff \quad & K_1 \geq \frac{I_1}{1-p_2^i}.
\end{aligned} \tag{G.21}$$

If equity wants to implement $\widetilde{P}(K_1) = 1$, they optimally make this bind at $p_2^i = p_2^H$, implying

$$K_1^* = K_1^{\text{Trad}} := \frac{I_1}{1-p_2^H}. \tag{G.22}$$

Moving back to $t = 0$, the bank rationally expects $A(K_1^*) = 1$. Recall that we assume $C^H \geq K_0^* + K_1^* > C^L$ and $C^H \geq K_0^* > C^L$ and verify these inequalities later. Then (G.2) implies that

$$\begin{aligned}
I_0 &= (1-p_1)K_0^* + p_1\Big[\overline{p}_2 C^L + (1-\overline{p}_2)(K_0^* + K_1^*) - I_1\Big] \\
&= (1-p_1\overline{p}_2)K_0^* + p_1\Big[\overline{p}_2 C^L + \frac{I_1(1-\overline{p}_2)}{1-p_2^H} - I_1\Big].
\end{aligned}$$

Rearranging, we obtain

$$K_0^* = K_0^{\text{Trad}} := \frac{I_0 - p_1\Big[\overline{p}_2 C^L + \frac{p_2^H - \overline{p}_2}{1-p_2^H} I_1\Big]}{1-p_1\overline{p}_2}. \tag{G.23}$$

According to Lemma 6, we have $K_0^{\text{Trad}} + K_1^{\text{Trad}} < C^H$ and $K_0^{\text{Trad}} > C^L$. Since $K_1^{\text{Trad}} > 0$, we have $C^L < K_0^{\text{Trad}} < K_0^{\text{Trad}} + K_1^{\text{Trad}} < C^H$.

Next, we consider the alternative where equity could set the lowest K_1 that ensures $\widetilde{P}(K_1) = 1-q$. Using $\widetilde{P}(K_1) = 1-q$, (G.19) and (G.20), we can see that the offer is accepted if and only if

$$
\begin{aligned}
&\frac{(1-p_2^i)K_0^* + p_2^i C^L}{2} + \Big(1 - \frac{1-q}{2}\Big)\big[(1-p_2^i)K_1 - I_1\big] \geq \big[q + (1-q)p_2^i\big]\frac{C^L}{2} + (1-q)\frac{(1-p_2^i)K_0^*}{2} \\
\iff & q\frac{(1-p_2^i)K_0^* + p_2^i C^L}{2} + \frac{1+q}{2}\big[(1-p_2^i)K_1 - I_1\big] \geq q\frac{C^L}{2} \\
\iff & q\frac{(1-p_2^i)(K_0^* - C^L)}{2} + \frac{(1+q)(1-p_2^i)}{2}\Big[K_1 - \frac{I_1}{1-p_2^i}\Big] \geq 0 \\
\iff & K_1 \geq \frac{I_1}{1-p_2^i} - \frac{q}{1+q}(K_0^* - C^L).
\end{aligned}
$$

If equity wants to achieve $\widetilde{P}(K_1) = 1-q$, they optimally make this bind at p_2^L:

$$
K_1^{\text{Alt}} = \frac{I_1}{1-p_2^L} - \frac{q}{1+q}(K_0^* - C^L). \tag{G.24}
$$

We have $K_0^* + K_1^{\text{Alt}} - C^L = \frac{I_1}{1-p_2^L} + \frac{1}{1+q}(K_0^* - C^L) > 0$ using $K_0^* = K_0^{\text{Trad}} > C^L$.

If $C^H - K_0^{\text{Trad}} - K_1^{\text{Trad}} \geq (1-q^2)\big(C^H - K_0^{\text{Trad}} - K_1^{\text{Alt}}\big)$, then it is not profitable to deviate to K_1^{Alt} when $C^H - K_0^{\text{Trad}} - K_1^{\text{Alt}} \geq 0$, and it is not feasible to deviate to K_1^{Alt} when $C^H - K_0^{\text{Trad}} - K_1^{\text{Alt}} < 0$. If $C^H - K_0^{\text{Trad}} - K_1^{\text{Trad}} < (1-q^2)\big(C^H - K_0^{\text{Trad}} - K_1^{\text{Alt}}\big)$, using $C^H - K_0^{\text{Trad}} - K_1^{\text{Trad}} > 0$, we conclude that $C^H - K_0^{\text{Trad}} - K_1^{\text{Alt}} > 0$, and deviating to K_1^{Alt} is feasible and profitable.

Thus, it is optimal to follow the equilibrium strategy $K_1^* = K_1^{\text{Trad}}$ given in (G.22) rather than deviating to K_1^{Alt} given in (G.24) if and only if

$$
\begin{aligned}
& C^H - K_0^* - K_1^* \geq (1-q^2)(C^H - K_0^* - K_1^{\text{Alt}}) \\
\iff & q^2(C^H - K_0^*) \geq \frac{I_1}{1-p_2^H} - (1-q^2)\left(\frac{I_1}{1-p_2^L} - \frac{q}{1+q}(K_0^* - C^L)\right) \\
\iff & q^2(C^H - K_0^*) \geq \frac{I_1}{1-p_2^H} - \frac{(1-q^2)I_1}{1-p_2^L} + q(1-q)(K_0^* - C^L) \\
\iff & q^2(C^H - C^L) \geq \frac{I_1}{1-p_2^H} - \frac{(1-q^2)I_1}{1-p_2^L} + q(K_0^* - C^L) \\
\iff & C^H - C^L \geq \frac{I_1}{q^2}\left(\frac{1}{1-p_2^H} - \frac{1-q^2}{1-p_2^L}\right) + \frac{1}{q}\left(\frac{I_0 - C^L - p_1\frac{p_2^H - \overline{p}_2}{1-p_2^H}I_1}{1-p_1\overline{p}_2}\right).
\end{aligned}
$$

The right side is $\mathbb{B}^{\text{Trad}}_{\text{belief}}$.

Finally, we show that

$$(1-\overline{p}_2)\mathbb{B}^{\text{Trad}}_{\text{belief}} > I_1\left(1+\frac{1-p_1}{1-p_1\overline{p}_2}\frac{p_2^H-\overline{p}_2}{1-p_2^H}\right)+(I_0-C^L)\frac{1-\overline{p}_2}{1-p_1\overline{p}_2}. \tag{G.25}$$

We have

$$\begin{aligned}
&(1-\overline{p}_2)\mathbb{B}^{\text{Trad}}_{\text{belief}} - I_1\left(1+\frac{1-p_1}{1-p_1\overline{p}_2}\frac{p_2^H-\overline{p}_2}{1-p_2^H}\right)-(I_0-C^L)\frac{1-\overline{p}_2}{1-p_1\overline{p}_2}\\
=&\left(\frac{1-\overline{p}_2}{q^2}\left(\frac{1}{1-p_2^H}-\frac{1-q^2}{1-p_2^L}\right)-\frac{1-\overline{p}_2}{q}\frac{p_1}{1-p_1\overline{p}_2}\frac{p_2^H-\overline{p}_2}{1-p_2^H}-1-\frac{1-p_1}{1-p_1\overline{p}_2}\frac{p_2^H-\overline{p}_2}{1-p_2^H}\right)I_1\\
&+\left(\frac{1}{q}-1\right)(I_0-C^L)\frac{1-\overline{p}_2}{1-p_1\overline{p}_2}\\
>&\left(\frac{1-\overline{p}_2}{q^2}\left(\frac{1}{1-p_2^H}-\frac{1-q^2}{1-p_2^L}\right)-\frac{1-\overline{p}_2}{q}\frac{p_1}{1-p_1\overline{p}_2}\frac{p_2^H-\overline{p}_2}{1-p_2^H}-1-\frac{1-p_1}{1-p_1\overline{p}_2}\frac{p_2^H-\overline{p}_2}{1-p_2^H}\right)I_1.
\end{aligned} \tag{G.26}$$

Denote $f(p_1)=\frac{1-\overline{p}_2}{q}\frac{p_1}{1-p_1\overline{p}_2}+\frac{1-p_1}{1-p_1\overline{p}_2}$. We have $f'(p_1)=\left(\frac{1}{q}-1\right)\frac{1-\overline{p}_2}{(1-p_1\overline{p}_2)^2}>0$. Thus, $f(p_1)<f(1)=\frac{1}{q}$. It follows that

$$\begin{aligned}
&\frac{1-\overline{p}_2}{q^2}\left(\frac{1}{1-p_2^H}-\frac{1-q^2}{1-p_2^L}\right)-\frac{1-\overline{p}_2}{q}\frac{p_1}{1-p_1\overline{p}_2}\frac{p_2^H-\overline{p}_2}{1-p_2^H}-1-\frac{1-p_1}{1-p_1\overline{p}_2}\frac{p_2^H-\overline{p}_2}{1-p_2^H}\\
>&\frac{1-\overline{p}_2}{q^2}\left(\frac{1}{1-p_2^H}-\frac{1-q^2}{1-p_2^L}\right)-\frac{1}{q}\frac{p_2^H-\overline{p}_2}{1-p_2^H}-1\\
=&\frac{1-\overline{p}_2}{q^2}\left(\frac{1}{1-p_2^H}-\frac{1}{1-p_2^L}\right)-\frac{1}{q}\frac{p_2^H-\overline{p}_2}{1-p_2^H}+\frac{1-\overline{p}_2}{1-p_2^L}-1=g(1/q),
\end{aligned} \tag{G.27}$$

where $g(x)=x^2(1-\overline{p}_2)\left(\frac{1}{1-p_2^H}-\frac{1}{1-p_2^L}\right)-x\frac{p_2^H-\overline{p}_2}{1-p_2^H}+\frac{1-\overline{p}_2}{1-p_2^L}-1$. For $x>1$, we have

$$\begin{aligned}
g'(x)=&2x(1-\overline{p}_2)\left(\frac{1}{1-p_2^H}-\frac{1}{1-p_2^L}\right)-\frac{p_2^H-\overline{p}_2}{1-p_2^H}\\
>&(1-\overline{p}_2)\left(\frac{1}{1-p_2^H}-\frac{1}{1-p_2^L}\right)-\frac{p_2^H-\overline{p}_2}{1-p_2^H}\\
=&\frac{1-p_2^H}{1-p_2^H}-\frac{1-\overline{p}_2}{1-p_2^L}=1-\frac{1-\overline{p}_2}{1-p_2^L}>0.
\end{aligned}$$

Thus, for any $x > 1$, we have

$$
\begin{aligned}
g(x) > g(1) = (1-\overline{p}_2)\left(\frac{1}{1-p_2^H} - \frac{1}{1-p_2^L}\right) - \frac{p_2^H - \overline{p}_2}{1-p_2^H} + \frac{1-\overline{p}_2}{1-p_2^L} - 1 \\
= \frac{(1-\overline{p}_2) - (p_2^H - \overline{p}_2)}{1-p_2^H} - 1 = 0.
\end{aligned}
$$

It follows that (G.27) is positive. Then (G.26) is positive and (G.25) holds.

G.3.3 Proof of Proposition 10

Consider a subgame in which equity offers K_1. Each investor i forms a rational expectation of the probability $\widetilde{P}(K_1)$ that the competing investor accepts. Now, suppose LMEs are allowed. Following the proof of Proposition 2, we only need to consider K_1 which yields the accepting investor's payoff:

$$
R^{\text{LME}}_{\text{accept}}(p_2^i) = \frac{(1-p_2^i)K_0^*}{2} + \left(1 - \frac{\widetilde{P}(K_1)}{2}\right)\left[p_2^i C^L + (1-p_2^i)K_1 - I_1\right]. \tag{G.28}
$$

and rejecting investor's payoff:

$$
R^{\text{LME}}_{\text{reject}}(p_2^i) = \left(1 - \widetilde{P}(K_1)\right)\frac{C^L}{2} + \widetilde{P}(K_1)\frac{(1-p_2^i)K_0^*}{2}. \tag{G.29}
$$

First, we consider K_1 such that $\widetilde{P}(K_1) = 1$. Investor i accepts the offer if and only if $R^{\text{LME}}_{\text{accept}}(p_2^i) \geq R^{\text{LME}}_{\text{reject}}(p_2^i)$. Using $\widetilde{P}(K_1) = 1$, (G.28) and (G.29), we can see that the offer is accepted if and only if

$$
\begin{aligned}
& \frac{(1-p_2^i)K_0^*}{2} + \left(1 - \frac{1}{2}\right)\left[p_2^i C^L + (1-p_2^i)K_1 - I_1\right] \geq \frac{(1-p_2^i)K_0^*}{2} \\
\iff \quad & p_2^i C^L + (1-p_2^i)K_1 - I_1 \geq 0 \\
\iff \quad & K_1 \geq \frac{I_1 - p_2^i C^L}{1-p_2^i}.
\end{aligned} \tag{G.30}
$$

Using (G.11), equity optimally makes this bind at the higher value:

$$
K_1^* = K_1^{\text{LME}} = \frac{I_1 - p_2^H C^L}{1-p_2^H}. \tag{G.31}
$$

Moving back to $t = 0$, the bank rationally expects $A(K_1^*) = 1$. Recall that we assume $C^H \geq K_0^* + K_1^* > C^L$ and $C^H \geq K_0^* > C^L$ and verify these inequalities later. Then (G.2) implies that

$$
\begin{aligned}
I_0 &= (1-p_1)K_0^* + p_1\Big[\overline{p}_2 C^L + (1-\overline{p}_2)(K_0^* + K_1^*) - I_1\Big] \\
&= (1-p_1\overline{p}_2)K_0^* + p_1\Big[\overline{p}_2 C^L + (1-\overline{p}_2)\frac{I_1 - p_2^H C^L}{1-p_2^H} - I_1\Big].
\end{aligned}
$$

Rearranging, we obtain

$$
K_0^* = K_0^{\text{LME}} = \frac{I_0 - p_1\Big[\overline{p}_2 C^L - (1-\overline{p}_2)\frac{p_2^H}{1-p_2^H}C^L + \frac{p_2^H - \overline{p}_2}{1-p_2^H} I_1\Big]}{1-p_1\overline{p}_2}. \tag{G.32}
$$

Comparing (G.23) with (G.32), we find that $K_0^{\text{LME}} > K_0^{\text{Trad}}$. According to Lemma 6, we have $K_0^{\text{LME}} + K_1^{\text{LME}} < C^H$ and $K_0^{\text{LME}} > C^L$. Denote $f(x) = \frac{I_1 - xC^L}{1-x}$. Using (G.11), we have $f(p_2^H) > f(p_2^L)$, which together with $p_2^H > p_2^L$ and $f'(x) = \frac{I_1 - C^L}{(1-x)^2}$ implies that $I_1 - C^L > 0$. Then we derive from (G.31) that $K_1^{\text{LME}} > C^L > 0$. Thus $C^L < K_0^{\text{LME}} < K_0^{\text{LME}} + K_1^{\text{LME}} < C^H$.

Next, we consider the alternative where equity could set the lowest K_1 that ensures $\widetilde{P}(K_1) = 1 - q$. Using $\widetilde{P}(K_1) = 1 - q$, (G.28) and (G.29), we can see that the offer is accepted if and only if

$$
\begin{aligned}
&\frac{(1-p_2^i)K_0^*}{2} + \Big(1 - \frac{1-q}{2}\Big)\big[p_2^i C^L + (1-p_2^i)K_1 - I_1\big] \geq q\frac{C^L}{2} + (1-q)\frac{(1-p_2^i)K_0^*}{2} \\
\iff\ & q\frac{(1-p_2^i)(K_0^* - C^L)}{2} + \frac{p_2^i C^L}{2} + \frac{1+q}{2}\big[(1-p_2^i)K_1 - I_1\big] \geq 0 \\
\iff\ & q\frac{K_0^* - C^L}{1+q} + \frac{p_2^i C^L}{(1+q)(1-p_2^i)} + K_1 - \frac{I_1}{1-p_2^i} \geq 0 \\
\iff\ & K_1 \geq \frac{I_1}{1-p_2^i} - \frac{p_2^i C^L}{(1+q)(1-p_2^i)} - \frac{q}{1+q}(K_0^* - C^L).
\end{aligned} \tag{G.33}
$$

Note that

$$
\begin{aligned}
&\frac{I_1}{1-p_2^H} - \frac{p_2^H C^L}{(1+q)(1-p_2^H)} > \frac{I_1}{1-p_2^L} - \frac{p_2^L C^L}{(1+q)(1-p_2^L)} \\
\iff\ & I_1\Big(\frac{1}{1-p_2^H} - \frac{1}{1-p_2^L}\Big) > \frac{C^L}{1+q}\Big(\frac{p_2^H}{1-p_2^H} - \frac{p_2^L}{1-p_2^L}\Big),
\end{aligned}
$$

and the inequality (G.11) implies this holds for $q = 0$ so it must hold for $q > 0$. Thus, if equity wants to achieve $\widetilde{P}(K_1) = 1 - q$, they optimally make (G.33) bind at p_2^L:

$$K_1^{\text{Alt}} = \frac{I_1}{1 - p_2^L} - \frac{p_2^L C^L}{(1+q)(1-p_2^L)} - \frac{q}{1+q}(K_0^* - C^L). \tag{G.34}$$

We have

$$\begin{aligned} K_0^* + K_1^{\text{Alt}} - C^L &= \frac{I_1}{1-p_2^L} + \frac{1}{1+q}(K_0^* - C^L) - \frac{p_2^L C^L}{(1+q)(1-p_2^L)} \\ > &\frac{I_1}{1-p_2^L} - \frac{p_2^L C^L}{(1+q)(1-p_2^L)} > 0, \end{aligned} \tag{G.35}$$

where the first inequality uses $K_0^* = K_0^{\text{LME}} > C^L$, and the second inequality uses $I_1 - C^L > 0$.

Similar to the proof in Proposition 9, it is optimal to follow the equilibrium strategy $K_1^* = K_1^{\text{LME}}$ given in (G.31) rather than deviating to K_1^{Alt} given in (G.34) if and only if

$$\begin{aligned} & C^H - K_0^* - K_1^* \geq (1-q^2)(C^H - K_0^* - K_1^{\text{Alt}}) \\ \iff & q^2(C^H - K_0^*) \geq \frac{I_1 - p_2^H C^L}{1-p_2^H} - (1-q^2)\left(\frac{I_1}{1-p_2^L} - \frac{p_2^L C^L}{(1+q)(1-p_2^L)} - \frac{q}{1+q}(K_0^* - C^L) \right) \\ \iff & q^2(C^H - K_0^*) \geq \frac{I_1 - p_2^H C^L}{1-p_2^H} - \frac{(1-q^2) I_1}{1-p_2^L} + \frac{(1-q)p_2^L C^L}{1-p_2^L} + q(1-q)(K_0^* - C^L) \\ \iff & q^2(C^H - C^L) \geq \frac{I_1 - p_2^H C^L}{1-p_2^H} - \frac{(1-q^2) I_1}{1-p_2^L} + \frac{(1-q)p_2^L C^L}{1-p_2^L} + q(K_0^* - C^L) \\ \iff & C^H - C^L \geq \frac{I_1}{q^2}\left(\frac{1}{1-p_2^H} - \frac{1-q^2}{1-p_2^L} \right) + \frac{1}{q^2}\Big[- \frac{p_2^H C^L}{1-p_2^H} + \frac{(1-q)p_2^L C^L}{1-p_2^L} + q(K_0^* - C^L) \Big]. \end{aligned}$$

The right side is

$$
\begin{aligned}
&\frac{I_1}{q^2}\left(\frac{1}{1-p_2^H}-\frac{1-q^2}{1-p_2^L}\right)+\frac{1}{q^2}\left[-\frac{p_2^H C^L}{1-p_2^H}+\frac{(1-q)p_2^L C^L}{1-p_2^L}\right] \\
&+\frac{1}{q}\left(\frac{I_0-p_1\left[\overline{p}_2 C^L-(1-\overline{p}_2)\frac{p_2^H}{1-p_2^H}C^L+\frac{p_2^H-\overline{p}_2}{1-p_2^H}I_1\right]}{1-p_1\overline{p}_2}-C^L\right) \\
=&\frac{I_1}{q^2}\left(\frac{1}{1-p_2^H}-\frac{1-q^2}{1-p_2^L}\right)+\frac{1}{q}\left(\frac{I_0-C^L-p_1\frac{p_2^H-\overline{p}_2}{1-p_2^H}I_1}{1-p_1\overline{p}_2}\right) \\
&+\frac{C^L}{q^2}\left[\frac{qp_1(1-\overline{p}_2)\frac{p_2^H}{1-p_2^H}}{1-p_1\overline{p}_2}-\frac{p_2^H}{1-p_2^H}+\frac{(1-q)p_2^L}{1-p_2^L}\right] \\
=&\mathbb{B}_{\text{belief}}^{\text{Trad}}+\frac{C^L}{q^2}\left[\frac{p_2^H}{1-p_2^H}\left(\frac{qp_1(1-\overline{p}_2)}{1-p_1\overline{p}_2}-1\right)+\frac{(1-q)p_2^L}{1-p_2^L}\right]=\mathbb{B}_{\text{belief}}^{\text{LME}}.
\end{aligned}
$$

Note that

$$
\begin{aligned}
&\frac{p_2^H}{1-p_2^H}\left(\frac{qp_1(1-\overline{p}_2)}{1-p_1\overline{p}_2}-1\right)+\frac{(1-q)p_2^L}{1-p_2^L} \\
<&\frac{p_2^H}{1-p_2^H}\left(\frac{qp_1(1-\overline{p}_2)}{1-p_1\overline{p}_2}-1\right)+\frac{(1-q)p_2^H}{1-p_2^H} \\
=&\frac{p_2^H}{1-p_2^H}\left(\frac{qp_1(1-\overline{p}_2)}{1-p_1\overline{p}_2}-q\right) \\
=&\frac{p_2^H q}{1-p_2^H}\left(\frac{p_1-p_1\overline{p}_2}{1-p_1\overline{p}_2}-1\right)=\frac{p_2^H q}{1-p_2^H}\left(\frac{p_1-1}{1-p_1\overline{p}_2}\right)<0.
\end{aligned}
$$

Thus $\mathbb{B}_{\text{belief}}^{\text{LME}}<\mathbb{B}_{\text{belief}}^{\text{Trad}}$.

H Details for the Simple Continuous Time Capital Structure Model

In this Appendix, we provide the mathematical details for the model described in Subsection 5.7.

H.1 Formal model setup

All agents share a common discount rate $r > 0$. At time $t = 0$, a firm can raise I_0 dollars to initiate a project, which produces a constant pre-tax cash flow $\mu_0 > rI_0$ when business is as usual. However, the project is subject to a (downward) jump shock that arrives at a constant rate $\lambda > 0$. When this shock hits at stochastic time T, the firm must raise liquidity. Absent this liquidity injection, the firm's pre-tax cash flow decreases permanently to $\mu_L \in (0, rI_0)$. The firm has an option to inject liquidity I_T at T to potentially avoid the negative consequences of the jump shock. The size of the jump shock is random in the following sense. By injecting liquidity I_T at T, with probability $1 - p$ the firm succeeds by permanently maintaining its cash flow at μ_0 per unit time, but with probability p it fails and its cash flow permanently decreases to μ_L per unit time. Similar to Condition 1 in our baseline model, we impose the following condition, implying that the firm is insolvent if it uses debt financing to pay the initial expense I_0:

Condition 9. $p\frac{\mu_L}{r} + (1-p)\frac{\mu_0}{r} < I_0 + I_T$.

Furthermore, we assume conditions implying that liquidity injection is efficient and satisfies the limited liability constraint.

Condition 10. $(1-p)\frac{\mu_0 - \mu_L}{r} > \mathbb{B}^{\text{Commit}}$, *where*

$$\mathbb{B}^{\text{Commit}} = \max\Big\{\frac{I_T}{1-\pi}, I_T\Big(1 + \frac{\overline{\theta}}{q}\Big), I_T(1+\overline{\theta}) + (I_0 - \widehat{C}^L)\frac{(r+\lambda)(1-p)}{r + \lambda(1-p)}\Big\}, \tag{H.1}$$

and $\widehat{C}^L := (1-\alpha)(1-\pi)\mu_L/r$ *is the bankruptcy value.*

Condition 11. $I_T > \max\Big\{\frac{p}{1+\overline{\theta}}, \frac{p + (1-p)q\left[\frac{1}{(1-\alpha)(1-\pi)} - 1\right]}{1+q}\Big\}\widehat{C}^L$.

At time zero, the firm decides whether or not to issue debt to cover the initial expense I_0. The benefit of debt at time zero is that it allows the firm to deduct coupons from its tax payments.

Otherwise, the firm pays taxes at rate π on its cash flow. The downside of debt is that the firm incurs bankruptcy costs if it fails at time T: the cash flow falls to $(1-\alpha)\mu_L$ rather than μ_L, where the parameter α captures bankruptcy costs.

We now consider three different scenarios for firm financing: (1) an all equity firm; (2) a firm that issues secured debt at $t = 0$ that bans LMEs at T; or (3) a firm that issues secured debt at $t = 0$ and enables LMEs at T. We characterize parameters such that option (3) increases ex-ante firm value.

H.2 Firm value with only equity financing

To begin, we characterize a firm that cannot issue debt at time zero or T. At time zero, external equity investors inject I_0 to cover the initial expense.

At time T, the equity investors have two choices. First, they can forgo new equity investment. In this case the firm produces post-tax cash flow $(1-\pi)\mu_L$ in perpetuity and the firm value is

$$\frac{(1-\pi)\mu_L}{r}. \tag{H.2}$$

Alternatively, the firm can issue equity. In this case, the firm value is

$$E_T = -I_T + \frac{1-\pi}{r}\Big[(1-p)\mu_0 + p\mu_L\Big]. \tag{H.3}$$

We see that equity investors optimally cover the expense at T if and only if

$$\frac{(1-p)(\mu_0-\mu_L)}{r} > \frac{I_T}{1-\pi}. \tag{H.4}$$

Given our Condition 10, this is satisfied. Then the value of the existing equity at T is (H.3). At time 0, immediately after paying the expense I_0, the firm value is equal to the equity value, which is

$$\int_0^\infty \left[\int_0^T e^{-rs}(1-\pi)\mu_0 ds + e^{-rT}E_T\right]\lambda e^{-\lambda T}dT = \frac{(1-\pi)\mu_0}{r+\lambda} + \frac{\lambda E_T}{r+\lambda}. \tag{H.5}$$

Thus, under Condition 10, we can plug in (H.3) to see that the time-zero firm value with only equity financing is

$$E_0^{\mathrm{EQ}} = -I_0 + \frac{(1-\pi)\mu_0}{r+\lambda} + \frac{\lambda}{r+\lambda}\Big[-I_T + \frac{1-\pi}{r}\Big[(1-p)\mu_0 + p\mu_L\Big]\Big]. \tag{H.6}$$

H.3 Firm value with debt

We follow the standard tradeoff theory setting (Leland, 1994).[51] Interest payments are tax deductible and the firm incurs distress costs in bankruptcy. As we show later, in equilibrium, the firm only files for bankruptcy at T when its cash flow is low and equals μ_L. In this case, creditors receive the entire bankruptcy value:

$$\widehat{C}^L := (1-\alpha)(1-\pi)\mu_L/r, \tag{H.7}$$

where α is a parameter measuring distress costs and π denotes the corporate profit tax rate.

At $t=0$, equity holders issue interest-only debt with face value I_0 (as in Leland (1994)) by promising to pay a stream of coupons m_0 per unit time to debt holders in perpetuity until the firm defaults. At stochastic time T, should the firm choose to raise liquidity I_T, it makes a take-it-or-leave-it offer with a promised coupon payment m_T for the newly issued perpetual debt to existing debtholders.[52] Due to the limited liability constraint, we assume $m_0 \le \mu_0$ and $m_0 + m_T \le \mu_0$.

As in our baseline model of Section 3, we analyze both financing with and without LMEs. In both financing arrangements, there are two debt holders who potentially provide liquidity at T, each of whom has an independent, privately observed funding cost equal to $\overline{\theta}$ with probability q and zero with probability $1-q$. In each arrangement, creditors form expectations accordingly and act strategically. Specifically, each creditor i forms a belief $\widetilde{P}(m_T)$ that the other creditor j will

[51] Abel (2018) develops a tradeoff theory of capital structure with downward earnings jump shocks.

[52] As the project's cash flow is constant (μ_0) before T and (conditionally) constant after T, we focus on interest-only perpetual debt issuance at $t=0$ and $t=T$, as it matches the firm's cash flow and liability payment so that both leverage (debt-to-value ratio) and the debt-service coverage ratio are constant before and after the liquidity shocks arrives, and leverage and thus the debt-service coverage ratio only change at T.

accept. Given this, in the absence of an LME, creditor i's expected payoff for accepting is

$$\frac{1}{2}\Big[(1-p)\frac{m_0}{r}+p\widehat{C}^L\Big]+\left(1-\frac{\widetilde{P}(m_T)}{2}\right)\big[(1-p)\frac{m_T}{r}-I_T(1+\theta_i)\big] \tag{H.8}$$

and investor i's expected payoff from rejecting a non-LME offer is

$$(1-\widetilde{P}(m_T)+p\widetilde{P}(m_T))\frac{\widehat{C}^L}{2} \;+\; \widetilde{P}(m_T)(1-p)\frac{1}{2}\frac{m_0}{r}. \tag{H.9}$$

As in our baseline model, creditor i's expected payoff from accepting an LME offer is

$$(1-p)\frac{1}{2}\frac{m_0}{r}+\left(1-\frac{\widetilde{P}(m_T)}{2}\right)\big[p\widehat{C}^L+(1-p)\frac{m_T}{r}-I_T(1+\theta_i)\big], \tag{H.10}$$

and the payoff from rejecting an LME offer is

$$(1-\widetilde{P}(m_T))\frac{\widehat{C}^L}{2}+\widetilde{P}(m_T)(1-p)\frac{1}{2}\frac{m_0}{r}. \tag{H.11}$$

Investors accept an offer if and only if the expected payoff from accepting weakly exceeds the expected payoff from rejecting. The probability $\widetilde{P}(m_T)$ solves a fixed point problem such that it is the actual probability that the other investor accepts.

The original coupon m_0 is endogenously determined to make creditors break even when they provide liquidity at $t=0$. The coupon m_T is chosen strategically by equity holders at time T to maximize their objective: letting $A(m_T)=1-(1-\widetilde{P}(m_T))^2$ denote the probability that an offer m_T is accepted, equity holders choose m_T at time T to solve

$$\max_{m_T} E_T(m_0,m_T)=A(m_T)\frac{(1-\pi)(1-p)(\mu_0-m_0-m_T)}{r}. \tag{H.12}$$

Note that this objective implicitly assumes that equity holders choose to file for bankruptcy if the cash flow falls to μ_L. We will verify $m_0^*+m_T>\mu_L$ for equilibrium values of m_0^* and any m_T such that $A(m_T)>0$ in Internet Appendices H.4-H.6.

Lemma 7. *Suppose $\mu_0 \geq m_0+m_T$, and Condition 9 holds. If lenders break even at $t=0$, then $m_0>\mu_L$.*

When the liquidity shock hits at $t = T$, in a given debt financing arrangement with coupons m_0 and m_T, the time-T market value of bank debt issued at $t = 0$ is given by

$$D_T(m_0, m_T) = \widehat{C}^L + A(m_T)\Bigg[\underbrace{(1-p)\Big(\frac{m_0 + m_T}{r} - \widehat{C}^L\Big) - I_T}_{\text{expected increase of bank debt value}}\Bigg] - I_T\mathbb{E}[\Theta], \tag{H.13}$$

where $A(\cdot)$ is the probability that the restructuring plan is accepted, $\Theta = \frac{\sum_i \theta_i \mathbf{1}_{\text{accept}}(m_T,\theta_i)}{1+\prod_i \mathbf{1}_{\text{accept}}(m_T,\theta_i)}$ is liquidity cost given investors' participation decisions, and $\widehat{C}^L$ is the firm's bankruptcy value at T when the firm's cash flow is permanently low (μ_L). The second term equals the liquidity injection probability at T, $A(\cdot)$, multiplied by the expected increase in the value of bank debt from avoiding bankruptcy. The last term is the average funding cost among participating investors at T.

Equity value immediately after the liquidity shock hits at time T, $E_T(m_0, m_T)$, is given by (H.12). This expression is intuitive as equity value in equilibrium equals the probability that debt restructuring takes place and is successful, $A(m_T)(1 - p)$, multiplied by the present value of its after-tax after-interest cashflows: $(1 - \pi)(\mu_0 - m_0 - m_T)/r$.

Next, consider the valuation problem before the liquidity shock arrives. Debt value at time $t < T$ is

$$\begin{aligned} D_t(m_0, m_T) &= \int_0^\infty \left[\int_0^T e^{-rs} m_0 ds + e^{-rT} D_T(m_0, m_T)\right] \lambda e^{-\lambda T} dT \\ &= \frac{m_0}{r+\lambda} + \frac{\lambda D_T(m_0, m_T)}{r+\lambda}. \end{aligned} \tag{H.14}$$

Note that liquidity shocks shorten the duration for debtholders to collect coupons, thus increasing the denominator from r to $r + \lambda$. However, liquidity shocks also partially compensate debtholders by providing them with a market value of $D_T(m_0, m_T)$ at T. The break even condition requires that this debt value is equal to I_0 at time 0:

$$\frac{m_0 + \lambda D_T(m_0, m_T)}{r+\lambda} = I_0. \tag{H.15}$$

After time 0 but before time T, the equity value is

$$
\begin{aligned}
E_t(m_0, m_T) = &\int_0^\infty \left[\int_0^T e^{-rs}(1-\pi)(\mu_0 - m_0)ds + e^{-rT} E_T(m_0, m_T) \right] \lambda e^{-\lambda T} dT \\
= &\frac{(1-\pi)(\mu_0 - m_0)}{r+\lambda} + \frac{\lambda E_T(m_0, m_T)}{r+\lambda}.
\end{aligned} \tag{H.16}
$$

Combining these equations, we prove the following lemma, which is an analog of Lemma 2.

Lemma 8. *The date-0 value of equity is given by*

$$
\begin{aligned}
E_0(m_0, m_T) = &(1-\pi)\Big(\frac{\mu_0}{r+\lambda} - I_0 + \frac{\lambda}{r+\lambda}\widehat{C}^L\Big) \\
&+ \frac{\lambda(1-\pi)}{r+\lambda}\left[A(m_T)\Big[(1-p)\Big(\frac{\mu_0}{r} - \widehat{C}^L\Big) - I_T + p\Big(\frac{\mu_L}{r} - \widehat{C}^L\Big)\boldsymbol{1}_{\mu_L \geq m_0 + m_T}\Big] - I_T \mathbb{E}[\Theta] \right].
\end{aligned} \tag{H.17}
$$

If Condition 10 holds, then $E_0(m_0, m_T)$ is maximized at m_T^* with $A(m_T^*) = 1$.

Conditional on issuing debt, equity holders maximize this ex ante objective when they choose at date 0 whether or not to enable LMEs. We now characterize this choice by characterizing the equilibrium with LMEs and the equilibrium without LMEs.

H.4 Comparing LMEs to traditional renegotiation

As in Lemma 3, we first present a lemma about the limited liability condition.

Lemma 9. *Suppose Condition 10 holds. Then we have $\mu_0 > m_0 + m_T$ for any (m_0, m_T) satisfying the break-even condition, $A(m_T) = 1$, and $m_0 \geq m_0^{\text{Trad}}$, where*

$$
m_0^{\text{Trad}} = r\frac{I_0(r+\lambda) - \lambda\Big[I_T\overline{\theta}(1-q) + p\widehat{C}^L\Big]}{r + \lambda(1-p)}. \tag{H.18}
$$

We now characterize the equilibrium with only traditional renegotiation and the equilibrium with LMEs. We then provide a sufficient condition on parameters such that enabling LMEs shifts the optimal date-0 capital structure choice from all equity to issuing secured debt.

Equilibrium with only traditional restructuring. Similar to Proposition 1, the following proposition characterizes the equilibrium when equity holders issue debt at date 0 that bans LMEs. The proposition provides the condition under which renegotiation always succeeds when LMEs are not available.

Proposition 11. *Suppose Conditions 9-11 hold and define*

$$\mathbb{B}^{\mathrm{Trad}} = I_T\left(1+\frac{\overline{\theta}}{q^2}\right) + \frac{1-p}{q}\Big(\frac{m_0^{\mathrm{Trad}}}{r} - \widehat{C}^L\Big), \tag{H.19}$$

$$m_0^{\mathrm{Trad}} = r\frac{I_0(r+\lambda) - \lambda\Big[I_T\overline{\theta}(1-q) + p\widehat{C}^L\Big]}{r+\lambda(1-p)}. \tag{H.20}$$

Then:

1. *Assume* $(1-p)(\frac{\mu_0}{r} - \widehat{C}^L) \geq \mathbb{B}^{\mathrm{Trad}}$*. Then* $m_T^{\mathrm{Trad}} = r\frac{I_T(1+\overline{\theta})}{1-p}$ *is the optimal solution to* (H.12) *given* m_0^{Trad}*, and* $A(m_T^{\mathrm{Trad}}) = 1$ *. Also,* m_0^{Trad} *satisfies the break even condition given* m_T^{Trad}*. Finally,* $\mu_L < m_0^{\mathrm{Trad}} < m_0^{\mathrm{Trad}} + m_T^{\mathrm{Trad}} < \mu_0$*.*
2. *Assume* $(1-p)(\frac{\mu_0}{r} - \widehat{C}^L) < \mathbb{B}^{\mathrm{Trad}}$*. Then in any equilibrium* (m_0^*, m_T^*) *with only traditional renegotiation,* $A(m_T^*) \leq 1-q^2$*.*

Similar to Proposition 1, this proposition shows that traditional renegotiation fails with probability q^2 for certain parameters, even when it is efficient ex ante to ensure a successful renegotiation. We now show that LMEs can help solve this renegotiation failure.

Equilibrium with LMEs. Similar to Proposition 2, the following proposition provides the condition under which renegotiation always succeeds when using LMEs.

Proposition 12. *Suppose Conditions 9-11 hold and define*

$$\mathbb{B}^{\mathrm{LME}} = I_T\Big(1+\frac{\overline{\theta}}{q^2}\Big) + \frac{1-p}{q}\frac{m_0^{\mathrm{LME}}}{r} - \frac{\widehat{C}^L}{q} = \mathbb{B}^{\mathrm{Trad}} - \frac{p\widehat{C}^L}{q}\frac{r}{r+\lambda(1-p)} < \mathbb{B}^{\mathrm{Trad}}, \tag{H.21}$$

$$m_0^{\mathrm{LME}} = r\frac{I_0(r+\lambda) - \lambda I_T\overline{\theta}(1-q)}{r+\lambda(1-p)}. \tag{H.22}$$

Then:

1. *Assume* $(1-p)(\frac{\mu_0}{r}-\widehat{C}^L) \geq \mathbb{B}^{\text{LME}}$. *Then* $m_T^{\text{LME}} = r\frac{I_T(1+\bar{\theta})-p\widehat{C}^L}{1-p}$ *is the optimal solution to* (H.12) *given* m_0^{LME}, *and* $A(m_T^{\text{LME}}) = 1$. *Also,* m_0^{LME} *satisfies the break even condition given* m_T^{LME}. *Finally,* $\mu_L < m_0^{\text{LME}} < m_0^{\text{LME}} + m_T^{\text{LME}} < \mu_0$.

2. *Assume* $(1-p)(\frac{\mu_0}{r}-\widehat{C}^L) < \mathbb{B}^{\text{LME}}$. *Then in any equilibrium* (m_0^*, m_T^*) *with LMEs,* $A(m_T^*) \leq 1-q^2$.

H.5 Comparing values

Putting the above arguments together, we have shown that: (1) the ex-ante firm value with only equity financing is given by (H.6); (2) the ex-ante firm value with secured debt that bans LMEs is characterized by Lemma 8 and Proposition 11; and (3) the ex-ante firm value with secured debt that enables LMEs is characterized by Lemma 8 and Proposition 12. Using these formulas for ex ante firm values, we can derive conditions under which the ability to conduct an LME improves firm value and shifts the optimal financing structure from all equity to secured debt with LMEs. The following proposition provides these conditions:

Proposition 13. *Suppose that the following conditions hold.*

1. *Conditions 9-11 hold.*

2. *There is no efficient equilibrium without LMEs, and there is an efficient equilibrium with LMEs:*

$$\mathbb{B}^{\text{LME}} \leq (1-p)(\frac{\mu_0}{r}-\widehat{C}^L) < \mathbb{B}^{\text{Trad}} \tag{H.23}$$

   where $\mathbb{B}^{\text{Trad}}$ *is given in* (H.19), *and* $\mathbb{B}^{\text{LME}}$ *is given in* (H.21).

3. *The all-equity firm value is between the firm value in the efficient LME equilibrium and the firm value in the inefficient traditional-negotiation equilibrium:* $E_0^{\text{Trad}} < E_0^{\text{EQ}} < E_0^{\text{LME}}$, *where* E_0^{EQ} *is given in* (H.6),

$$\begin{aligned} E_0^{\text{LME}} = {} & (1-\pi)\Big(\frac{\mu_0}{r+\lambda} - I_0 + \frac{\lambda}{r+\lambda}\widehat{C}^L\Big) \\ & + \frac{\lambda(1-\pi)}{r+\lambda}\Big[(1-p)\Big(\frac{\mu_0}{r}-\widehat{C}^L\Big) - I_T(1+q\bar{\theta})\Big], \end{aligned} \tag{H.24}$$

and

$$
\begin{aligned}
E_0^{\text{Trad}} &= (1-\pi)\Big(\frac{\mu_0}{r+\lambda} - I_0 + \frac{\lambda}{r+\lambda}\widehat{C}^L\Big) \\
&\quad + \frac{\lambda(1-\pi)}{r+\lambda}\Bigg[(1-q^2)\Big[(1-p)\Big(\frac{\mu_0}{r} - \widehat{C}^L\Big) - I_T\Big]\Bigg], \qquad \text{if } A(m_T^*) = 1-q^2, \\
E_0^{\text{Trad}} &= (1-\pi)\Big(\frac{\mu_0}{r+\lambda} - I_0 + \frac{\lambda}{r+\lambda}\widehat{C}^L\Big), \qquad \text{if } A(m_T^*) = 0. \qquad \text{(H.25)}
\end{aligned}
$$

Then enabling LMEs improves ex ante firm value. Further, the ability to conduct an LME shifts the optimal capital structure from all equity to debt financing.

The conditions of the proposition hold for the following parameter values: $r = 0.06$, $\mu_0 = 1.15$, $\mu_L = 0.19$, $I_0 = 6$, $p = 0.7$, $I_T = 2$, $q = 0.8$, $\overline{\theta} = 0.02$, $\pi = 0.21$, $\alpha = 0.4$, $\lambda = 0.15$.

Thus, we have proved in a simple dynamic trade off model that LMEs can (1) improve ex-ante firm value and (2) shift the optimal ex-ante capital structure to one with higher leverage and more secured debt. In the rest of this Appendix, we provide proofs for all of the propositions and lemmas stated above.

H.6 Proofs

H.6.1 Proof of Lemma 7

At time T, equity holders default if and only if the cash flow is less than the total coupon: m_0 if there is no renegotiation or $m_0 + m_T$ if there is renegotiation. Recall that we impose as a constraint that $\mu_0 \geq m_0 + m_T$ when equity holders choose m_T. Thus, they do not default if the cash flow remains μ_0. For any coupons $m_0 > 0$ and $m_T > 0$, the value of debt at time T that accounts for all possible default scenarios is

$$D_T(m_0,m_T) = \underbrace{-I_T A(m_T) - I_T\mathbb{E}[\Theta]}_{\text{liquidity injection}} + \underbrace{A(m_T)(1-p)\frac{m_0+m_T}{r}}_{\text{constraint that } \mu_0\geq m_0+m_T}$$
$$+ (1-A(m_T))\underbrace{\left(\frac{m_0}{r}\mathbf{1}_{\mu_L\geq m_0} + \widehat{C}^L\mathbf{1}_{\mu_L<m_0}\right)}_{\text{default if } \mu_L<m_0}$$
$$+ A(m_T)p\underbrace{\left(\frac{m_0+m_T}{r}\mathbf{1}_{\mu_L\geq m_0+m_T} + \widehat{C}^L\mathbf{1}_{\mu_L<m_0+m_T}\right)}_{\text{default if } \mu_L<m_0+m_T}. \tag{H.26}$$

Note that $-I_T\mathbb{E}[\Theta]\leq 0$ and $\widehat{C}^L\leq \mu_L/r$, so

$$D_T(m_0,m_T) \leq -I_T A(m_T) + A(m_T)(1-p)\frac{m_0+m_T}{r}$$
$$+ (1-A(m_T))\left(\frac{m_0}{r}\mathbf{1}_{\mu_L\geq m_0} + \frac{\mu_L}{r}\mathbf{1}_{\mu_L<m_0}\right)$$
$$+ A(m_T)p\left(\frac{m_0+m_T}{r}\mathbf{1}_{\mu_L\geq m_0+m_T} + \frac{\mu_L}{r}\mathbf{1}_{\mu_L<m_0+m_T}\right).$$

Replacing m_0/r and $(m_0+m_T)/r$ with μ_L/r when μ_L/r is larger, we see that

$$\begin{aligned}
D_T(m_0,m_T) &\leq \big(1-A(m_T)+A(m_T)p\big)\frac{\mu_L}{r} + A(m_T)\Big[(1-p)\frac{m_0+m_T}{r} - I_T\Big] \\
&= \frac{\mu_L}{r} + A(m_T)\Big[(1-p)\frac{m_0+m_T-\mu_L}{r} - I_T\Big] \\
&\leq \frac{\mu_L}{r} + A(m_T)\Big[(1-p)\frac{\mu_0-\mu_L}{r} - I_T\Big] \\
&\leq \frac{\mu_L}{r} + A(m_T)\Big[I_0 - \frac{\mu_L}{r}\Big] \\
&\leq \frac{\mu_L}{r} + \Big[I_0 - \frac{\mu_L}{r}\Big] = I_0,
\end{aligned} \tag{H.27}$$

where the second inequality uses $\mu_0 \geq m_0+m_T$, the third inequality uses Condition 9, the fourth inequality uses $I_0 > \frac{\mu_L}{r}$.

Using $I_0 \geq D_T(m_0,m_T)$ and the break even condition (H.15), we have

$$I_0 = \frac{m_0+\lambda D_T(m_0,m_T)}{r+\lambda} \leq \frac{m_0+\lambda I_0}{r+\lambda}, \tag{H.28}$$

which implies that $I_0 \leq \frac{m_0}{r}$. Using $\frac{\mu_L}{r} < I_0$, we have $m_0 > \mu_L$.

H.6.2 Proof of Lemma 8

For any coupons $m_0 > 0$ and $m_T > 0$, the equity value at time T that accounts for all possible default scenarios is

$$E_T(m_0, m_T) = (1-\pi)\Bigg[\underbrace{A(m_T)(1-p)\frac{\mu_0 - m_0 - m_T}{r}}_{\text{constraint that } \mu_0 \geq m_0 + m_T} \\ + (1 - A(m_T))\underbrace{\left(\frac{\mu_L - m_0}{r}\mathbf{1}_{\mu_L \geq m_0}\right)}_{\text{default if } \mu_L < m_0} + A(m_T)p\underbrace{\left(\frac{\mu_L - m_0 - m_T}{r}\mathbf{1}_{\mu_L \geq m_0 + m_T}\right)}_{\text{default if } \mu_L < m_0 + m_T}\Bigg]. \tag{H.29}$$

Using (H.14) and (H.16), we have

$$\begin{aligned} & E_0(m_0, m_T) + (1-\pi)D_0(m_0, m_T) \\ = & \frac{1}{r+\lambda}\Big[(1-\pi)\mu_0 + \lambda(E_T(m_0, m_T) + (1-\pi)D_T(m_0, m_T))\Big]. \end{aligned} \tag{H.30}$$

Recall that we impose as a constraint that $\mu_0 \geq m_0 + m_T$. According to Lemma 7, we have $m_0 > \mu_L$. Then using (H.29) and (H.26), we have

$$\begin{aligned} & E_T(m_0, m_T) + (1-\pi)D_T(m_0, m_T) \\ = & (1-\pi)\Bigg[A(m_T)p\Big(\frac{\mu_L}{r} - \widehat{C}^L\Big)\mathbf{1}_{\mu_L \geq m_0 + m_T} + \widehat{C}^L + A(m_T)\Big[(1-p)\Big(\frac{\mu_0}{r} - \widehat{C}^L\Big) - I_T\Big] - I_T\mathbb{E}[\Theta]\Bigg]. \end{aligned} \tag{H.31}$$

Substituting the above into (H.30) and using $D_0(m_0, m_T) = I_0$, we obtain

$$\begin{aligned} & E_0(m_0, m_T) + (1-\pi)I_0 = E_0(m_0, m_T) + (1-\pi)D_0(m_0, m_T) \\ = & (1-\pi)\Big(\frac{\mu_0}{r+\lambda} + \frac{\lambda}{r+\lambda}\widehat{C}^L\Big) \\ & + \frac{\lambda(1-\pi)}{r+\lambda}\Bigg[A(m_T)\Big[(1-p)\Big(\frac{\mu_0}{r} - \widehat{C}^L\Big) - I_T + p\Big(\frac{\mu_L}{r} - \widehat{C}^L\Big)\mathbf{1}_{\mu_L \geq m_0 + m_T}\Big] - I_T\mathbb{E}[\Theta]\Bigg]. \end{aligned} \tag{H.32}$$

Then we obtain (H.17).

Thus, we only need to choose m_T to maximize

$$A(m_T)\Big[(1-p)\big(\frac{\mu_0}{r}-\widehat{C}^L\big)-I_T+p\Big(\frac{\mu_L}{r}-\widehat{C}^L\Big)\mathbf{1}_{\mu_L\geq m_0+m_T}\Big]-I_T\mathbb{E}[\Theta]. \tag{H.33}$$

According to Condition 10 and $\frac{\mu_L}{r}\geq\widehat{C}^L$, we have

$$(1-p)\Big(\frac{\mu_0}{r}-\widehat{C}^L\Big)>I_T\Big(1+\frac{\overline{\theta}}{q}\Big). \tag{H.34}$$

Replacing p_2 with p, C^H with $\frac{\mu_0}{r}$, C^L with $\widehat{C}^L$, and I_1 with I_T in the proof of Lemma 2, we can show that under condition (H.34), the following is maximized when $A(m_T^*)=1$:

$$A(m_T)\Big[(1-p)\big(\frac{\mu_0}{r}-\widehat{C}^L\big)-I_T\Big]-I_T\mathbb{E}[\Theta]. \tag{H.35}$$

Since $\frac{\mu_L}{r}\geq\widehat{C}^L$, then (H.33) is maximized when $A(m_T^*)=1$.

H.6.3 Proof of Lemma 9

If $m_0+m_T\leq\mu_L$, then we have $\mu_0>\mu_L\geq m_0+m_T$. Thus, we only need to consider the case $m_0+m_T>\mu_L$.

Comparing (H.8) and (H.9), and setting $\widetilde{P}=1$, $\theta_i=\overline{\theta}$, we can see $A(m_T)=1$ for any $m_T\geq m_T^{\text{Trad}}=r\frac{I_T(1+\overline{\theta})}{1-p}$ under traditional restructuring.

Substituting $A(m_T)=1$ and $\mathbb{E}[\Theta]=\overline{\theta}q$ into (H.26), and using $m_0+m_T>\mu_L$, we obtain

$$D_T(m_0,m_T)=\widehat{C}^L+(1-p)\Big(\frac{m_0+m_T}{r}-\widehat{C}^L\Big)-I_T-I_T\overline{\theta}q. \tag{H.36}$$

Thus, $\mu_0>m_0+m_T$ if and only if

$$D_T(m_0,m_T)<\widehat{C}^L+(1-p)\Big(\frac{\mu_0}{r}-\widehat{C}^L\Big)-I_T-I_T\overline{\theta}q. \tag{H.37}$$

Using (H.15), we have $D_T(m_0,m_T)=\frac{(r+\lambda)I_0-m_0}{\lambda}$. Thus (H.37) is equivalent to

$$(1-p)\Big(\frac{\mu_0}{r}-\widehat{C}^L\Big)>I_T(1+\overline{\theta}q)+\frac{(r+\lambda)I_0-m_0}{\lambda}-\widehat{C}^L. \tag{H.38}$$

We see that the right side of (H.38) is decreasing in m_0, so if (H.38) holds for m_0^{Trad}, then it holds for any $m_0 \geq m_0^{\text{Trad}}$.

Substituting $m_0 = m_0^{\text{Trad}} = r\frac{I_0(r+\lambda)-\lambda\left[I_T\overline{\theta}(1-q)+p\widehat{C}^L\right]}{r+\lambda(1-p)}$ into (H.38), a sufficient condition for $\mu_0 > m_0^{\text{Trad}} + m_T^{\text{Trad}}$ is

$$\begin{aligned}
&(1-p)\Big(\frac{\mu_0}{r}-\widehat{C}^L\Big) > I_T(1+\overline{\theta}q) + \frac{(r+\lambda)I_0 - m_0^{\text{Trad}}}{\lambda} - \widehat{C}^L \\
=&I_T(1+\overline{\theta}q) + \frac{r(1-q)}{r+\lambda(1-p)}I_T\overline{\theta} + \frac{(r+\lambda)I_0}{\lambda}\Big[1-\frac{r}{r+\lambda(1-p)}\Big] - \widehat{C}^L\Big[1-\frac{rp}{r+\lambda(1-p)}\Big] \\
=&I_T\left(1+\overline{\theta}\Big(q+\frac{r(1-q)}{r+\lambda(1-p)}\Big)\right) + (I_0-\widehat{C}^L)\frac{(r+\lambda)(1-p)}{r+\lambda(1-p)}.
\end{aligned} \tag{H.39}$$

Since $\frac{r}{r+\lambda(1-p)} \in (0,1)$, we have $q + \frac{r(1-q)}{r+\lambda(1-p)} < q+1-q = 1$. According to Condition 10, we have

$$(1-p)\Big(\frac{\mu_0}{r}-\widehat{C}^L\Big) \geq (1-p)\frac{\mu_0-\mu_L}{r} > I_T(1+\overline{\theta}) + (I_0-\widehat{C}^L)\frac{(r+\lambda)(1-p)}{r+\lambda(1-p)}. \tag{H.40}$$

Then we derive from the above inequality that (H.39) holds.

Therefore, we proved that $m_T^{\text{Trad}} = r\frac{I_T(1+\overline{\theta})}{1-p}$, $m_0^{\text{Trad}} = r\frac{I_0(r+\lambda)-\lambda\left[I_T\overline{\theta}(1-q)+p\widehat{C}^L\right]}{r+\lambda(1-p)}$ satisfies $\mu_0 > m_0^{\text{Trad}} + m_T^{\text{Trad}}$.

H.6.4 Proof of Proposition 11

Recall that $\widetilde{P}(m_T)$ denotes the probability that the other investor accepts the offer. Fix an equilibrium in which investors and lenders anticipate the acceptance probability $\widetilde{P}(m_T) = 1$. Following the proof of Proposition 1, we only need to consider m_T that yield the accepting investor's payoff as given in (H.8), and the rejecting investor's payoff as given in (H.9). Setting $\widetilde{P}(m_T) = 1$, $\theta_i = \overline{\theta}$ in (H.8) and (H.9), we can see that a high cost investor is indifferent between accepting and rejecting if (H.8) equals (H.9), implying that equity optimally offers

$$\frac{m_T^{\text{Trad}}}{r} = \frac{I_T(1+\overline{\theta})}{1-p}. \tag{H.41}$$

Recall that we assume $\mu_0 \geq m_0^* + m_T^* > \mu_L$ and $\mu_0 \geq m_0^* > \mu_L$ and verify these inequalities later. Substituting $A(m_T) = 1$ and $\mathbb{E}[\Theta] = q\overline{\theta}$ into (H.13), we obtain

$$\begin{aligned} D_T(m_0, m_T) &= \widehat{C}^L + (1-p)\Big(\frac{m_0 + m_T}{r} - \widehat{C}^L\Big) - I_T(1 + q\overline{\theta}) \\ &= (1-p)\frac{m_0}{r} + (1-p)\frac{m_T}{r} - I_T(1 + q\overline{\theta}) + p\widehat{C}^L. \end{aligned} \tag{H.42}$$

Substituting (H.41) into (H.42), we obtain

$$D_T(m_0, m_T^{\text{Trad}}) = (1-p)\frac{m_0}{r} + I_T\overline{\theta}(1-q) + p\widehat{C}^L. \tag{H.43}$$

Substituting (H.43) into $I_0 = D_0(m_0, m_T^{\text{Trad}}) = \frac{m_0 + \lambda D_T(m_0, m_T^{\text{Trad}})}{r+\lambda}$, we obtain

$$m_0 = I_0(r+\lambda) - \lambda\left[(1-p)\frac{m_0}{r} + I_T\overline{\theta}(1-q) + p\widehat{C}^L\right]. \tag{H.44}$$

Rearranging, we obtain

$$\frac{m_0^{\text{Trad}}}{r} = \frac{I_0(r+\lambda) - \lambda\Big[I_T\overline{\theta}(1-q) + p\widehat{C}^L\Big]}{r + \lambda(1-p)}. \tag{H.45}$$

According to Lemma 7 and Lemma 9, we have $m_0^{\text{Trad}} + m_T^{\text{Trad}} < \mu_0$ and $m_0^{\text{Trad}} > \mu_L$. Since $m_T^{\text{Trad}} > 0$, we have $\mu_L < m_0^{\text{Trad}} < m_0^{\text{Trad}} + m_T^{\text{Trad}} < \mu_0$.

Now, suppose equity deviates to a lower coupon m_T^{Alt} at T such that the acceptance probability is $\widetilde{P}(m_T^{\text{Alt}}) = 1 - q$. If investor i accepts, he gets

$$\frac{(1-p)\frac{m_0^{\text{Trad}}}{r} + p\widehat{C}^L}{2} + \Big(1 - \frac{1-q}{2}\Big)\Big[(1-p)\frac{m_T}{r} - I_T(1+\theta_i)\Big]. \tag{H.46}$$

If investor i rejects, he gets

$$\frac{(1-q)(1-p)\frac{m_0^{\text{Trad}}}{r} + (q + (1-q)p)\widehat{C}^L}{2}. \tag{H.47}$$

Thus, a low cost investor is indifferent between accepting and rejecting if (H.47) is equal to (H.46) with $\theta_i = 0$, implying that equity optimally offers the following to induce $\widetilde{P}(m_T^{\text{Alt}}) = 1 - q$:

$$\frac{m_T^{\text{Alt}}}{r} = \frac{I_T}{1-p} - \frac{q}{1+q}\Big(\frac{m_0^{\text{Trad}}}{r} - \widehat{C}^L\Big). \tag{H.48}$$

We have

$$\begin{aligned}
&\frac{m_0^{\text{Trad}} + m_T^{\text{Alt}} - \mu_L}{r} = \frac{m_0^{\text{Trad}} + m_T^{\text{Alt}}}{r} - \widehat{C}^L + \widehat{C}^L - \frac{\mu_L}{r} \\
=&\frac{I_T}{1-p} + \frac{1}{1+q}\Big(\frac{m_0^{\text{Trad}}}{r} - \widehat{C}^L\Big) + \widehat{C}^L - \frac{\mu_L}{r} \\
>&\frac{I_T}{1-p} + \frac{1}{1+q}\Big(\frac{\mu_L}{r} - \widehat{C}^L\Big) + \widehat{C}^L - \frac{\mu_L}{r} \\
=&\frac{I_T}{1-p} - \frac{q}{1+q}\Big(\frac{\mu_L}{r} - \widehat{C}^L\Big) \\
=&\frac{I_T}{1-p} - \widehat{C}^L\frac{q}{1+q}\Big[\frac{1}{(1-\alpha)(1-\pi)} - 1\Big] > 0,
\end{aligned} \tag{H.49}$$

where the first inequality uses $m_0^{\text{Trad}} > \mu_L$, the last equality uses $\widehat{C}^L = (1-\alpha)(1-\pi)\mu_L/r$, the last inequality uses Condition 11.

Next, we compare the value of the date 1 equity objective under these two strategies. If $\mu_0 - m_0^{\text{Trad}} - m_T^{\text{Trad}} \geq (1-q^2)\big(\mu_0 - m_0^{\text{Trad}} - m_T^{\text{Alt}}\big)$, then it is not profitable to deviate to m_T^{Alt} when $\mu_0 - m_0^{\text{Trad}} - m_T^{\text{Alt}} \geq 0$, and it is not feasible to deviate to m_T^{Alt} when $\mu_0 - m_0^{\text{Trad}} - m_T^{\text{Alt}} < 0$. If $\mu_0 - m_0^{\text{Trad}} - m_T^{\text{Trad}} < (1-q^2)\big(\mu_0 - m_0^{\text{Trad}} - m_T^{\text{Alt}}\big)$, using $\mu_0 - m_0^{\text{Trad}} - m_T^{\text{Trad}} > 0$, we conclude that $\mu_0 - m_0^{\text{Trad}} - m_T^{\text{Alt}} > 0$, and deviating to m_T^{Alt} is feasible and profitable.

Thus, it is optimal to follow $m_T^* = m_T^{\text{Trad}}$, rather than deviating to m_T^{Alt}, if and only if

$$\mu_0 - m_0^{\text{Trad}} - m_T^{\text{Trad}} \geq (1-q^2)(\mu_0 - m_0^{\text{Trad}} - m_T^{\text{Alt}}). \tag{H.50}$$

Following the proof of Proposition 1, we can show that (H.50) is equivalent to

$$(1-p)\Big(\frac{\mu_0}{r} - \widehat{C}^L\Big) \geq \mathbb{B}^{\text{Trad}} = I_T\Big(1 + \frac{\overline{\theta}}{q^2}\Big) + \frac{1-p}{q}\Big(\frac{m_0^{\text{Trad}}}{r} - \widehat{C}^L\Big). \tag{H.51}$$

H.6.5 Proof of Proposition 12

Recall that $\widetilde{P}(m_T)$ denotes the probability that the other investor accepts the offer. Fix an equilibrium in which investors and lenders anticipate the acceptance probability $\widetilde{P}(m_T) = 1$. Following the proof of Proposition 2, we only need to consider m_T that yield the accepting investor's payoff as given in (H.10), and the rejecting investor's payoff as given in (H.11). Setting $\widetilde{P}(m_T) = 1$, $\theta_i = \overline{\theta}$ in (H.10) and (H.11), we can see that a high cost investor is indifferent between accepting and rejecting if (H.10) equals (H.11), implying that equity optimally offers

$$\frac{m_T^{\text{LME}}}{r} = \frac{I_T(1+\overline{\theta}) - p\widehat{C}^L}{1-p}. \tag{H.52}$$

Under Condition 11, we have $m_T^{\text{LME}} > 0$.

Recall that we assume $\mu_0 \geq m_0^* + m_T^* > \mu_L$ and $\mu_0 \geq m_0^* > \mu_L$ and verify these inequalities later. Substituting (H.52) into (H.42), we obtain

$$D_T(m_0, m_T^{\text{LME}}) = (1-p)\frac{m_0}{r} + I\overline{\theta}(1-q). \tag{H.53}$$

Substituting (H.53) into $I_0 = D_0(m_0, m_T^{\text{LME}}) = \frac{m_0 + \lambda D_T(m_0, m_T^{\text{LME}})}{r+\lambda}$, we obtain

$$m_0 = I_0(r+\lambda) - \lambda\left[(1-p)\frac{m_0}{r} + I_T\overline{\theta}(1-q)\right]. \tag{H.54}$$

Rearranging, we obtain

$$\frac{m_0^{\text{LME}}}{r} = \frac{I_0(r+\lambda) - \lambda I_T\overline{\theta}(1-q)}{r+\lambda(1-p)}. \tag{H.55}$$

We can see that m_0^{LME} is larger than m_0^{Trad} given in (H.18). Then we derive from Lemma 9 that $\mu_0 > m_0^{\text{LME}} + m_T^{\text{LME}}$. Then, according to Lemma 7, we have $m_0^{\text{LME}} > \mu_L$. Since $m_T^{\text{LME}} > 0$, we have $\mu_0 > m_0^{\text{LME}} + m_T^{\text{LME}} > m_0^{\text{LME}} > \mu_L$.

Now, suppose equity deviates to a lower coupon m_T^{Alt} at T such that acceptance probability

$\widetilde{P}(m_T^{\text{Alt}}) = 1 - q$. If investor i accepts, he gets

$$\frac{(1-p)\frac{m_0^{\text{LME}}}{r}}{2} + \Big(1 - \frac{1-q}{2}\Big)[p\widehat{C}^L + (1-p)\frac{m_T}{r} - I_T(1+\theta_i)]. \tag{H.56}$$

If investor i rejects, they get

$$\frac{(1-q)(1-p)\frac{m_0^{\text{LME}}}{r} + q\widehat{C}^L}{2}. \tag{H.57}$$

Thus, a low cost investor is indifferent between accepting and rejecting if (H.57) equals (H.56) with $\theta_i = 0$, implying that equity optimally offers the following to induce $\widetilde{P}(m_T^{\text{Alt}}) = 1 - q$:

$$\frac{m_T^{\text{Alt}}}{r} = \frac{I_T}{1-p} + \frac{q}{1+q}\Big(\widehat{C}^L - \frac{m_0^{\text{LME}}}{r}\Big) - \frac{p\widehat{C}^L}{(1+q)(1-p)}. \tag{H.58}$$

We have

$$\begin{aligned}
&\frac{m_0^* + m_T^{\text{Alt}} - \mu_L}{r} = \frac{m_0^* + m_T^{\text{Alt}}}{r} - \widehat{C}^L + \widehat{C}^L - \frac{\mu_L}{r} \\
=&\frac{I_T}{1-p} + \frac{1}{1+q}\Big(\frac{m_0^{\text{LME}}}{r} - \widehat{C}^L\Big) - \frac{p\widehat{C}^L}{(1+q)(1-p)} + \widehat{C}^L - \frac{\mu_L}{r} \\
>&\frac{I_T}{1-p} + \frac{1}{1+q}\Big(\frac{\mu_L}{r} - \widehat{C}^L\Big) - \frac{p\widehat{C}^L}{(1+q)(1-p)} + \widehat{C}^L - \frac{\mu_L}{r} \\
=&\frac{I_T}{1-p} - \left[\frac{p}{(1+q)(1-p)} + \frac{q}{1+q}\Big(\frac{1}{(1-\alpha)(1-\pi)} - 1\Big)\right]\widehat{C}^L \\
=&\frac{I_T}{1-p} - \left[p + (1-p)q\Big(\frac{1}{(1-\alpha)(1-\pi)} - 1\Big)\right]\frac{\widehat{C}^L}{(1+q)(1-p)} > 0,
\end{aligned} \tag{H.59}$$

where the first inequality uses $m_0^{\text{LME}} > \mu_L$, the third equality uses $\widehat{C}^L = (1-\alpha)(1-\pi)\mu_L/r$, the last inequality uses Condition 11.

Next, we compare the value of the date 1 equity objective under these two strategies. If $\mu_0 - m_0^{\text{LME}} - m_T^{\text{LME}} \geq (1-q^2)\big(\mu_0 - m_0^{\text{LME}} - m_T^{\text{Alt}}\big)$, then it is not profitable to deviate to m_T^{Alt} when $\mu_0 - m_0^{\text{LME}} - m_T^{\text{Alt}} \geq 0$, and it is not feasible to deviate to m_T^{Alt} when $\mu_0 - m_0^{\text{LME}} - m_T^{\text{Alt}} < 0$. If $\mu_0 - m_0^{\text{LME}} - m_T^{\text{LME}} < (1-q^2)\big(\mu_0 - m_0^{\text{LME}} - m_T^{\text{Alt}}\big)$, using $\mu_0 - m_0^{\text{LME}} - m_T^{\text{LME}} > 0$, we conclude that $\mu_0 - m_0^{\text{LME}} - m_T^{\text{Alt}} > 0$, and deviating to m_T^{Alt} is feasible and profitable.

Thus, it is optimal to follow $m_T^* = m_T^{\text{LME}}$, rather than deviating to m_T^{Alt}, if and only if

$$\mu_0 - m_0^{\text{LME}} - m_T^{\text{LME}} \geq (1-q^2)(\mu_0 - m_0^{\text{LME}} - m_T^{\text{Alt}}). \tag{H.60}$$

Following the proof of Proposition 2, this is equivalent to

$$(1-p)\Big(\frac{\mu_0}{r} - \widehat{C}^L\Big) \geq \mathbb{B}^{\text{LME}} = I_T\Big(1 + \frac{\overline{\theta}}{q^2}\Big) + \frac{1-p}{q}\frac{m_0^{\text{LME}}}{r} - \frac{\widehat{C}^L}{q}. \tag{H.61}$$

In addition, we have

$$\begin{aligned}\mathbb{B}^{\text{LME}} - \mathbb{B}^{\text{Trad}} &= \frac{1-p}{q}\frac{m_0^{\text{LME}} - m_0^{\text{Trad}}}{r} - \frac{p\widehat{C}^L}{q} = \frac{p\widehat{C}^L}{q}\Big[\frac{(1-p)\lambda}{r+\lambda(1-p)} - 1\Big] \\ &= -\frac{p\widehat{C}^L}{q}\frac{r}{r+\lambda(1-p)}.\end{aligned} \tag{H.62}$$

H.7 Proof of Proposition 13

Recall from Lemma 8 that ex-ante firm value with debt is given by (H.17), and it is maximized when $A(m_T) = 1$. Under the stated conditions of Proposition 13, Propositions 11 and 12 imply that $A(m_T^*) = 1$ if and only if LMEs are enabled. Thus, equity holders prefer to enable LMEs if they choose to issue debt at date 0. It remains to show that the all-equity firm value is higher than the levered firm value with no LMEs but lower than the levered firm value with LMEs.

We first consider the traditional renegotiation equilibrium (m_0^*, m_T^*) such that $A(m_T^*) = 1 - q^2$. According to Lemma 7, we have $m_0^* > \mu_L$. The following (H.48) and (H.49), we have $m_0^* + m_T^* > \mu_L$. Then we derive from (H.17) that the levered firm value with no LMEs is

$$\begin{aligned}E_0^{\text{Trad}} = (1-\pi)&\Big(\frac{\mu_0}{r+\lambda} - I_0 + \frac{\lambda}{r+\lambda}\widehat{C}^L\Big) \\ &+ \frac{\lambda(1-\pi)}{r+\lambda}\bigg[(1-q^2)\Big[(1-p)\Big(\frac{\mu_0}{r} - \widehat{C}^L\Big) - I_T\Big]\bigg]\end{aligned} \tag{H.63}$$

when $A(m_T^*) = 1 - q^2$.

For the traditional renegotiation with $A(m_T^*) = 0$, we derive from (H.17) that

$$E_0^{\text{Trad}} = (1-\pi)\Big(\frac{\mu_0}{r+\lambda} - I_0 + \frac{\lambda}{r+\lambda}\widehat{C}^L\Big). \tag{H.64}$$

The levered firm value with LMEs is

$$\begin{aligned} E_0^{\text{LME}} = {} & (1-\pi)\Big(\frac{\mu_0}{r+\lambda} - I_0 + \frac{\lambda}{r+\lambda}\widehat{C}^L\Big) \\ & + \frac{\lambda(1-\pi)}{r+\lambda}\Bigg[\Big[(1-p)\Big(\frac{\mu_0}{r} - \widehat{C}^L\Big) - I_T\Big] - I_T q\bar{\theta}\Bigg]. \end{aligned} \tag{H.65}$$

This follows from $A(m_T^{\text{LME}}) = 1$, $m_0^{\text{LME}} + m_T^{\text{LME}} > \mu_L$, and the fact that $\mathbb{E}[\Theta] = q\bar{\theta}$.

Finally, the time-zero firm value with only equity financing is given by (H.6). Thus, under the stated conditions of Proposition 13, the levered firm value is higher than the equity-only firm value if and only if LMEs are available.

I Details for Section 6

In this Appendix, we provide details related to the continuous-time model described in Section 6. To begin, we characterize the equity and debt values at time T, along with the associated optimizations.

I.1 Optimization at time T

For any coupons $m_0 > 0$ and $m_T > 0$, the equity value at time T that accounts for all possible scenarios is

$$\begin{aligned} E_T(W_T, m_0, m_T) =& (1 - A(W_T, m_T)) \left[\frac{(1-\pi)(\mu_L - m_0)}{r} + W_T \right]^+ \\ &+ A(W_T, m_T) p \left[\frac{(1-\pi)(\mu_L - m_0 - m_T)}{r} + (W_T - I_T)^+ \right]^+ \\ &+ A(W_T, m_T)(1-p) \left[\frac{(1-\pi)(\mu_G - m_0 - m_T)}{r} + (W_T - I_T)^+ \right]^+, \end{aligned} \tag{I.1}$$

where $x^+ = \max\{x, 0\}$ and $A(W_T, m_T)$ denotes the probability that an offer m_T is accepted. Note that $A(W_T, m_T)$ is same as $A(m_T)$ defined in Internet Appendix H.3 but with I_T replaced by $I_T - W_T$ when $W_T < I_T$, and $A(W_T, m_T) = 1$ when $W_T \geq I_T$. Let $m_T^*(W_T, m_0)$ denote the optimal coupon to maximize $E_T(W_T, m_0, m_T)$ given in (I.1) and let $E_T^*(W_T, m_0) = E_T(W_T, m_0, m_T^*(W_T, m_0))$ denote this optimal equity value.

Similarly, the debt value at time T that accounts for all possible scenarios is

$$\begin{aligned} &D_T(W_T, m_0, m_T) = -(I_T - W_T)^+ A(W_T, m_T) - (I_T - W_T)^+ \mathbb{E}[\Theta] \\ &+ (1 - A(W_T, m_T)) \left[\frac{m_0}{r} \mathbf{1}_{\frac{(1-\pi)(\mu_L - m_0)}{r} + W_T \geq 0} + (\widehat{C}^L + W_T) \mathbf{1}_{\frac{(1-\pi)(\mu_L - m_0)}{r} + W_T < 0} \right] \\ &+ A(W_T, m_T) p \left[\frac{m_0 + m_T}{r} \mathbf{1}_{E_T^{\text{low}} \geq 0} + (\widehat{C}^L + (W_T - I_T)^+) \mathbf{1}_{E_T^{\text{low}} < 0} \right] \\ &+ A(W_T, m_T)(1-p) \left[\frac{m_0 + m_T}{r} \mathbf{1}_{E_T^{\text{high}} \geq 0} + (\widehat{C}^L + (W_T - I_T)^+) \mathbf{1}_{E_T^{\text{high}} < 0} \right], \end{aligned} \tag{I.2}$$

where $E_T^{\text{low}} = \frac{(1-\pi)(\mu_L - m_0 - m_T)}{r} + (W_T - I_T)^+$ and $E_T^{\text{high}} = \frac{(1-\pi)(\mu_G - m_0 - m_T)}{r} + (W_T - I_T)^+$. We let

$D^*_T(W_T, m_0) = D_T(W_T, m_0, m^*_T(W_T, m_0))$.

Note that the problem facing debt holders and equity holders at time T is nearly identical to the analogous problem in Internet Appendix H. If $I_T \leq W_T$, then the choice of m_T is irrelevant since $A = 1$. If $I_T > W_T$, then the problem at time T is identical to the problem at time T in Internet Appendix H, except that I_T in that appendix is replaced with $I_T - W_T$. This is true with or without LMEs. Thus, for any fixed m_0 and W_T, we use the calculations from Internet Appendix H to calculate the relevant strategies and associated values $D^*_T(W_T, m_0)$ and $E^*_T(W_T, m_0)$.

I.2 Solving for equilibrium strategies before time T

Next, we solve the firm's optimization problem at time $t \in (0, T)$ for given values of m_0 and m^*_T, where m^*_T is the optimal solution that maximizes $E_T(W_T, m_0, m_T)$ in (I.1), and m_0 is determined by the break even condition at time 0.[53] In particular, we (1) provide a verification theorem showing that the solution to the Hamilton-Jacobi Bellman (HJB) equation is the firm value under the optimal strategy and (2) prove that our numerical algorithm converges to the relevant solution of the HJB equation.

In the following sections, we use $S = (T_{\mathcal{D}}, \{U_t, N_t, i_t; t \in (0, T \wedge T_{\mathcal{D}})\})$ to denote the strategy in the period $t \in (0, T \wedge T_{\mathcal{D}})$, where $a \wedge b = min\{a, b\}$, $T_{\mathcal{D}}$ is the firm's strategic default time,[54] U_t is a non-decreasing process representing the cumulative dividends paid to shareholders, N_t is a non-decreasing process representing the cumulative external equity issuance, and the investment strategy i_t is a deterministic function of the firm's cash W_t. We call strategy S a feasible strategy if the cash process W_t, defined by equation (22), has a unique strong solution on the interval $[0, \infty)$, and the equity value remains nonnegative at all times.

For a given coupon m_0, and a feasible strategy S, we denote the equity value at time $t < T_G \wedge T$ by $\mathcal{P}(W_t; m_0, S)$, and the equity value at time $t \in [T_G, T)$ by $\mathcal{P}_G(W_t; m_0, S)$.

[53] We focus on the case $\overline{\theta} < h_1$, i.e., the proportional cost of equity financing is higher than the proportional cost of injection at time T. Thus, firm won't issue equity at time T.

[54] It is suboptimal for the firm to strategically default with positive cash because it can make a payout. Thus $T_{\mathcal{D}} = \inf\{s : W_s = 0\}$ if firm chooses to strategically default when it runs out of cash, and $T_{\mathcal{D}} = \infty$ if firm chooses to issue equity when it runs out of cash.

I.2.1 Verification Theorem

For a given coupon m_0, and a candidate equilibrium strategy S^*, let $E_G(W_t) = \mathcal{P}_G(W_t; m_0, S^*)$ denote the equilibrium equity value at time $t \in [T_G, T)$, and let $E(W_t) = \mathcal{P}(W_t; m_0, S^*)$ denote the equilibrium equity value at time $t < T_G \wedge T$.

After T_G, the firm has no incentive to invest, so $i = 0$. By the dynamic programming principle, the equilibrium equity value $E_G(w)$ satisfies the following variational inequality[55]

$$\min\{-\mathcal{L}_G E_G(w),\ E_G'(w) - 1,\ E_G(w) - E_G^N(w)\} = 0 \tag{I.3}$$

for $w \geq 0$, where $E_G^N(w)$ is defined by

$$E_G^N(w) = \sup_{\Delta w \geq 0} E_G(w + \Delta w) - h_0 - (1 + h_1)\Delta w, \tag{I.4}$$

and the generator $\mathcal{L}_G E_G(w)$ is defined by

$$\begin{aligned} \mathcal{L}_G E_G(w) = \lambda(E_T^*(w, m_0) - E_G(w)) - rE_G(w) + \frac{(1-\pi)^2\sigma^2}{2} E_G''(w) \\ + \bigg((1-\pi)(\mu_G + rw - m_0) - \xi w \bigg) E_G'(w). \end{aligned} \tag{I.5}$$

We will prove that the equation (I.3) ensures the optimality of the strategy S^* after time T_G in Theorem 1.

By the dynamic programming principle, the equilibrium equity value $E(w)$ satisfies the following variational inequality[56]

$$\min\Big\{ -\sup_i \mathcal{L}^i E(w),\ E'(w) - 1,\ E(w) - E^N(w) \Big\} = 0 \tag{I.6}$$

for $w \geq 0$, where $E^N(w)$ is defined by

$$E^N(w) = \sup_{\Delta w \geq 0} E(w + \Delta w) - h_0 - (1 + h_1)\Delta w, \tag{I.7}$$

[55] Boundary conditions are $\lim_{w\to\infty} E_G'(w) - 1 = 0$, and $\min\big\{E_G(0) - E_G^N(0),\ E_G(0)\big\} = 0$.
[56] Boundary conditions are $\lim_{w\to\infty} E'(w) - 1 = 0$, and $\min\big\{E(0) - E^N(0),\ E(0)\big\} = 0$.

and $\mathcal{L}^i E(w)$ is defined by

$$\mathcal{L}^i E(w) = \lambda(E_T^*(w, m_0) - E(w)) - rE(w) + \frac{(1-\pi)^2\sigma^2}{2} E''(w)$$
$$+ \left((1-\pi) \left(\mu_0 + rw - m_0 - \phi i^2 \right) - \xi w \right) E'(w) + i(E_G(w) - E(w)). \qquad \text{(I.8)}$$

Compared with (I.5), (I.8) has an investment term i and a lower drift μ_0. We will prove that the equation (I.6) ensures the optimality of the strategy S^* during the period $t \in (0, T_G)$ in Theorem 1.

Next, we summarize the verification theorem.

Theorem 1. *Given a coupon m_0 and Markov strategy S^*, suppose that $E_G(w) = \mathcal{P}_G(w; m_0, S^*)$ satisfies (I.3) and $E(w) = \mathcal{P}(w; m_0, S^*)$ satisfies (I.6). Then S^* is optimal at any time $t \in (0, T)$:*

$$\mathcal{P}(W_t; m_0, S^*) \geq \mathcal{P}(W_t; m_0, S), \quad t \in (0, T_G \wedge T), \qquad \text{(I.9)}$$

$$\mathcal{P}_G(W_t; m_0, S^*) \geq \mathcal{P}_G(W_t; m_0, S), \quad t \in [T_G, T) \qquad \text{(I.10)}$$

for any feasible strategy S.

I.2.2 Numerical analysis of the variational inequality

As explained in Bolton, Chen, and Wang (2011), it is suboptimal for the firm to raise external funds when cash holdings are strictly positive. This is because (1) cash within the firm earns a below-market interest rate, i.e. $(1-\pi)r - \xi < r$, (2) the firm can always pay for any level of investment it desires using cash as long as as long as $W_t > 0$, and (3) the condition $\overline{\theta} < h_1$ makes the firm unwilling to issue equity to cover part of the investment I_T.

As a result, we only need to analyze a simplified version of equation (I.3):

$$\min\{-\mathcal{L}_G E_G(w),\ E'_G(w) - 1\} = 0, \qquad \text{(I.11)}$$

and (I.6):

$$\min\{-\sup_i \mathcal{L}^i E(w),\ E'(w) - 1\} = 0. \qquad \text{(I.12)}$$

The penalized version of the problem (I.11) is given by[57]

$$-\mathcal{L}_G E_G(w) + \min_{l\in[0,\beta]} l(E'_G(w) - 1) = 0, \tag{I.13}$$

and the penalized version of the problem (I.12) is given by[58]

$$-\sup_{i\in[0,\beta]} \mathcal{L}^i E(w) + \min_{l\in[0,\beta]} l(E'(w) - 1) = 0, \tag{I.14}$$

where $\beta > 0$ is an arbitrary large "penalty" constant that ensures the variational inequalities hold.

Let $\mathbb{W} = \{w_0, w_1, \cdots, w_n\}$ denote the discretized domain of computation, where $w_j = j\Delta w$, $j = 0, 1, \cdots, n$, $\Delta w > 0$ is a small number, and n is a sufficiently large integer. Let $\mathcal{M}^l_G(w, \widetilde{w})$ denote the coefficient of $E_G(\widetilde{w})$ in the discretization of $-\mathcal{L}_G E_G(w) + l(E'_G(w) - 1)$,[59] and let $\mathcal{M}^{i,l}(w, \widetilde{w})$ denote the coefficient of $E(\widetilde{w})$ in the discretization of $-\mathcal{L}^i E(w) + l(E'(w) - 1)$,[60] where $w, \widetilde{w} \in \mathbb{W}$.

To solve our model numerically, we first fix a value m_0. Then, we compute $D^*_T(W_T, m_0)$ and $E^*_T(W_T, m_0)$ for each possible value of W_T on the discretized grid described above. Next, we follow the policy iteration algorithms described below to calculate the equity and debt value functions associated with m_0. We repeat this for a grid of potential m_0 values. Finally, we search along this grid numerically to find the optimal initial capital structure such that (1) $m^*_0(D_0)$ satisfies the break even condition described below and (2) ex-ante firm value $E(D_0 - I_0; m^*_0(D_0))$ is maximized.

We now describe the policy iteration algorithms. We then provide proofs that these numerical solutions converge to the relevant solutions of the variational inequalities. Next, we describe the debt calculations, then we conclude with proofs of the convergence and verification theorems.

For a fixed m_0, and the associated time T value functions $D^*_T(w, m_0)$ and $E^*_T(w, m_0)$, our policy iteration algorithms entail the following steps:

[57] Boundary conditions are $\lim_{w\to\infty} E'_G(w) - 1 = 0$, and $\min_{l\in[0,\beta]} E_G(0) + l[E_G(0) - E^N_G(0)] = 0$.

[58] Boundary conditions are $\lim_{w\to\infty} E'(w) - 1 = 0$, and $\min_{l\in[0,\beta]} E(0) + l[E(0) - E^N(0)] = 0$.

[59] When $w = w_n$, $\mathcal{M}^l_G(w, \widetilde{w})$ denotes the coefficient of $E_G(\widetilde{w})$ in the discretization of $E'_G(w) - 1$. When $w = 0$, $\mathcal{M}^l_G(w, \widetilde{w})$ denotes the coefficient of $E_G(\widetilde{w})$ in the discretization of $E_G(0) + l[E_G(0) - E^N_G(0)]$.

[60] When $w = w_n$, $\mathcal{M}^{i,l}(w, \widetilde{w})$ denotes the coefficient of $E(\widetilde{w})$ in the discretization of $E'(w) - 1$. When $w = 0$, $\mathcal{M}^{i,l}(w, \widetilde{w})$ denotes the coefficient of $E(\widetilde{w})$ in the discretization of $E(0) + l[E(0) - E^N(0)]$.

Policy iteration algorithm for solving $E_G(w)$.

1. Initialize $l_0(w) \in [0, \beta]$ for any $w \in \mathbb{W}$.

2. Compute $V_k(w)$ for any $w \in \mathbb{W}$, $k = 0, 1, 2 \cdots$, such that

$$\sum_{\widetilde{w} \in \mathbb{W}} \mathcal{M}_G^{l_k(w)}(w, \widetilde{w}) V_k(\widetilde{w}) = 0, \quad \forall\ w \in \mathbb{W}. \tag{I.15}$$

3. For each $w \in \mathbb{W}$, compute $l_{k+1}(w)$ according to

$$l_{k+1}(w) \in \arg \min_{l \in [0,\beta]} \sum_{\widetilde{w} \in \mathbb{W}} \mathcal{M}_G^{l}(w, \widetilde{w}) V_k(\widetilde{w}). \tag{I.16}$$

4. Repeat steps 2 and 3 until $|V_k(w) - V_{k+1}(w)|$ is sufficiently small for any $w \in \mathbb{W}$.

5. Set $E_G(w) = V_{k+1}(w)$ as the numerical solution.

Policy iteration algorithm for solving $E(w)$.

1. Initialize investment $i_0(w) \in [0, \beta]$ and $l_0(w) \in [0, \beta]$ for any $w \in \mathbb{W}$.

2. Compute $V_k(w)$ for any $w \in \mathbb{W}$, $k = 0, 1, 2 \cdots$, such that

$$\sum_{\widetilde{w} \in \mathbb{W}} \mathcal{M}^{i_k(w), l_k(w)}(w, \widetilde{w}) V_k(\widetilde{w}) = 0, \quad \forall\ w \in \mathbb{W}. \tag{I.17}$$

3. For each $w \in \mathbb{W}$, compute $i_{k+1}(w)$ and $l_{k+1}(w)$ according to

$$(i_{k+1}(w), l_{k+1}(w)) \in \arg \min_{(i,l) \in [0,\beta] \times [0,\beta]} \sum_{\widetilde{w} \in \mathbb{W}} \mathcal{M}^{i,l}(w, \widetilde{w}) V_k(\widetilde{w}). \tag{I.18}$$

4. Repeat steps 2 and 3 until $|V_k(w) - V_{k+1}(w)|$ is sufficiently small for any $w \in \mathbb{W}$.

5. Set $E(w) = V_{k+1}(w)$ as the numerical solution.

Next, we summarize the comparison theorems for equations (I.11) and (I.12). As we formalize below, these comparison theorems imply that our numerical algorithms converge to the viscosity solutions of the associated variational inequalities.

Theorem 2. *Let $u(w)$ be an upper semicontinuous, viscosity subsolution of* (I.11)*, and let $v(w)$ be a lower semicontinuous, viscosity supersolution of* (I.11)*. Assume that there exists a constant $M > 0$ such that $u(w)$ and $v(w)$ take values in $[w, w + M]$. Then $u(w) \leq v(w)$ for any $w \geq 0$.*

Theorem 3. *Let $u(w)$ be an upper semicontinuous, viscosity subsolution of* (I.12)*, and let $v(w)$ be a lower semicontinuous, viscosity supersolution of* (I.12)*. Assume that there exists a constant $M > 0$ such that $u(w)$ and $v(w)$ take values in $[w, w + M]$. Then $u(w) \leq v(w)$ for any $w \geq 0$.*

According to Barles and Souganidis (1991), the convergence of the numerical schemes is the result of the above comparison theorems.

Corollary 3. *The numerical scheme* (I.15)-(I.16) *converges to the viscosity solution to* (I.11) *as the discretization gets finer. The numerical scheme* (I.17)-(I.18) *converges to the viscosity solution to* (I.12) *as the discretization gets finer.*

I.2.3 Debt values

Finally, we determine the coupon m_0 by using the break even condition for the debt value. Let $D_G(W_t)$ denote the equilibrium debt value at time $t \in [T_G, T)$, and let $D(W_t)$ denote the equilibrium debt value at time $t < T_G \wedge T$.

By the dynamic programming principle, the debt value $D_G(w)$ satisfies the following equation[61]

$$
\begin{aligned}
&rD_G(w) = m_0 + \lambda(D_T^*(W_T, m_0) - D_G(w)) + \frac{(1-\pi)^2\sigma^2}{2} D_G''(w) \\
&+ \bigg((1-\pi)(\mu_G + rw - m_0) - \xi w \bigg) D_G'(w),
\end{aligned}
\tag{I.19}
$$

and the debt value $D(w)$ satisfies the following equation[62]

$$
\begin{aligned}
&rD(w) = m_0 + \lambda(D_T^*(W_T, m_0) - D(w)) + \frac{(1-\pi)^2\sigma^2}{2} D''(w) \\
&+ \bigg((1-\pi)(\mu_0 + rw - m_0 - \phi i^*(w)^2) - \xi w \bigg) D'(w) + i^*(w)(D_G(w) - D(w)),
\end{aligned}
\tag{I.20}
$$

[61]Boundary conditions are $\lim_{w\to\infty} D_G'(w) = 0$, and $D_G(0) = \widehat{C}^L \mathbf{1}_{E_G(0)<E_G^N(0)} + D_G(w_G^*)\mathbf{1}_{E_G(0)=E_G^N(0)}$, where $w_G^* = \arg\sup_{\Delta w \geq 0} E_G(\Delta w) - h_0 - (1+h_1)\Delta w$.

[62]Boundary conditions are $\lim_{w\to\infty} D'(w) = 0$, and $D(0) = \widehat{C}^L \mathbf{1}_{E(0)<E^N(0)} + D(w^*)\mathbf{1}_{E(0)=E^N(0)}$, where $w^* = \arg\sup_{\Delta w \geq 0} E(\Delta w) - h_0 - (1+h_1)\Delta w$.

where $i^*(w) = \arg\max_{i \geq 0} \mathcal{L}^i E(w)$.

The comparison theorems for equations (I.19) and (I.20) can also be established, analogous to Theorems 2 and 3.

The coupon $m_0 = m_0^*(D_0)$ is determined by the break even condition $D_0 = D(W_0; m_0) = W_0 + I_0$. Finally, at time 0, equity holders choose $D_0 \geq I_0$ to maximize ex-ante firm value $E(D_0 - I_0; m_0^*(D_0))$.

I.3 Proofs

I.3.1 Proof of Theorem 1

Let $S = (T_{\mathcal{D}}, \{U_t, N_t, i_t; t \in (0, T \wedge T_{\mathcal{D}})\})$ be a feasible strategy. Since U_t and N_t are nondecreasing processes, we have the following decomposition:

$$U_t = U_t^c + \sum_{s \in [0,t]} \Delta U_s,$$

$$N_t = N_t^c + \sum_{s \in [0,t]} \Delta N_s,$$

where U_t^c denotes the continuous component of U_t, $\Delta U_s := U_s - U_{s-}$ denotes the jump of U_s, N_t^c denotes the continuous component of N_t, and $\Delta N_s := N_s - N_{s-}$ denotes the jump of N_s. Then we have following decomposition for W_t:

$$W_t = W_t^c + \sum_{s \in [0,t]} \Delta W_s,$$

where W_t^c denotes the continuous component of W_t, $\Delta W_s := W_s - W_{s-} = \Delta N_s - \Delta U_s$ denotes the jump of W_s.

The presence of equity financing costs makes it suboptimal for the firm to simultaneously make a payout and issue equity. Thus, $\Delta U_t \Delta N_t = 0$ for all t.[63] In addition, it is suboptimal for the firm to strategically default with positive cash because it can pay all cash as divident before its default.

[63]If $\Delta U_t \geq \Delta N_t$, then it is better for the firm to payout $\Delta U_t - \Delta N_t$ without issuing equity. If $\Delta U_t < \Delta N_t$, then it is better for the firm to issue equity $\Delta N_t - \Delta U_t$ without marking payout. Therefore, we only need to consider strategy S in which payout and equity financing do not occur at the same time.

Thus, $T_{\mathcal{D}} = \inf\{s : W_s = 0\}$ if firm chooses to strategically default when it runs out of cash, and $T_{\mathcal{D}} = \infty$ if firm chooses to issue equity when it runs out of cash.

We first show that

$$E_G(w) \geq \mathcal{P}_G(w; m_0, S), \quad \forall\, w \geq 0. \tag{I.21}$$

Note that firm has no incentive to invest after time T_G, so $i_t = 0$ for $t \geq T_G$. Using the generalized Ito's formula, we can write:

$$\begin{aligned}
& dE_G(W_t) \\
=& \left[\Big((1-\pi)(\mu_G + rW_t - m_0) - \xi W_t\Big) E'_G(W_t) + \frac{(1-\pi)^2\sigma^2}{2} E''_G(W_t)\right] dt \\
& + (1-\pi)\sigma E'_G(W_t) dB_t + E'_G(W_t)(dN_t^{\mathrm{c}} - dU_t^{\mathrm{c}}) + E_G(W_{t-} + \Delta W_t) - E_G(W_{t-}) \\
=& \Bigg[\Big((1-\pi)(\mu_G + rW_t - m_0) - \xi W_t\Big) E'_G(W_t) + \frac{(1-\pi)^2\sigma^2}{2} E''_G(W_t) \\
& + \lambda\left(E_T^*(W_t, m_0) - E_G(W_t)\right)\Bigg] dt - \lambda\left(E_T^*(W_t, m_0) - E_G(W_t)\right) dt \\
& + (1-\pi)\sigma E'_G(W_t) dB_t + E'_G(W_t)(dN_t^{\mathrm{c}} - dU_t^{\mathrm{c}}) + E_G(W_{t-} + \Delta W_t) - E_G(W_{t-}).
\end{aligned}$$

We derive from above that

$$\begin{aligned}
& d\left(e^{-rt} E_G(W_t)\right) = e^{-rt} \mathcal{L}_G E_G(W_t) dt - \lambda e^{-rt}\left(E_T^*(W_t, m_0) - E_G(W_t)\right) dt \\
& + e^{-rt}\Big[(1-\pi)\sigma E'_G(W_t) dB_t + E'_G(W_t)(dN_t^{\mathrm{c}} - dU_t^{\mathrm{c}}) + E_G(W_{t-} + \Delta W_t) - E_G(W_{t-})\Big],
\end{aligned} \tag{I.22}$$

where $\mathcal{L}_G E_G(w)$ is given in (I.5).

Integrating the equation (I.22) from t to $\tau := T_{\mathcal{D}} \wedge T$ and using $\Delta W_t = -\Delta U$ or ΔN, we obtain

$$
\begin{aligned}
& e^{-r(\tau-t)} E_G(W_\tau) + \lambda \int_t^\tau e^{-r(s-t)} \left[E_T^*(W_s, m_0) - E_G(W_s)\right] ds \\
=& E_G(W_{t-}) + \int_t^\tau e^{-r(s-t)} \mathcal{L}_G E_G(W_s) ds + \int_t^\tau e^{-r(s-t)} (1-\pi)\sigma E_G'(W_s) dB_s \\
&+ \int_t^\tau e^{-r(s-t)} E_G'(W_s)(-dU^{\mathrm{c}}) + \sum_{t \le s < \tau} e^{-r(s-t)} \big(E_G(W_{s-} - \Delta U_s) - E_G(W_{s-})\big) \\
&+ \int_t^\tau e^{-r(s-t)} E_G'(W_s) dN^{\mathrm{c}} + \sum_{t \le s < \tau} e^{-r(s-t)} \big(E_G(W_{s-} + \Delta N_s) - E_G(W_{s-})\big).
\end{aligned} \tag{I.23}
$$

Because $\lim_{w\to\infty} E_G'(w) - 1 = 0$ and $E_G(w)$ is $\mathcal{C}^1$ in $w \ge 0$, we can see that $E_G'(w)$ is bounded in $w \ge 0$. Since $E_G'(w)$ is bounded, the third term in the right of (I.23), the integral with respect dB_s, is a square integrable martingale. Taking conditional expectations at time t in (I.23), we obtain:

$$
\begin{aligned}
E_G(W_{t-}) =& \mathbb{E}_t\Big[e^{-r(\tau-t)} E_G(W_\tau) + \lambda \int_t^\tau e^{-r(s-t)} \left[E_T^*(W_s, m_0) - E_G(W_s)\right] ds\Big] \\
&- \mathbb{E}_t\left[\int_t^\tau e^{-r(s-t)} \mathcal{L}_G E_G(W_s) ds\right] \\
&+ \mathbb{E}_t\left[\int_t^\tau e^{-r(s-t)} E_G'(W_s) dU_s^{\mathrm{c}} + \sum_{t \le s < \tau} e^{-r(s-t)} \big(E_G(W_{s-}) - E_G(W_{s-} - \Delta U_s)\big)\right] \\
&+ \mathbb{E}_t\left[-\int_t^\tau e^{-r(s-t)} E_G'(W_s) dN_s^{\mathrm{c}} - \sum_{t \le s < \tau} e^{-r(s-t)} \big(E_G(W_{s-} + \Delta N_s) - E_G(W_{s-})\big)\right].
\end{aligned} \tag{I.24}
$$

Since T is the first arrival time of a Poisson process with the arrival rate λ, we have

$$
\begin{aligned}
& \mathbb{E}_t\Big[\lambda \int_t^\tau e^{-r(s-t)} \left[E_T^*(W_s, m_0) - E_G(W_s)\right] ds\Big] \\
=& \mathbb{E}_t\Big[\lambda \int_t^\infty \int_t^{T_{\mathcal{D}} \wedge u} e^{-r(s-t)} \left[E_T^*(W_s, m_0) - E_G(W_s)\right] ds \lambda e^{-\lambda(u-t)} du\Big] \\
=& \mathbb{E}_t\Big[\lambda \int_t^{T_{\mathcal{D}}} \int_s^\infty \lambda e^{-\lambda(u-t)} du e^{-r(s-t)} \left[E_T^*(W_s, m_0) - E_G(W_s)\right] ds\Big] \\
=& \mathbb{E}_t\Big[\lambda \int_t^{T_{\mathcal{D}}} e^{-(r+\lambda)(s-t)} \left[E_T^*(W_s, m_0) - E_G(W_s)\right] ds\Big] \\
=& \mathbb{E}_t\Big[\mathbf{1}_{T<T_{\mathcal{D}}} e^{-r(T-t)} \left[E_T^*(W_T, m_0) - E_G(W_T)\right]\Big],
\end{aligned} \tag{I.25}
$$

where the last equality uses $T \neq T_{\mathcal{D}}$ almost surely. It follows that

$$
\begin{aligned}
&\mathbb{E}_t\Big[e^{-r(\tau-t)}E_G(W_\tau)+\lambda\int_t^\tau e^{-r(s-t)}\left[E_T^*(W_s,m_0)-E_G(W_s)\right]ds\Big]\\
=&\mathbb{E}_t\Big[\mathbf{1}_{T>T_{\mathcal{D}}}e^{-r(T_{\mathcal{D}}-t)}E_G(W_\tau)+\mathbf{1}_{T<T_{\mathcal{D}}}e^{-r(T-t)}E_T^*(W_T,m_0)\Big].
\end{aligned}
\tag{I.26}
$$

Substituting (I.26) into (I.24), we obtain:

$$
\begin{aligned}
&E_G(W_{t-})=\mathbb{E}_t\Big[\mathbf{1}_{T>T_{\mathcal{D}}}e^{-r(T_{\mathcal{D}}-t)}E_G(W_\tau)+\mathbf{1}_{T<T_{\mathcal{D}}}e^{-r(T-t)}E_T^*(W_T,m_0)\Big]\\
&-\mathbb{E}_t\left[\int_t^\tau e^{-r(s-t)}\mathcal{L}_G E_G(W_s)ds\right]\\
&+\mathbb{E}_t\left[\int_t^\tau e^{-r(s-t)}E_G'(W_s)dU_s^{\mathrm{c}}+\sum_{t\le s<\tau}e^{-r(s-t)}\big(E_G(W_{s-})-E_G(W_{s-}-\Delta U_s)\big)\right]\\
&+\mathbb{E}_t\left[-\int_t^\tau e^{-r(s-t)}E_G'(W_s)dN_s^{\mathrm{c}}-\sum_{t\le s<\tau}e^{-r(s-t)}\big(E_G(W_{s-}+\Delta N_s)-E_G(W_{s-})\big)\right].
\end{aligned}
\tag{I.27}
$$

Using $E_G'(w)\ge 1$ for any $w\ge 0$, we conclude that

$$
\begin{aligned}
&\mathbb{E}_t\left[\int_t^\tau e^{-r(s-t)}E_G'(W_s)dU_s^{\mathrm{c}}+\sum_{t\le s<\tau}e^{-r(s-t)}\big(E_G(W_{s-})-E_G(W_{s-}-\Delta U_s)\big)\right]\\
\ge&\mathbb{E}_t\left[\int_t^\tau e^{-r(s-t)}dU_s^{\mathrm{c}}+\sum_{t\le s<\tau}e^{-r(s-t)}\Delta U_s\right]=\mathbb{E}_t\left[\int_{t\le s<\tau}e^{-r(s-t)}dU_s\right].
\end{aligned}
\tag{I.28}
$$

Let H_t^{c} denote the continuous component of the equity financing cost H_t, and let $\Delta H_t := H_t - H_{t-}$ denote the jump in H_t. Using $E_G(w)\ge E_G^N(w)$, we have $E_G(W_{s-})\ge E_G(W_{s-}+\Delta N_s) - h_0-(1+h_1)\Delta N_s = E_G(W_{s-}+\Delta N_s)-\Delta N_s-\Delta H_s$. It follows that

$$
E_G(W_{s-})-E_G(W_{s-}+\Delta N_s)\ge -(\Delta N_s+\Delta H_s).
\tag{I.29}
$$

In the case $h_0>0$, we must have $dN_s^{\mathrm{c}}=0$ for almost every s. Otherwise, the equity issuance costs are infinite, which leads to negative equity value. In the case $h_0=0$, we have $E_G(w)\ge E_G^N(w)\ge E_G(w+\varepsilon)-(1+h_1)\varepsilon$ for any small $\varepsilon>0$, which yields that $E_G'(w)\le 1+h_1$ for $w\ge 0$. Hence, we have

$$
\begin{aligned}
&\mathbb{E}_t\left[-\int_t^{\tau} e^{-r(s-t)}E_G'(W_s)dN_s^{\mathrm{c}} - \sum_{t\le s<\tau} e^{-r(s-t)}\big(E_G(W_{s-}+\Delta N_s)-E_G(W_{s-})\big)\right]\\
\ge&\mathbb{E}_t\left[\mathbf{1}_{h_0=0}\int_t^{\tau} e^{-r(s-t)}(-1-h_1)dN_s^{\mathrm{c}} - \sum_{t\le s<\tau} e^{-r(s-t)}\big(\Delta N_s+\Delta H_s\big)\right]\\
=&\mathbb{E}_t\left[\mathbf{1}_{h_0=0}\int_t^{T_{\mathcal{D}}} e^{-r(s-t)}(-dN_s^{\mathrm{c}}-dH_s^{\mathrm{c}}) - \sum_{t\le s<\tau} e^{-r(s-t)}\big(\Delta N_s+\Delta H_s\big)\right]\\
=&-\mathbb{E}_t\left[\int_{t\le s<\tau} e^{-r(s-t)}(dN_s+dH_s)\right],
\end{aligned}
\tag{I.30}
$$

where the inequality uses (I.29), and $E_G'(w)\le 1+h_1$ for $w\ge 0$.

Using boundary condition $\min\{E_G(0)-E_G^N(0),\ E_G(0)\}=0$, we have $E_G(0)\ge 0$. Combining it with $E_G'(w)\ge 1$, we have $E_G(w)\ge w$. Substituting (I.28) and (I.30) into (I.27), and using $E_G(w)\ge w\ge 0$, $\mathcal{L}_G E_G(w)\le 0$ for any $w\ge 0$, we obtain

$$
\begin{aligned}
E_G(W_{t-})\ge&\mathbb{E}_t\Big[\mathbf{1}_{T<T_{\mathcal{D}}}e^{-r(T-t)}E_T^*(W_T,m_0)\Big]+\mathbb{E}_t\left[\int_{t\le s<\tau} e^{-r(s-t)}(dU_s-dN_s-dH_s)\right]\\
=&\mathcal{P}_G(W_{t-};m_0,S),
\end{aligned}
\tag{I.31}
$$

where the equality uses $T_{\mathcal{D}}=\inf\{s: W_s=0\}$ or $T_{\mathcal{D}}=\infty$. Thus, (I.21) holds.

Next, we prove

$$
E(w)\ge \mathcal{P}(w;m_0,S),\quad \forall\ w\ge 0. \tag{I.32}
$$

Similar to (I.22), using the generalized Ito's formula, we have

$$
\begin{aligned}
&d\left(e^{-rt}E(W_t)\right)=e^{-rt}\mathcal{L}^{i_t}E(W_t)dt\\
&-\lambda e^{-rt}\left(E_T^*(W_t,m_0)-E(W_t)\right)dt - i_t e^{-rt}\left(E_G(W_t)-E(W_t)\right)dt\\
&+e^{-rt}\Big[(1-\pi)\sigma E'(W_t)dB_t+E'(W_t)(dN_t^{\mathrm{c}}-dU_t^{\mathrm{c}})+E(W_{t-}+\Delta W_t)-E(W_{t-})\Big],
\end{aligned}
\tag{I.33}
$$

where $\mathcal{L}^i E(w)$ is defined in (I.8), $i_t=i^*(W_t)$, and $i^*(w)$ is the value of i that maximizes $\mathcal{L}^i E(w)$.

Integrating the equation (I.33) from t to $\widetilde{\tau} := T_G \wedge T_{\mathcal{D}} \wedge T$ and using $\Delta W_t = -\Delta U$ or ΔN, we obtain

$$\begin{aligned}
& e^{-r(\widetilde{\tau}-t)} E(W_{\widetilde{\tau}}) + \lambda \int_t^{\widetilde{\tau}} e^{-r(s-t)} \left[E_T^*(W_s, m_0) - E(W_s) \right] ds + \int_t^{\widetilde{\tau}} e^{-r(s-t)} i_s \left[E_G(W_s) - E(W_s) \right] ds \\
=& E(W_{t-}) + \int_t^{\widetilde{\tau}} e^{-r(s-t)} \mathcal{L}^{i_s} E(W_s) ds + \int_t^{\widetilde{\tau}} e^{-r(s-t)} (1-\pi)\sigma E'(W_s) dB_s \\
& + \int_t^{\widetilde{\tau}} e^{-r(s-t)} E'(W_s)(-dU^{\mathrm{c}}) + \sum_{t \le s < \widetilde{\tau}} e^{-r(s-t)} \big(E(W_{s-} - \Delta U_s) - E(W_{s-}) \big) \\
& + \int_t^{\widetilde{\tau}} e^{-r(s-t)} E'(W_s) dN^{\mathrm{c}} + \sum_{t \le s < \widetilde{\tau}} e^{-r(s-t)} \big(E(W_{s-} + \Delta N_s) - E(W_{s-}) \big).
\end{aligned} \tag{I.34}$$

Replacing $T_{\mathcal{D}}$ with $T_G \wedge T_{\mathcal{D}}$ in (I.25), we obtain

$$\begin{aligned}
& \mathbb{E}_t \Big[\lambda \int_t^{\widetilde{\tau}} e^{-r(s-t)} \left[E_T^*(W_s, m_0) - E(W_s) \right] ds \Big] \\
=& \mathbb{E}_t \Big[\mathbf{1}_{T < T_G \wedge T_{\mathcal{D}}} e^{-r(T-t)} \left[E_T^*(W_T, m_0) - E(W_T) \right] \Big].
\end{aligned} \tag{I.35}$$

Similarly, we have

$$\begin{aligned}
& \mathbb{E}_t \Big[\int_t^{\widetilde{\tau}} e^{-r(s-t)} i_s \left[E_G(W_s) - E(W_s) \right] ds \Big] \\
=& \mathbb{E}_t \Big[\int_t^{\infty} \int_t^{T_{\mathcal{D}} \wedge T \wedge u} e^{-r(s-t)} i_s \left[E_G(W_s) - E(W_s) \right] ds \ i_u e^{-\int_t^u i_z dz} du \Big] \\
=& \mathbb{E}_t \Big[\int_t^{T_{\mathcal{D}} \wedge T} \int_s^{\infty} i_u e^{-\int_t^u i_z dz} du e^{-r(s-t)} \left[E_G(W_s) - E(W_s) \right] i_s ds \Big] \\
=& \mathbb{E}_t \Big[\int_t^{T_{\mathcal{D}} \wedge T} e^{-\int_t^s (r+i_z) dz} \left[E_G(W_s) - E(W_s) \right] i_s ds \Big] \\
=& \mathbb{E}_t \Big[\mathbf{1}_{T_G < T_{\mathcal{D}} \wedge T} e^{-r(T_G-t)} \left[E_G(W_{T_G}) - E(W_{T_G}) \right] \Big].
\end{aligned} \tag{I.36}$$

Combining (I.35) and (I.36), we obtain

$$\begin{aligned}
& \mathbb{E}_t \Big[e^{-r(\widetilde{\tau}-t)} E(W_{\widetilde{\tau}}) + \lambda \int_t^{\widetilde{\tau}} e^{-r(s-t)} \left[E_T^*(W_s, m_0) - E(W_s) \right] ds + \int_t^{\widetilde{\tau}} e^{-r(s-t)} i_s \left[E_G(W_s) - E(W_s) \right] ds \Big] \\
=& \ \mathbb{E}_t \Bigg[e^{-r(\widetilde{\tau}-t)} \Big(\mathbf{1}_{T_{\mathcal{D}} < T_G \wedge T} E(W_{T_{\mathcal{D}}}) + \mathbf{1}_{T < T_{\mathcal{D}} \wedge T_G} E_T^*(W_T, m_0) + \mathbf{1}_{T_G < T_{\mathcal{D}} \wedge T} E_G(W_{T_G}) \Big) \Bigg].
\end{aligned} \tag{I.37}$$

Taking expectations in (I.34) and using (I.37), we obtain:

$$
\begin{aligned}
E(W_{t-}) = {} & \mathbb{E}_t\left[e^{-r(\widetilde{\tau}-t)}\Big(\mathbf{1}_{T_{\mathcal{D}}<T_G\wedge T}E(W_{T_{\mathcal{D}}}) + \mathbf{1}_{T<T_{\mathcal{D}}\wedge T_G}E^*_T(W_T, m_0) + \mathbf{1}_{T_G<T_{\mathcal{D}}\wedge T}E_G(W_{T_G})\Big)\right] \\
& - \mathbb{E}_t\left[\int_t^{\tau} e^{-r(s-t)}\mathcal{L}^{i_s}E(W_s)ds\right] \\
& + \mathbb{E}_t\left[\int_t^{\widetilde{\tau}} e^{-r(s-t)}E'(W_s)dU^{\mathrm{c}}_s + \sum_{t\le s<\widetilde{\tau}} e^{-r(s-t)}\big(E(W_{s-}) - E(W_{s-}-\Delta U_s)\big)\right] \\
& + \mathbb{E}_t\left[-\int_t^{\widetilde{\tau}} e^{-r(s-t)}E'(W_s)dN^{\mathrm{c}}_s - \sum_{t\le s<\widetilde{\tau}} e^{-r(s-t)}\big(E(W_{s-}+\Delta N_s) - E(W_{s-})\big)\right]. \qquad \text{(I.38)}
\end{aligned}
$$

Similar to (I.28), we have

$$
\begin{aligned}
& \mathbb{E}_t\left[\int_t^{\widetilde{\tau}} e^{-r(s-t)}E'(W_s)dU^{\mathrm{c}}_s + \sum_{t\le s<\widetilde{\tau}} e^{-r(s-t)}\big(E(W_{s-}) - E(W_{s-}-\Delta U_s)\big)\right] \\
\ge & \mathbb{E}_t\left[\int_{t\le s<\widetilde{\tau}} e^{-r(s-t)}dU_s\right]. \qquad \text{(I.39)}
\end{aligned}
$$

Similar to (I.30), we have

$$
\begin{aligned}
& \mathbb{E}_t\left[-\int_t^{\widetilde{\tau}} e^{-r(s-t)}E'(W_s)dN^{\mathrm{c}}_s - \sum_{t\le s<\widetilde{\tau}} e^{-r(s-t)}\big(E(W_{s-}+\Delta N_s) - E(W_{s-})\big)\right] \\
\ge & -\mathbb{E}_t\left[\int_{t\le s<\widetilde{\tau}} e^{-r(s-t)}(dN_s + dH_s)\right]. \qquad \text{(I.40)}
\end{aligned}
$$

Using the boundary condition $\min\{E(0) - E^N(0),\ E(0)\} = 0$, we have $E(0) \ge 0$. Combining it with $E'(w) \ge 1$, we have $E(w) \ge w$. Substituting (I.39)-(I.40) into (I.38), and using $E(w) \ge w \ge 0$, $\mathcal{L}^i E(w) \le \max_i \mathcal{L}^i E(w) \le 0$ for any $w \ge 0$, we obtain

$$
\begin{aligned}
E(W_{t-}) \ge & \mathbb{E}_t\left[e^{-r(\widetilde{\tau}-t)}\Big(\mathbf{1}_{T<T_{\mathcal{D}}\wedge T_G}E^*_T(W_T, m_0) + \mathbf{1}_{T_G<T_{\mathcal{D}}\wedge T}E_G(W_{T_G})\Big)\right] \\
& + \mathbb{E}_t\left[\int_{t\le s<\widetilde{\tau}} e^{-r(s-t)}(dU_s - dN_s - dH_s)\right]. \qquad \text{(I.41)}
\end{aligned}
$$

Substituting (I.21) into above inequality, we obtain

$$
\begin{aligned}
E(W_{t-}) \geq & \mathbb{E}_t\left[e^{-r(\widetilde{\tau}-t)}\Big(\mathbf{1}_{T<T_{\mathcal{D}}\wedge T_G} E_T^*(W_T, m_0) + \mathbf{1}_{T_G<T_{\mathcal{D}}\wedge T}\mathcal{P}_G(W_{T_G}; m_0, S)\Big)\right] \\
& + \mathbb{E}_t\left[\int_{t\leq s<\widetilde{\tau}} e^{-r(s-t)}(dU_s - dN_s - dH_s)\right] \\
= & \mathcal{P}(W_{t-}; m_0, S),
\end{aligned}
\tag{I.42}
$$

Therefore, (I.32) holds.

I.3.2 Proof of Theorems 2-3

In the following proof, we denote $\widetilde{u}(w) = e^{-\eta g(w)}u(w)$ and $\widetilde{v}(w) = e^{-\eta g(w)}v(w)$, where $\eta > 0$ is a constant, and the function $g(w)$ is smooth and satisfies $g(w) > 1$, $0 < g'(w) \leq M/(1+w)$, $|g''(w)| \leq M$, $w > 0$, for some constant $M > 0$. We denote $\widetilde{u}^N(w) = sup_{\Delta w\geq 0}\widetilde{u}(w+\Delta w) - h_0 - (1+h_1)\Delta w$, and $\widetilde{v}^N(w) = sup_{\Delta w\geq 0}\widetilde{v}(w+\Delta w) - h_0 - (1+h_1)\Delta w$.

For the sake of contradiction, assume that $\widetilde{u}(w) > \widetilde{v}(w)$ for some $w \geq 0$. Then for any small $\zeta > 0$, there exists a point $\widetilde{w} \geq 0$ and $\delta_\zeta > 0$ such that $\widetilde{u}(\widetilde{w}) - \widetilde{v}(\widetilde{w}) > \delta_\zeta \geq \sup_{w\geq 0}\widetilde{u}(w) - \widetilde{v}(w) - \zeta > 0$.

We begin by showing that if $\widetilde{w} = 0$, then there is another point with the same property on the interior. Consider the case $\widetilde{w} = 0$. Using the boundary condition of the viscosity subsolution, we have $\min\{\widetilde{u}(0) - \widetilde{u}^N(0), \widetilde{u}(0)\} \leq 0$, which implies $\widetilde{u}(0) \leq 0$ or $\widetilde{u}(0) \leq \widetilde{u}^N(0)$. Using the boundary condition of the viscosity supersolution, we have $\min\{\widetilde{v}(0) - \widetilde{v}^N(0), \widetilde{v}(0)\} \geq 0$, which implies $\widetilde{v}(0) \geq \widetilde{v}^N(0)$ and $\widetilde{v}(0) \geq 0$. When $\widetilde{u}(0) \leq 0$, we have $\widetilde{u}(0) - \widetilde{v}(0) \leq 0$, which contradicts $\widetilde{u}(0) - \widetilde{v}(0) > \delta_\zeta > 0$. When $\widetilde{u}(0) \leq \widetilde{u}^N(0)$, we have $\widetilde{u}^N(0) = sup_{\Delta w\geq 0}[\widetilde{u}(\Delta w) - h_0 - (1+h_1)\Delta w] = sup_{\Delta w>0}[\widetilde{u}(\Delta w) - h_0 - (1+h_1)\Delta w]$ as $h_0 > 0$. Then we have

$$
\begin{aligned}
& \widetilde{u}(0) - \widetilde{v}(0) \leq \widetilde{u}^N(0) - \widetilde{v}^N(0) \\
\leq & sup_{w_1>0}[\widetilde{u}(w_1) - h_0 - (1+h_1)w_1] - sup_{w_2>0}[\widetilde{v}(w_2) - h_0 - (1+h_1)w_2] \\
\leq & sup_{\Delta w>0}[\widetilde{u}(\Delta w) - \widetilde{v}(\Delta w)],
\end{aligned}
\tag{I.43}
$$

where the second inequality uses $\widetilde{v}^N(0) \geq sup_{w_2>0}[\widetilde{v}(w_2) - h_0 - (1+h_1)w_2]$. It follows that there exists another point $\widetilde{w} > 0$ such that $\widetilde{u}(\widetilde{w}) - \widetilde{v}(\widetilde{w}) > \delta_\zeta \geq \sup_{w\geq 0} \widetilde{u}(w) - \widetilde{v}(w) - \zeta > 0$.

Define

$$\Phi^\varepsilon(w,z) = \widetilde{u}(w) - \widetilde{v}(z) - \beta(w-\widetilde{w})^4 - \frac{1}{2\varepsilon}(w-z)^2, \tag{I.44}$$

where $\beta > 0$ can be any large number, $\varepsilon > 0$ can be any small number. We have

$$\sup_{w>0,z>0} \Phi^\varepsilon(w,z) \geq \Phi^\varepsilon(\widetilde{w},\widetilde{w}) = \widetilde{u}(\widetilde{w}) - \widetilde{v}(\widetilde{w}) > \delta_\zeta. \tag{I.45}$$

According to the semicontinuity and growth conditions in $\widetilde{u}$ and $\widetilde{v}$, we conclude that $\Phi^\varepsilon(w,z)$ has a maximizer $(w_\varepsilon, z_\varepsilon)$. Moreover, the growth conditions in $\widetilde{u}$ and $\widetilde{v}$ give an upper bound for this maximizer. Thus, there is a subsequence of ε, such that $(w_\varepsilon, z_\varepsilon)$ converges as $\varepsilon \to 0$. Here and hereafter, we only consider ε along this subsequence. Because the lower bound in (I.45) is independent of ε, we have

$$\liminf_{\varepsilon\to 0} \Phi^\varepsilon(w_\varepsilon, z_\varepsilon) > \delta_\zeta. \tag{I.46}$$

It follows that

$$\limsup_{\varepsilon\to 0} \frac{1}{2\varepsilon}(w_\varepsilon - z_\varepsilon)^2 < \infty. \tag{I.47}$$

Consequently, $\limsup_{\varepsilon\to 0} |w_\varepsilon - z_\varepsilon| = 0$, and there is a subsequence of ε, such that w_ε and z_ε converges to the same point w_* as $\varepsilon \to 0$.

Rearranging terms in (I.44) and using (I.45), we obtain

$$\begin{aligned}
&\lim_{\varepsilon\to 0} \beta(w_\varepsilon - \widetilde{w})^4 + \lim_{\varepsilon\to 0} \frac{1}{2\varepsilon}(w_\varepsilon - z_\varepsilon)^2 \\
\leq &\limsup_{\varepsilon\to 0}\ [\widetilde{u}(w_\varepsilon) - \widetilde{v}(z_\varepsilon)] - \delta_\zeta \\
\leq &\widetilde{u}(w_*) - \widetilde{v}(w_*) - \delta_\zeta < \zeta,
\end{aligned} \tag{I.48}$$

where the second inequality uses semicontinuity in u and v, the last inequality uses $\delta_\zeta \geq \sup_{w>0} u(w) -$

$v(w) - \zeta$. Because $\beta > 0$ can be any large number, by choosing β such that $\beta \widetilde{w}^4 > \zeta$, we derive from (I.48) and $w_* > 0$. Then $w_\varepsilon > 0$ and $z_\varepsilon > 0$ for small $\varepsilon > 0$.

Because the maxima of $\Phi^\varepsilon(w, z)$ are attained in interior points $(w_\varepsilon, z_\varepsilon)$, we derive from Ishii's lemma (Crandall, Ishii, and Lions, 1992) that

$$(\rho_\varepsilon, X_\varepsilon) \in \overline{J}^{2,+}(\widetilde{u}(w_\varepsilon)), \tag{I.49}$$

$$(\rho_\varepsilon, Y_\varepsilon) \in \overline{J}^{2,-}(\widetilde{v}(z_\varepsilon)), \tag{I.50}$$

and

$$X_\varepsilon \le Y_\varepsilon + o(1), \tag{I.51}$$

where $o(1)$ converges to 0 as $\varepsilon \to 0$, and $\rho_\varepsilon = \frac{1}{\varepsilon}(w_\varepsilon - z_\varepsilon)$.

Recall the generator $\mathcal{L}_G$ defined in (I.5). For any smooth function $f(w)$ and $\widetilde{f}(w) = e^{-\eta g(w)} f(w)$, we have

$$\begin{aligned}
& - e^{-\eta g(w)} \mathcal{L}_G f(w) = (r + \lambda) \widetilde{f}(w) - e^{-\eta g(w)} \lambda E_T^*(w, m_0) \\
& - [(1-\pi)(\mu_G + rw - m_0) - \xi w][\widetilde{f}'(w) + \eta g'(w) \widetilde{f}(w)] \\
& - \frac{(1-\pi)^2 \sigma^2}{2} \Big[\widetilde{f}''(w) + 2\eta g(w) \widetilde{f}'(w) + \big(\eta^2 g'(w)^2 + \eta g''(w)\big) \widetilde{f}(w) \Big] \\
& = b_0(w) + b_1(w) \widetilde{f}(w) + b_2(w) \widetilde{f}'(w) + b_3 \widetilde{f}''(w),
\end{aligned} \tag{I.52}$$

where $b_0(w) = -e^{-\eta g(w)} \lambda E_T^*(w, m_0)$, $b_1(w) = r + \lambda - [(1-\pi)(\mu_G + rw - m_0) - \xi w]\eta g'(w) - \frac{(1-\pi)^2\sigma^2}{2}\big(\eta^2 g'(w)^2 + \eta g''(w)\big)$, $b_2(w) = -[(1-\pi)(\mu_G + rw - m_0) - \xi w] - (1-\pi)^2 \sigma^2 \eta g(w)$, $b_3 = -\frac{(1-\pi)^2\sigma^2}{2}$. Since $\lambda > 0$ and $0 < g'(w) \le M/(1+w)$, $|g''(w)| \le M$, we can choose a small η such that $b_1(w) \ge r > 0$.

Using (I.52), and the viscosity properties of $\widetilde{u}$ and $\widetilde{v}$ yields

$$\min\{b_0(w_\varepsilon) + b_1(w_\varepsilon)\widetilde{u}(w_\varepsilon) + b_2(w_\varepsilon)\rho_\varepsilon + b_3 X_\varepsilon, \eta g'(w_\varepsilon)\widetilde{u}(w_\varepsilon) + \rho_\varepsilon - e^{-\eta g(w_\varepsilon)}\} \le 0, \tag{I.53}$$

and

$$\min\{b_0(w_\varepsilon) + b_1(w_\varepsilon)\widetilde{v}(w_\varepsilon) + b_2(w_\varepsilon)\rho_\varepsilon + b_3 Y_\varepsilon, \eta g'(w_\varepsilon)\widetilde{v}(w_\varepsilon) + \rho_\varepsilon - e^{-\eta g(w_\varepsilon)}\} \geq 0, \tag{I.54}$$

Next, we prove that $\limsup_{\varepsilon\to 0} \left[\widetilde{u}(w_\varepsilon) - \widetilde{v}(z_\varepsilon)\right] \leq 0$ by considering two cases, depending on whether

$$b_0(w_\varepsilon) + b_1(w_\varepsilon)\widetilde{u}(w_\varepsilon) + b_2(w_\varepsilon)\rho_\varepsilon + b_3 X_\varepsilon \leq 0, \tag{I.55}$$

or

$$\eta g'(w_\varepsilon)\widetilde{u}(w_\varepsilon) + \rho_\varepsilon - e^{-\eta g(w_\varepsilon)} \leq 0. \tag{I.56}$$

We first consider the case in which (I.55) holds. Using (I.54), we have

$$b_0(w_\varepsilon) + b_1(w_\varepsilon)\widetilde{v}(w_\varepsilon) + b_2(w_\varepsilon)\rho_\varepsilon + b_3 Y_\varepsilon \geq 0. \tag{I.57}$$

Combining (I.57) and (I.55), we have $b_1(w_\varepsilon)\widetilde{v}(w_\varepsilon) + b_3 Y_\varepsilon \geq b_1(w_\varepsilon)\widetilde{u}(w_\varepsilon) + b_3 X_\varepsilon$, which implies that

$$b_1(w_\varepsilon)(\widetilde{v}(w_\varepsilon) - \widetilde{u}(w_\varepsilon)) \geq -b_3(Y_\varepsilon - X_\varepsilon) \geq -o(1). \tag{I.58}$$

where the second inequality uses $-b_3 = \frac{(1-\pi)^2\sigma^2}{2} > 0$ and (I.51). As $b_1(w) \geq r > 0$, we derive from (I.58) that $\widetilde{u}(w_\varepsilon) - \widetilde{v}(w_\varepsilon) \leq o(1)$. Sending ε to 0, we obtain $\limsup_{\varepsilon\to 0} \left[\widetilde{u}(w_\varepsilon) - \widetilde{v}(z_\varepsilon)\right] \leq 0$.

Next, we consider the case in which (I.56) holds. Using (I.54), we have

$$\eta g'(w_\varepsilon)\widetilde{v}(w_\varepsilon) + \rho_\varepsilon - e^{-\eta g(w_\varepsilon)} \geq 0. \tag{I.59}$$

Combining (I.56) and (I.59), we obtain

$$\eta g'(w_\varepsilon)(\widetilde{v}(w_\varepsilon) - \widetilde{u}(w_\varepsilon)) \geq 0. \tag{I.60}$$

Since $g'(w) > 0$ and $\eta > 0$, we have $\widetilde{u}(w_\varepsilon) - \widetilde{v}(w_\varepsilon) \leq 0$. Sending ε to 0, we obtain $\limsup_{\varepsilon\to 0} \left[\widetilde{u}(w_\varepsilon) - \right.$

$\widetilde{v}(z_\varepsilon)\big] \leq 0$.

Substituting $\limsup_{\varepsilon\to 0} \big[\widetilde{u}(w_\varepsilon) - \widetilde{v}(z_\varepsilon)\big] \leq 0$ into the first inequality in (I.48), we obtain

$$\begin{aligned}
&\lim_{\varepsilon\to 0} \beta(w_\varepsilon - \widetilde{w})^4 + \lim_{\varepsilon\to 0} \frac{1}{2\varepsilon}(w_\varepsilon - z_\varepsilon)^2 \\
\leq &\limsup_{\varepsilon\to 0} \ [\widetilde{u}(w_\varepsilon) - \widetilde{v}(z_\varepsilon)] - \delta_\zeta \leq -\delta_\zeta.
\end{aligned} \tag{I.61}$$

As $\delta_\zeta > 0$, we arrive at a contradiction.

Thus, $\widetilde{u}(w) \leq \widetilde{v}(w)$ for any $w \geq 0$. Then we complete the proof of Theorem 2.

To prove Theorem 3, we follow nearly identical steps, except that we must first bound the optimal investment policy. Applying the first-order condition, we can obtain the optimal investment policy as follows

$$i^*(w) = \frac{E_G(w) - E(w)}{2E'(w)(1-\pi)\phi}\mathbf{1}_{E_G(w)>E(w)}.$$

Since $E'(w) \geq 1$, we have $i^*(w) \leq \frac{|E_G(w)-E(w)|}{2(1-\pi)\phi}$. Growth conditions imply that there exists a constant $M > 0$, such that $E(w) \in [w, w+M]$, $E_G(w) \in [w, w+M]$; see the proof of Corollary 3 for details regarding these growth conditions. Hence, we have $i^*(w) \leq \frac{M}{2(1-\pi)\phi}$.

Then we can restrict ourselves to the case where there is a maximal permitted investment rate $i_{\max} < \infty$. In other words, it suffices to analyze the following equation:

$$\min\{-\sup_{i\in[0,i_{\max}]} \mathcal{L}^i E(w),\ E'(w) - 1\} = 0. \tag{I.62}$$

The remaining proof of Theorem 3 is similar to that of Theorem 2.

I.3.3 Proof of Corollary 3

According to Barles and Souganidis (1991), the comparison principle that we have established implies the stability property for viscosity solutions, which implies the convergence of monotone numerical schemes to a relevant solution.

Thus, we only need to verify that the equilibrium equity value satisfies the growth conditions in Theorems 2-3. That is, there exists a constant $M > 0$, such that $E_G(w) \in [w, w+M]$ and $E(w) \in [w, w+M]$ for any $w \geq 0$.

Because one admissible strategy for the firm is to (1) pay out cash holdings as a dividend then (2) default, we have $E_G(w) \geq w$ and $E(w) \geq w$ for any $w \geq 0$.

Denote $\tau = T_{\mathcal{D}} \wedge T$. Using (22), we can see that for $t \in [T_G, \tau)$, we have

$$\begin{aligned}
&\mathbb{E}_t\Big[\int_{t\leq s<\tau} e^{-r(s-t)}(dU_s - dN_s - dH_s)\Big]\\
=&\mathbb{E}\Big[\int_{t\leq s<\tau} e^{-r(s-t)}((r-\pi r-\xi)W_s ds - dW_s + (1-\pi)(\mu_s - m_0)ds - dH_s)\Big]\\
=&W_t - \mathbb{E}_t\Big[e^{-r(\tau-t)}W_\tau\Big] + \mathbb{E}_t\Big[\int_{t\leq s<\tau} e^{-r(s-t)}((1-\pi)(\mu_s - m_0)ds - (\pi r+\xi)W_s ds - dH_s)\Big]\\
\leq&W_t - \mathbb{E}_t\Big[\mathbf{1}_{T<T_{\mathcal{D}}}e^{-r(T-t)}W_T\Big] + (1-\pi)\mu_G/r,
\end{aligned} \tag{I.63}$$

where the inequality uses $\mu_s \leq \mu_G$, $m_0 \geq 0$, and $W_s \geq 0$ for any s.

It follows that

$$\begin{aligned}
E_G(W_t) =&\mathbb{E}_t\Big[\mathbf{1}_{T<T_{\mathcal{D}}}e^{-r(T-t)}E_T^*(W_T, m_0)\Big] + \mathbb{E}_t\left[\int_{t\leq s<\tau} e^{-r(s-t)}(dU_s - dN_s - dH_s)\right]\\
\leq&W_t + \mathbb{E}_t\Big[\mathbf{1}_{T<T_{\mathcal{D}}}e^{-r(T-t)}(E_T^*(W_T, m_0) - W_T)\Big] + (1-\pi)\mu_G/r\\
\leq&W_t + E_T^*(0, m_0)\mathbb{E}_t\Big[\mathbf{1}_{T<T_{\mathcal{D}}}e^{-r(T-t)}\Big] + (1-\pi)\mu_G/r,
\end{aligned} \tag{I.64}$$

where the first inequality follows from (I.63), and the second inequality uses the result that $E_T^*(w, m_0) - w$ is non-increasing in w (see (I.1)). Therefore, $E_G(w) \leq w + M_0$ for any $w \geq 0$, where $M_0 = \max\{E_T^*(0, m_0), 0\} + (1-\pi)\mu_G/r$.

Denote $\widetilde{\tau} = T_G \wedge T_{\mathcal{D}} \wedge T$. Using (22), we can see that for $t \in [0, \widetilde{\tau})$, we have

$$\begin{aligned}
&\mathbb{E}_t\Big[\int_{t\leq s<\widetilde{\tau}} e^{-r(s-t)}(dU_s - dN_s - dH_s)\Big]\\
=&\mathbb{E}\Big[\int_{t\leq s<\widetilde{\tau}} e^{-r(s-t)}((r-\pi r-\xi)W_s ds - dW_s + (1-\pi)\big(\mu_0 - m_0 - \phi i_s^2\big)ds - dH_s)\Big]\\
=&W_t - \mathbb{E}_t\Big[e^{-r(\widetilde{\tau}-t)}W_{\widetilde{\tau}}\Big] + \mathbb{E}_t\Big[\int_{t\leq s<\widetilde{\tau}} e^{-r(s-t)}((1-\pi)\big(\mu_0 - m_0 - \phi i_s^2\big)ds - (\pi r+\xi)W_s ds - dH_s)\Big]\\
\leq&W_t - \mathbb{E}_t\Big[(\mathbf{1}_{T<T_{\mathcal{D}}\wedge T_G} + \mathbf{1}_{T_G<T_{\mathcal{D}}\wedge T})e^{-r(\widetilde{\tau}-t)}W_{\widetilde{\tau}}\Big] + (1-\pi)\mu_0/r.
\end{aligned} \tag{I.65}$$

It follows that

$$
\begin{aligned}
E(W_t) =& \mathbb{E}_t\left[e^{-r(\widetilde{\tau}-t)}\Big(\mathbf{1}_{T<T_{\mathcal{D}}\wedge T_G}E_T^*(W_T,m_0)+\mathbf{1}_{T_G<T_{\mathcal{D}}\wedge T}E_G(W_{T_G})\Big)\right]\\
&+\mathbb{E}_t\Big[\int_{t\le s<\widetilde{\tau}}e^{-r(s-t)}(dU_s-dN_s-dH_s)\Big]\\
\le& W_t+\mathbb{E}_t\Big[\mathbf{1}_{T<T_{\mathcal{D}}\wedge T_G}e^{-r(T-t)}(E_T^*(W_T,m_0)-W_T)\\
&+\mathbf{1}_{T_G<T_{\mathcal{D}}\wedge T}e^{-r(T_G-t)}(E_G(W_{T_G})-W_{T_G})\Big]+(1-\pi)\mu_0/r\\
\le& W_t+\max\{E_T^*(0,m_0),0\}+M_0+(1-\pi)\mu_0/r,
\end{aligned}
\tag{I.66}
$$

where the first inequality follows from (I.65), and the second inequality uses $E_T^*(W_T,m_0)-W_T\le E_T^*(0,m_0)\le\max\{E_T^*(0,m_0),0\}$ and $E_G(w)\le w+M_0$ for any $w\ge 0$. Thus, there exists a constant $M>M_0$, such that $E(w)\le w+M$ for any $w\ge 0$. Therefore, our earlier results imply the comparison principles that imply the desired convergence results.

J Further Institutional Details

This Appendix provides further institutional details.

J.1 Details on J. Crew

While there were some earlier attempts, J. Crew is commonly credited as achieving the first drop-down transaction in 2017 (Ayotte and Scully, 2021; Ivashina and Vallee, 2025). As Ayotte and Scully (2021) describe, J. Crew's existing secured term-loan agreement included a "trapdoor" that allowed it to transfer up to $250 million in collateral between subsidiaries. First, J. Crew transferred $250 million in trademark collateral to a foreign "restricted non-loan-party" Cayman Islands subsidiary. As Ayotte and Scully (2021) explain, J. Crew's agreement likely allowed for this arrangement because the "restricted-subsidiary status protects lenders by allowing those subsidiaries to remain subject to the covenants in the loan documents, while the non-loan-party status prevents triggering the adverse tax consequences." However, once this first transfer was achieved, the trapdoor allowed for a second transfer of "the trademarks into a newly formed unrestricted subsidiary, freeing them from both the covenants and the debt obligations." (Ayotte and Scully, 2021). As Buccola and Nini (2024) explain, "collateral moved to an unrestricted subsidiary ceases to be collateral for the original loan." Finally, J. Crew sought consent from the term-loan holders to transfer additional collateral beyond what the contract allowed. As Ayotte and Scully (2021) describe,

> J. Crew then set about creating an offer to the term lenders. If it could get a majority of the lenders to agree, it could have the loan amended... and move the remaining 28% of the trademark value. To do so, the company created an exchange offer on a short timeline that subjected the individual term lenders to a prisoner's dilemma. The terms included partial repayment of the loan at par... if a majority consented, then those who refused to participate would be stuck with no repayment and no litigation right... over 88% of the lenders supported the amendments.

Once all of the trademark collateral was transferred, J. Crew issued new secured debt with a first lien on the trademarks.[64]

Thus, at a high level, J. Crew (1) offered a bribe (partial repayment) to coerce a majority of term-loan holders to yield collateral rights, then (2) bolstered its liquidity by creating new secured debt with the existing collateral. While each new dropdown differs slightly from the previous one, most dropdowns share these two features with the J. Crew LME. As Buccola and Nini (2024) summarize, "collateral for an existing loan is transferred downstream, free and clear of liens; the downstream entity borrows against the assets and uses the proceeds to relieve an upstream entity's capital needs; upstream creditors are subordinated."

J.2 Details on exclusive versus nonexclusive deals

In Revlon's dropdown, all term-loan holders could in principle have participated, but some chose not to. As Mugford and Gulick (2022) write, the nonparticipating investors "organized and signed a joint co-operation agreement that would bind them to vote against the deal," which was only meaningful because they had the chance to vote in favor. Likewise, in J. Crew's dropdown, 88% of term loan holders participated (Ayotte and Scully, 2021). The remaining 12% were not excluded, but rather "refused to consent" (LawJustia, 2018).

In contrast, other LMEs are exclusive. As Buccola (2023a) summarizes,

> In a conventional workout, the company offers ratable consideration to all consenting creditors in a facility whose contractual rights are to be waived or altered. In a non-pro rata transaction, by contrast, the inducement is offered only to a subset of creditors whose consent is sufficient to bind the facility.

Buccola (2023a) classifies many uptiers in this way, including Serta Simmons, TriMark, Boardriders, and TPC Group. The author also points out that dropdowns can in principle be exclusive (e.g., Envision Healthcare).

[64] Specifically, J. Crew created "superpriority notes backed by its iconic brands, which had theretofore been part of its lenders' collateral, and then exchanged the new notes for junior, but maturing, unsecured notes" (Buccola and Nini, 2024).

J.3 Evidence of concentrated term-loan ownership

LMEs usually target a type of syndicated term loan called an "institutional term loan." As Badawi, Buccola, and Nini (2024) write,

> Among the most remarkable developments in leveraged finance over the last two decades has been the emergence and proliferation of institutional term loans... These are loans made to speculative-grade and unrated borrowers, arranged by a bank with an eye toward syndicating the credit immediately upon closing to non-bank institutions such as collateralized loan obligations (CLOs), loan mutual funds, hedge funds, and other asset managers... An active secondary market is an important ingredient in the institutional term loan's viability.

Roughly 90% of these are secured first-lien loans (Nini and Smith, 2024).

As we discuss in the main text, by the time a firm enters its distress, it is common for the majority of the term loan to be owned by a small number of hedge funds.[65] This highly concentrated ownership is partly due to the nature of the secondary market for term loans: "trading is opaque and illiquid. The market is mediated by a small number of dealers" (Nini and Smith, 2024). It is also partly due to the fact that the Trust Indenture Act applies only to public bonds, not institutional term loans (Buccola, 2023a).

As an example of concentrated ownership, consider J. Crew's term loan that was impacted by its LME. At the time of the LME, one hedge fund called Anchorage Capital owned 28% of the term loan (Jessica DiNapoli, 2017) and another hedge fund called Eaton Vance owned 10% (LawJustia, 2018). There are many other examples, such as the following. As we discuss in the main text, three asset managers owned 74% of Neiman Marcus's term loan when it filed for bankruptcy.[66] When Robertshaw negotiated an LME with four asset managers, the coalition held 76% of its term loan.[67]

[65] Blickle et al. (2020) find that the lead arranger of an institutional syndicated loan sells its entire position in 70% of deals. Demiroglu and James (2015) show that firms frequently fail to restructure term loans out of court when they are originally funded by nonbank lenders (e.g., hedge funds). Hotchkiss and Mooradian (1997) also show vulture funds buy distressed debt.

[66] See docket entry 146 at `https://cases.stretto.com/public/X064/10214/PLEADINGS/1021405082080000000011.pdf`.

[67] See `https://globalrestructuringreview.com/review/restructuring-review-of-the-`

Consider the following additional examples. Three hedge funds jointly owned roughly 23% of Revlon's term loan at the time of its LME; another fund owned 10%.[68] Similarly, for the relevant "PTL" term loan in Serta's LME, a group of three hedge funds (Eaton Vance, Barings, and Farallon capital) owned roughly 44%.[69] When Wellpath filed for Chapter 11, five hedge funds owned 67% of its first-lien term loan.[70]

Perhaps the best evidence of concentrated term-loan ownership is the fact that firms are able to negotiate LMEs with term-loan holders in the first place. For example, representatives of the hedge funds Angelo Gordon and Centerbridge Partners met directly with Envision's legal team (Kirkland and Ellis) to negotiate the Envision dropdown (Eliza Ronalds-Hannon and Davide Scigliuzzo, 2022).

J.4 Court responses to LMEs

LMEs are often followed by litigation disputing the validity of the transaction. Overall, courts have typically validated LMEs, though there is a great deal of nuance. Dropdowns have been broadly validated: As Buccola and Nini (2024) write,

> The legality of a generic dropdown has not been seriously questioned. Although lenders have expressed a combination of surprise and outrage at borrowers' use of unrestricted subsidiaries to subordinate unwilling lenders, the contractual permissions underpinning such transactions are relatively clear.

In contrast, uptiers have been more contentious in court. Most notably, the U.S. Court of Appeals ruled that Serta Simmon's non-pro-rata uptier transaction, arguably the original uptier,

americas/2024/article/investment-fund-activity-in-us-debt-restructurings and https://caselaw.findlaw.com/court/us-ban-crt-s-d-tex-hou-div/116510401.html.

[68]See https://news.bloomberglaw.com/banking-law/citi-sues-hps-symphony-over-mistaken-payment-on-revlon-loan-1, which mentions three hedge funds held 46% of the 50% of the term loan that was not exchanged. See also docket entry 43 of Revlon's bankruptcy (stating Angelo Gordon held over $300 million in B-1 and B-2 term loans) and docket entry 30 (listing roughly $2.7 billion in term loans outstanding).

[69]See docket entry 1116 in Serta's bankruptcy showing these hedge funds owned roughly $450 million and docket entry 26 for the total amount of $1.027 billion.

[70]The investors collectively owned $321 million, see https://document.epiq11.com/document/getdocumentbycode?docId=4404529&projectCode=WPT&source=DM. The total amount outstanding was $475 million, see https://disclosure.spglobal.com/ratings/en/regulatory/article/-/view/sourceId/13054062.

was invalid. Serta had claimed that it was able to hand pick participating debt holders and exchange only their debt for new supersecured debt as an "open market transaction." As Holland & Knight (2025) summarize,

> The U.S. Court of Appeals for the Fifth Circuit has unanimously held that the debt exchange undertaken by Serta Simmons Bedding did not qualify as an "open market purchase" under the terms of Serta's 2016 credit agreement. Therefore, the "uptier transaction" was not permitted.

Importantly, this ruling is specific to the terms of Serta's credit agreement and did not invalidate all uptiers. Instead, it ruled out one approach for implementing an uptier. Indeed, "on the same day as the Fifth Circuit's decision, the New York Supreme Court's First Appellate Division issued a separate ruling ratifying an uptier transaction executed by Mitel Networks" (Holland & Knight, 2025).

J.5 Alternative interpretations of the contractual response to LMEs

Buccola and Nini (2024) show that new loan contracts continue to enable dropdowns. They conclude that dropdowns create value ex ante. However, it is important to note that there are alternative interpretations of these facts. Firms and lenders might not bother to block dropdowns because it is ultimately impossible to prevent LMEs: Ayotte and Scully (2021) write that

> A variety of "J. Crew blocker" terms emerged in response, but covenant analysts argued that most of them are only partially effective at preventing unrestricted subsidiary transfers. Many other contracts continued to leave them out entirely.

Likewise, Ayotte and Scully (2021) write "Contracting parties, no matter how sophisticated, cannot possibly imagine and contract to prevent all possible loopholes that other sophisticated parties might exploit."

In our view, it is instructive that *some* contracts block dropdowns and most loan contracts now intentionally ban exclusive LMEs. This pattern suggests that participants view these contractual defenses as meaningful, or they would not bother including these terms. Likewise, given that participants now know about "J. Crew blockers," anyone who believes LMEs are purely harmful would see some benefit and no cost in including such a clause. By this argument, the continued absence of J. Crew blockers in a large fraction of contracts suggests that dropdowns create value ex ante. Nonetheless, we acknowledge that this is not the only interpretation of Buccola and Nini (2024). At the very least, we show that these facts *can* be explained by a formal model in which optimal contracts enable LMEs. Likewise, we show that some firms can benefit ex ante from allowing for dropdowns in contracts, regardless of whether the observed choices to enable dropdowns are intentional.

J.6 LMEs are a continuing trend

While the January 2025 court ruling on Serta's uptier invalidated a specific type of exclusive LME, this ruling has had little impact on the overall trend of frequent LMEs. For example, on November 25, 2025, a leading distressed debt news outlet called Petition dedicated a special issue to LMEs.[71] In this November 2025 issue, they begin: "Liability management has become the dominant theme in restructuring over the past several years." They interview "Scott Greenberg, Partner and Global Chair of Gibson Dunn's Business Restructuring and Reorganization Practice group," who asserts:

> Lawyers, bankers and investors are smart — folks just figure out how to build a better mouse trap. So when Serta happened the world of LME didn't end as we know it — far cry — as it's been a busy '25 on the LME front.

[71] See `https://www.petition11.com/p/notice-of-appearance-scott-greenberg-gibson-dunn`.